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Robert Kaczmarek, Jean-Claude Vannier, Wan-Ying Huang

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Robert KACZMAREK, Wan-Ying HUANG and Jean-Claude VANNIER
Dept. Energie
Supélec
91192 Gif sur Yvette, France
robert.kaczmarek@supelec.fr

***Abstract*—The PWM-supplied permanent magnet synchronous machines are exposed to magnets' heating due to harmonic fields which rotate in relation to rotor. The corresponding losses can be calculated by FE simulation for each frequency, with superposition of all components of the losses. In certain conditions three dimensional modelling is required to get enough precision. However, time-stepping 3D finite element methods coupled with motion equations can lead to excessive computational time. Then a locked rotor approach with current sheet rotating may be interesting. In this case however, the superposition doesn't really work unless special procedure is applied. Magnetic loss dissipation cannot be correctly calculated with simple superposition of individual fields, e.g. the 5th and 7th harmonic rotating in opposite direction. This is because the resulting field has an alternative component with dissipative effects depending on its position. They will be more important if the component oscillates in the middle of magnets and less important in the middle of gap between them.**

Keywords-permanent magnet synchronous machine; hamonics; superposition ; finite elements ; 3D modeling

I. Introduction

Permanent magnet synchronous machine (PMSM), with its simplicity, high efficiency even at low speeds, strong power density and relatively low cost of maintenance can be suitable choice for an AC inverter drive. Usual way of treating non sinusoidal supply conditions is superposition of harmonic components, particularly for torque and magnetic losses calculation. Analytical methods are often verified by finite element (FE) modeling and simulation, as high precision is required with rigorously smooth torque applications or with minimal size machine design. In the latter case losses in magnets are of concern, because under PWM voltage supply the eddy current losses may significantly rise magnets temperature. This can be prevented by magnets' fragmentation, an operation mechanically delicate.

With FE application the problem of choice between 2D and 3D packages is crucial. Traditionally, when machines can be considered as essentially radial flux devices with stray flux in the end windings being negligibly small [1] then the 2D time-stepped simulation [2-4] is precise enough for parameter evaluation and torque calculation.

This axiom can not always be applied to PM SM where accuracy of calculation of magnets losses depends to a large extent on magnets dimensions. The 2D simulation assures good precision in case of magnets with ratio of axial and tangential lengths exceeding ten [5]. This condition however may be irrelevant in case of actual machines where preparation and mounting of long and thin magnet pieces on rotor's surface is delicate operation. Then 3D simulation can be required.

Here the time-stepped finite elements simulation becomes excessively time consuming. A solution to this problem comes with locked rotor and harmonic MMF rotating around it. Actual machine being designed with quasi sinusoidal winding distribution, the harmonic analysis is concerned with time harmonics of order 5 and 7, 11 and 13 etc. Calculation of magnets losses on rotor side implies then harmonics -/+6th, -/+12th etc. However, superposition of individual losses may differ substantially from losses dissipated by composed fields.

In order to fix conditions for superposition to work, we start with harmonic fields' analysis in stator and in rotor reference frame.

II. Sliding and Composed MMF

A. Stator reference

In inverter-fed PM SM the armature currents are usually non sinusoidal, whereas the spatial distribution of magneto-motive force can be assumed as sinusoidal in stator reference frame fixed ($\theta=0$) in geometrical axis of winding distribution of phase "a". This axis points to maximum of the cosine expression for MMF of phase "a" (1):

$$\begin{aligned} F_a(\theta) &= \frac{2}{\pi} n i_a \cos(\theta) \\ F_b(\theta) &= \frac{2}{\pi} n i_b \cos(\theta - 120°) \\ F_c(\theta) &= \frac{2}{\pi} n i_c \cos(\theta - 240°) \end{aligned} \quad (1)$$

If we start counting time from the moment the fundamental current in winding "a" is at its peak value

$$i_{a1}(t) = I_1 \cos(\omega t)$$
$$v_{a1}(t) = V_1 \cos(\omega t + \varphi_1) \quad (2)$$

then we find peak value of the fundamental composed magneto-motive force at $\theta=0$

$$F_1(\theta,t)=F_{a1}(\theta,t)+F_{b1}(\theta,t)+F_{c1}(\theta,t) \quad (3)$$

In this moment the rotor axis is lagging the stator axis, and obviously also the fundamental MMF, by the machine internal angle δ (Fig. 1).

For further analysis we take the 5th and the 7th armature current harmonics (4):

$$i_{a5}(t) = I_5 \cos(5\omega t + \phi_5)$$
$$i_{b5}(t) = I_5 \cos(5\omega t + \phi_5 + 120°)$$
$$i_{c5}(t) = I_5 \cos(5\omega t + \phi_5 + 240°)$$
$$i_{a7}(t) = I_7 \cos(7\omega t + \phi_7)$$
$$i_{b7}(t) = I_7 \cos(7\omega t + \phi_7 - 120°)$$
$$i_{c7}(t) = I_7 \cos(7\omega t + \phi_7 - 240°) \quad (4)$$

with initial phases ϕ_5 and ϕ_7 measured in relation to their corresponding harmonic voltages, which are all in-phase with supply PWM voltage and, obviously, with its fundamental component.

These currents form a pair of fields F_5, F_7 of classical form of sinusoidal distribution along the air gap, rotating in relation to stator at, respectively, -5ω:

$$F_5(\theta) = \frac{3nI_5}{\pi} \cos(5\omega t + \phi_5 + \theta)$$

and $+7\omega$:

$$F_7(\theta) = \frac{3nI_7}{\pi} \cos(7\omega t + \phi_7 - \theta) \quad (6)$$

At t=0 the $F_5(\theta)$ has its maximum in $\theta=-\phi_5$ and $F_7(\theta)$ at $\theta=\phi_7$ (Fig.1).

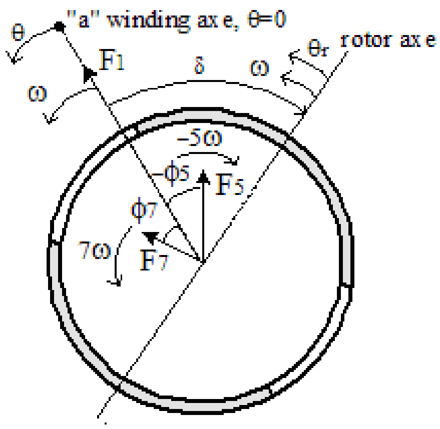

Fig. 1. The fundamental, the 5th and the 7th harmonics of MMF in relation to stator, represented here by its "a" winding at t=0. Rotor is considered with one pair of poles and magnets indicated in black.

We should now relate all these quantities to rotor.

B. Rotor reference

In rotor reference frame we will see these fields as created by phase currents of, respectively, negative and positive sequence (7):

$$i_{a-6}(t) = I_- \cos(6\omega t + \phi_{-6})$$
$$i_{b-6}(t) = I_- \cos(6\omega t + \phi_{-6} + 120°)$$
$$i_{c-6}(t) = I_- \cos(6\omega t + \phi_{-6} + 240°)$$
$$i_{a6}(t) = I_+ \cos(6\omega t + \phi_{+6})$$
$$i_{b6}(t) = I_+ \cos(6\omega t + \phi_{+6} - 120°)$$
$$i_{c6}(t) = I_+ \cos(6\omega t + \phi_{+6} - 240°) \quad (7)$$

these currents circulating in virtual 3-phases windings fixed on the rotor in position of δ ahead of the rotors axis; at t=0 this position is that of the stator "a" winding. Then the rotor related MMFs have the form (8):

$$F_{a6} = \frac{2}{\pi} n i_{a6} \cos(\theta_r - \delta)$$
$$F_{b6} = \frac{2}{\pi} n i_{b6} \cos(\theta_r - \delta - 120°)$$
$$F_{c6} = \frac{2}{\pi} n i_{c6} \cos(\theta_r - \delta - 240°) \quad (8)$$

The phase ϕ in current expression being nothing more than initial angles, we have $\phi_{-6} = \phi_5$ and $\phi_{+6} = \phi_7$. Amplitudes don't change, neither, when passing from stator to rotor reference frame and so we have $I_+ = I_7$ and $I_- = I_5$.

The magneto-motive force F_{+6} resulting from the positive sequence currents can be represented by vector rotating over rotor surface at $+6\omega$:

$$F_{+6}(\theta_r,t)=\frac{3nI_+}{\pi}\cos\left(6\omega t+\phi_{+6}-(\theta_r-\delta)\right) \qquad (9)$$

and similarly for negative sequence currents we have MMF rotating at -6ω:

$$F_{-6}(\theta_r,t)=\frac{3nI_-}{\pi}\cos\left(6\omega t+\phi_{-6}+(\theta_r-\delta)\right) \qquad (10)$$

At $t=0$ the F_{-6} has its maximum in $\theta_r=\delta-\varphi_{-6}$ and F_{+6} at $\theta_r=\delta+\varphi_{+6}$. They coincide with disposition of, respectively, the 5th and the 7th stator related components (Fig.1).

With 5th and 7th current harmonics having different amplitudes, for example $I_- > I_+$, the resulting MMF is composed of one oscillating and one sliding component

$$F_{-\cup+}(\theta_r,t)=$$
$$\frac{6nI_+}{\pi}\left[\cos\left(6\omega t+\frac{\phi_{+6}+\phi_{-6}}{2}\right)\cdot\cos\left(\frac{\phi_{+6}-\phi_{-6}}{2}-(\theta_r-\delta)\right)\right] \qquad (11)$$
$$+\frac{3n(I_--I_+)}{\pi}\cos\left(6\omega t+\phi_{-6}+(\theta_r-\delta)\right)$$

With the oscillating part growing, the dependence of losses on its angular position will grow. We will now establish a law on this dependence as well as conditions on the PM loss superposition.

III. SURPLUS COEFFICIENT FOR LOSSES AND ENERGIES

We assume eddy current dissipation as the only PM losses. Consequently, we take them as proportional to square of MMF, this proportionality being characterized by new loss coefficient C (12). For one sliding MMF it will be

$$C=\frac{1}{T}\int_0^T\int_\theta F^2 d\theta dt \qquad (12)$$

with θ - integration area of magnet's extent on rotor circumference.

In case where magnets cover two thirds of each pole we get loss coefficients corresponding to F_{-6} and F_{+6}

$$C_-=\frac{6n^2I_-^2}{\pi} \quad \text{and} \quad C_+=\frac{6n^2I_+^2}{\pi} \qquad (13)$$

Both are independent of the angle δ. This will not be the case of loss coefficient $C_{-\cup+}$ characterizing the oscillating MMF composed of F_{-6} and F_{+6}.

In case of different amplitudes of currents $I_-=I_++\delta I$ the composed MMF is a sum of oscillating and sliding components. The corresponding loss coefficient can be represented as

$$C_{-\cup+}=\frac{12n^2I_+{}^2}{\pi}+\frac{6n^2\delta I^2}{\pi}+\frac{12n^2I_+\delta I}{\pi}+\frac{9\sqrt{3}n^2I_+(I_++\delta I)}{\pi^2}\cos(2\delta) \qquad (14)$$

whereas the coefficients of individual losses are

$$C_++C_-=\frac{12n^2I_+{}^2}{\pi}+\frac{6n^2\delta I^2}{\pi}+\frac{12n^2I_+\delta I}{\pi}$$

With these coefficients we can define an analytic surplus parameter Δ_a which gives a measure of excess of losses calculated for oscillating MMF in comparison to sum of losses of its two sliding components:

$$\Delta_a=C_{-\cup+}-[C_++C_-]=\frac{9\sqrt{3}n^2}{\pi^2}\left(I_+^2+I_+\delta I\right)\cos(2\delta) \qquad (15)$$

Except for two positions of the oscillating field, the losses of the latter don't equal the sum of losses and superposition method can give erroneous results. In order to evaluate this error we introduce a relative surplus coefficient SC_a, with index a for "analytic":

$$SC_{a\_I_+\neq I}=\left[\frac{(\Delta_a)}{(C_++C_-)}\right]_{I_+\neq I_-}=\frac{3\sqrt{3}}{4\pi}\cos(2\delta)\cdot k \qquad (16)$$

with

$$k=\frac{I_+^2+I_+\Delta I}{I_+^2+I_+\Delta I+0.5\Delta I^2}=\frac{1}{1+0.5\frac{(I_--I_+)^2}{I_+I_-}}\leq 1 \qquad (17)$$

It confirms theoretical assumption that the sliding component, present in the composite MFF, lowers the surplus coefficient. As a matter of fact, the peak value of k corresponds to purely oscillating MMF. The equation (17) suggests also that any ratio I_-/I_+ gives the same coefficient that its inverse (Fig. 2).

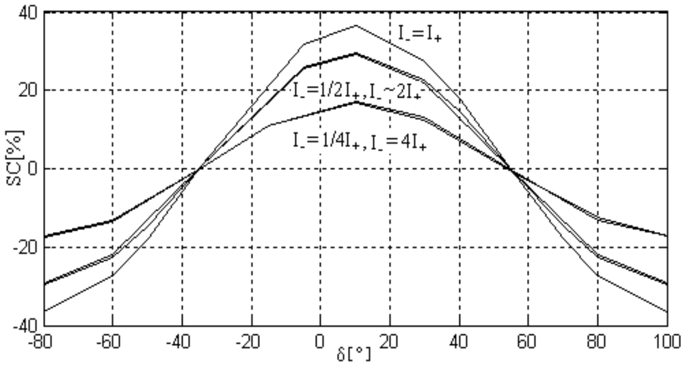

Fig. 2. The loss surplus coefficient is the same for a given currents ratio and for its inverse. Simulation by FE.

Amplitude of the loss surplus SC varies between 12% and 25% for current ratio between 4 and 1; the error of superposition diminishes when the oscillating part of MMF becomes less.

We can expect that if the magnet covers totally the rotor surface, then the superposition of losses generated by sliding fields gives correct results. The losses don't depend then on position of oscillating components. The FE simulation confirms this (Fig. 3).

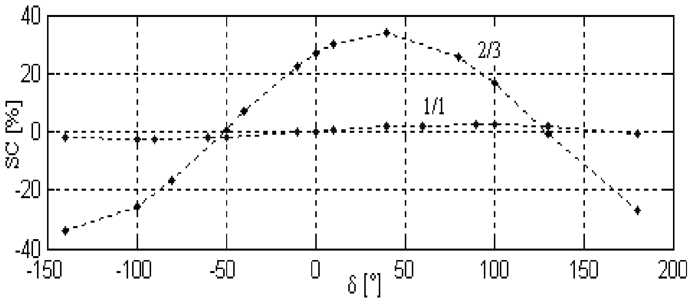

Fig. 3 With magnets covering 2/3 of the rotor circumference losses depend on position of oscillating field, whereas with rotor entirely covered by magnets (1/1) they don't. Simulation by FE.

IV. INFLUENCES OF LOAD

Load depending currents moderate amplitudes of analytic surplus parameter Δ_a (15). Actually, currents rise with load, and so does the angle δ. The most dissipating position of oscillating flux corresponds also to low, no-load value of currents, whereas with δ near 90°, i.e. in low dissipating position, currents are high. Losses are obviously higher in low dissipation position.

This moderation doesn't change the relative surplus coefficients SC. Being developed for the 5^{th} and the 7^{th} harmonics, it can be easily calculated for higher frequencies, like 11^{th} and 13^{th}, 19^{th} etc.

V. CONCLUSION

When locked rotor method is applied to PM harmonic losses then the losses superposition can deceive if adequate decomposition of field into sliding and oscillating fields is not operated. Error of losses estimation rises with square of MMF, and so it is more important for large machines.

The problem will disappear with future user-friendly FE packages permitting easy and efficient electro-dynamic modeling and simulation of rotor in movement and stator supplied with non sinusoidal voltage. This, however, doesn't seem to be near future.

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