

Influences of oversampling and analog imperfections on Hybrid Filter Bank A/D converters

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Abstract—Hybrid Filter Banks (HFB) are considered as good candidate for implementing wide-band, high-frequency A/D parallel converters. The performance of HFB (original and two-stage) structures is analyzed for different values of oversampling ratios leading to the choice of the optimal value. The electronic elements of the analysis filter bank of HFB structure are considered with real constraints (realization errors and drifts). Real analysis filters of HFB are considered with the analog elements for which the real values are randomly Gaussian-distributed. Sensitivity to the deviations from real values is simulated and the performance of two HFB (original and two-stage) structures are compared. The possible methods for decreasing the sensitivity to realization errors such as Total Least Squares (TLS) optimization are discussed. Original structure is found to be a better candidate rather than the two-stage one in practical cases. However, it is shown that a blind estimation method would be necessary to compensate the realization errors for practically implementing the HFB-based A/D converters.

I. INTRODUCTION

The idea of parallel A/D and D/A converters covering a wide frequency band has been considered for two decades [1]. Hybrid Filter banks (HFB) have been proposed to practically implement parallel A/D converters [2]. The digital output of HFB-based A/D converters is subject to the frequency distortions and aliasing [2]. The aliasing terms are related to the spectral overlapping at each branch of HFB structure originating from the undersampling process. The aliasing terms are the main restricting source of resolution in the realization of HFB-based converters [3]. Using a small ratio of oversampling, HFB-based A/D converters have shown a large reduction in the aliasing levels [4]. Two-stage HFB structure as a rearrangement of Perfect Reconstruction (PR) equations was proposed to achieve much lower aliasing levels considering simply implementable analog filters (RC and RLC circuits) along with FIR synthesis filters [5]. However, these performances were obtained in the absence of realization errors. The analysis filter bank of HFB includes analog circuits whose characteristics always deviate from the typical (or design) values. The deviations from typical values (or analog imperfections) are related either to the fabrication parameters or to the drifts (e.g. temperature). In this paper, the effects of analog imperfections and oversampling values on the performance of the HFB-based A/D converters are studied. For simulation purpose, an 8-channel HFB-based A/D converter has been used. Its analysis filter bank includes a first-order

RC circuit with seven second-order RLC circuits. Synthesis filters are FIR.

In this paper, the original and two-stage structures of HFB-based A/D converters are first reviewed and the PR equations are briefly discussed in section (II). Then, the influence of oversampling ratios is analyzed and the optimum value of oversampling ratio is extracted in subsection (III-A). The performance of the HFB structures is then studied in terms of sensitivity to realization errors in part (III-B). The results of the simulations are briefly reviewed and Total Least Squares (TLS) optimization method is proposed and tried for reducing the sensitivity of the structure to realization errors. The feasibility of HFB-based A/D converters in the presence of real analog imperfections is discussed in subsection (III-C). The results are briefly mentioned in the conclusion (section IV).

II. HFB-BASED A/D CONVERSION

Figure 1 represents the original HFB architecture for A/D conversion [2].

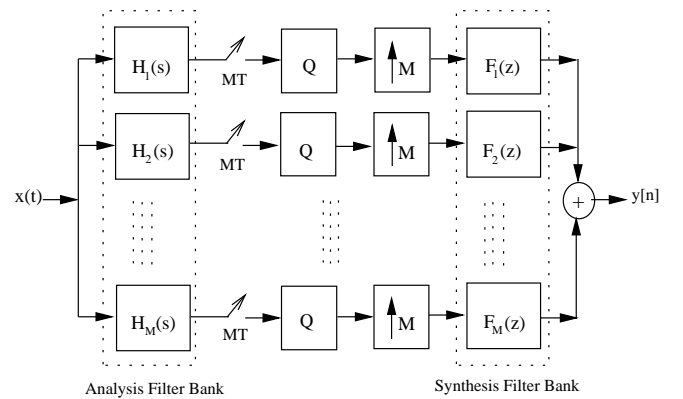


Fig. 1. The original HFB-based A/D. Q represents the quantization process in each branch of the A/D converter.

HFB structure uses M A/D converters with sampling rate $\frac{1}{MT}$ which is M times lower than the Nyquist rate $\frac{1}{T}$ of the input. To better show the effects of aliasing terms, quantization is neglected throughout the following analysis and $\tilde{H}_i(j\Omega)$ represents the periodic extension of the analysis filter $H_i(j\Omega)$ considering only the frequency interval $[-\frac{\pi}{T}, \frac{\pi}{T}]$. It is supposed that the input signal spectrum is null for the frequencies out of

this band. $\tilde{X}(j\Omega)$ similarly represents the periodic extension of the input signal with period $\frac{2\pi}{T}$. The output $y[n]$ can be represented in the frequency domain as follows [5]:

$$Y(e^{j\omega}) = \sum_{m=0}^{M-1} \tilde{X}(j\frac{\omega}{T} - j\frac{2\pi}{MT}m) \cdot T_m(e^{j\omega}) \quad (1)$$

$$T_m(e^{j\omega}) = \frac{1}{MT} \sum_{k=1}^M \tilde{H}_k(j\frac{\omega}{T} - j\frac{2\pi}{MT}m) F_k(e^{j\omega})$$

where $T_0(e^{j\omega})$ stands for the distortion function and $\{T_m(e^{j\omega}), 1 \leq m \leq M-1\}$ are the $(M-1)$ terms of aliasing functions.

If all aliasing terms are null and the distortion function is a pure delay with a possible scale factor, Perfect Reconstruction (PR) conditions are held. Thus, PR equations are:

$$\begin{cases} T_0(e^{j\omega}) = Me^{-j\omega n_d} \\ T_m(e^{j\omega}) = 0 \end{cases} \quad m = 1, 2, \dots, M-1 \quad (2)$$

where n_d is an arbitrary delay. Scale factor has been assumed to be unity. Selecting a group of points in the bandwidth and substituting in the above equations (2), the synthesis filter bank of the HFB structure are obtained from solving the equations [5]. Two-stage HFB structure is shown in figure 2. In two-stage HFB structure, anti-aliasing filter bank eliminates

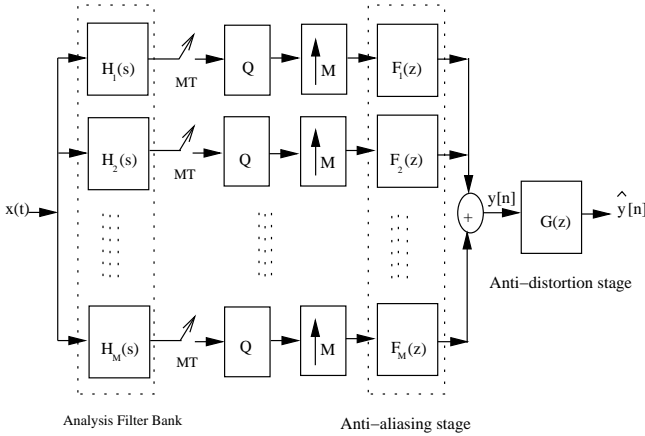


Fig. 2. Two-stage HFB structure of an A/D converter. Synthesis is accomplished through two stages called 'anti-aliasing' and 'anti-distortion'.

the aliasing terms in $y[n]$ while anti-distortion FIR filter compensates the total distortion to reconstruct the original signal. Solving separately the distortion- and aliasing-related equations of PR conditions, those two groups of synthesis filters may be obtained [5].

III. PERFORMANCE IN TERMS OF OVERSAMPLING RATIO AND REALIZATION ERRORS

A. Oversampling process

The analysis filter bank is generally assumed to be chosen firstly because of implementation reasons [3]. FIR digital

filters are employed to realize the synthesis filter bank. HFB-based A/D converters often show a large amount of aliasing which limits the resolution of the converter. If the total sampling rate $\frac{1}{T}$ is selected a bit higher than the necessary Nyquist rate $\frac{1}{T_N}$ (oversampling process), the aliasing terms are greatly reduced [4]. Oversampling ratio is $(1 - \frac{T}{T_N})$. Figure 3

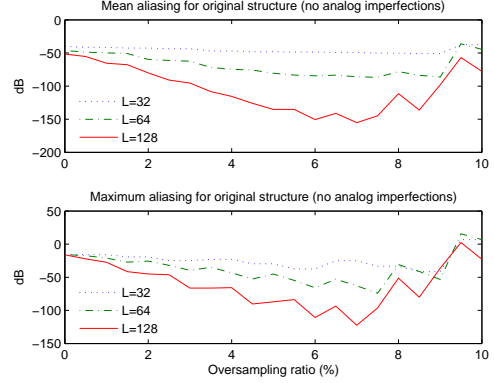


Fig. 3. Mean (above) and maximum (below) aliasing values versus oversampling ratio in percent for the original HFB structure. L represents the length of FIR synthesis filters.

illustrates the mean and maximum aliasing values for an 8-branch original HFB structure versus different oversampling ratios. It may be seen that the aliasing terms are minimum for an oversampling ratio equal to 7% of the bandwidth. Figure 4 shows the same for an 8-branch two-stage HFB structure. The two-stage HFB structure is not useful unless an oversampling process is used because the aliasing level is higher than 0dB for no oversampling case (fig.4). The performance of HFB structures deteriorates when the oversampling ratio is close to $\{k\frac{1}{M}, 0 \leq k \leq M-1\}$ in which M is the number of branches.

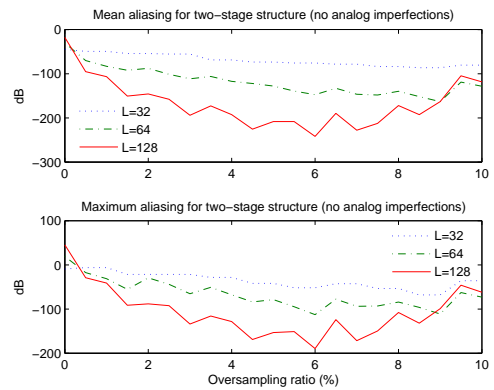


Fig. 4. Mean (above) and maximum (below) aliasing values versus oversampling ratio in percent for two-stage HFB structure. L represents the length of FIR synthesis filters.

B. Realization errors

Up to this point, it has been supposed that the analog part of system was perfect. In this subsection, we will consider the

influence of analog realization imperfections on the previously achieved performance. In the real world, the practical aliasing level is different from above mentioned simulation values because the analysis filter bank (analog circuits) includes the imperfections associated with fabrication phase or drifts (e.g. temperature drift). In practice, only the design (or nominal) values of the analysis filters are known. The synthesis filter bank is designed according to those values. So, the designed digital filter bank is not optimum for the actual analysis filter bank. To measure the sensitivity of HFB structure to realization errors, the relative deviation S_f of synthesis filter bank is defined as follows [6]:

$$S_f = \frac{\|\mathbf{f} - \mathbf{f}_o\|}{\|\mathbf{f}_o\|} \quad (3)$$

where \mathbf{f}_o and \mathbf{f} are the impulse responses of synthesis filter bank considering no realization error and practical (with realization errors) cases respectively. This measure would be almost independent from the length of FIR synthesis filters [6]. The electronic elements (R, C and L) of analysis filter bank are assumed to include Gaussian random deviations from their nominal values. In this paper, the standard deviation of the error distribution is used as the parameter of deviation from typical (or design) values. Using an 8-branch HFB structure, the simulations have been performed for 1000 trials of the Gaussian realization errors. The performance is studied versus the deviation from typical values.

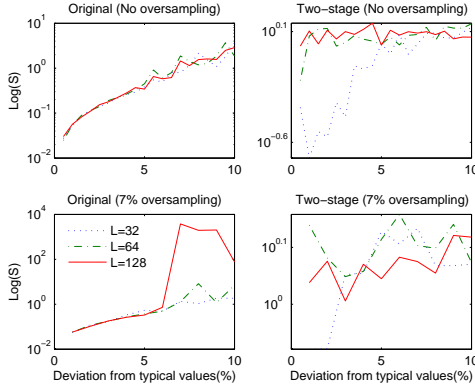


Fig. 5. Sensitivity S_f (logarithmic) versus the deviation from typical values (%) for the case of no oversampling (above) and with oversampling 7% (below). The curves are related to the original (left) and two-stage (right) structures. L represents the length of FIR synthesis filters.

Figure 5 demonstrates S_f versus the deviation from typical values for the original and two-stage HFB structures in logarithmic scale. It is seen that the two-stage HFB structure is more sensitive to analog imperfections. When the oversampling is not used, the sensitivity of the original HFB structure increases linearly versus the deviation ratios of electronic elements (logarithmic scale). When the oversampling ratio is equal to 7%, the sensitivity depends on the length of the FIR synthesis filters (fig.5). The sensitivity of the original HFB grows rapidly in the case of long FIR synthesis filters

(with 128 coefficients). Two-stage HFB are more sensitive to realization errors than the original structure (fig. 5). Figure 6

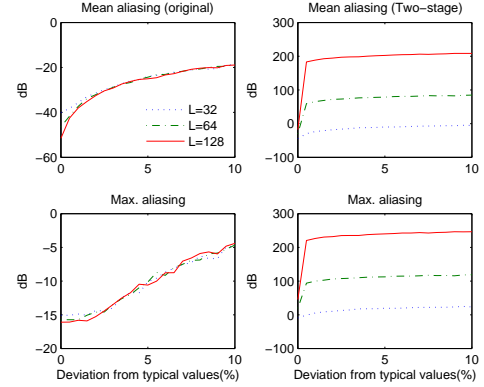


Fig. 6. Mean (above) and maximum (below) aliasing functions in dB versus the deviation from typical values (%). The curves belong to the original (left) and two-stage (right) structures and no oversampling has been used. L represents for the number of coefficients used for FIR synthesis filters.

shows the mean and maximum aliasing versus the deviations from typical values without oversampling process. Figure 7 shows the same when the oversampling ratio is equal to 7%. It may be seen that the two-stage HFB is not useful without oversampling because the aliasing level is too high. The original HFB appears less sensitive to analog imperfections. However, the performance of the original HFB is not practically tolerable in the presence of realization errors (fig.7).

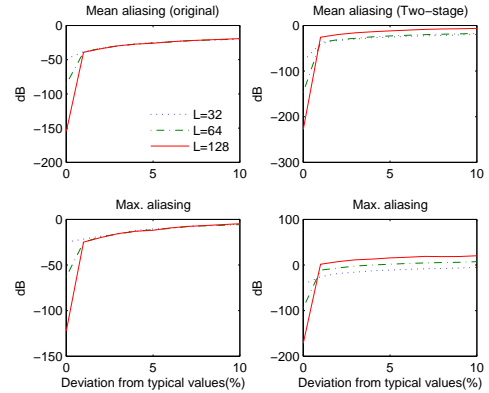


Fig. 7. Mean (above) and maximum (below) aliasing functions in dB versus the deviation from typical values (%) and the oversampling ratio 7% has been used. L shows the length of FIR synthesis filters.

C. Discussion

Oversampling process provides a lower aliasing level for the HFB-based A/D converters, but augments the sensitivity to realization errors. The oversampling process nullifies some coefficients of the equations and provides a lower level of aliasing, but increases the sensitivity originated from a higher condition number associated with the coefficient matrix. Practically, the original HFB is less sensitive to analog imperfections

than the two-stage one. Nevertheless, it needs a compensation method for being used in the architecture of wide-band A/D conversion. It may be seen that the performance is no longer acceptable when the electronic circuits of the analysis filter bank are subject to deviations from typical values higher than 1% (fig. 7).

Total Least Squares (TLS) optimization method is a candidate for decreasing the sensitivity to deviations of the coefficient matrix (or errors in variables) [6]. TLS was used for designing the synthesis filters of original HFB instead of Least Squares (LS). The sensitivity to analog imperfections associated with TLS and LS optimization methods is shown in figure 8 for comparison. The performances are approximately equal and the TLS optimization technique shows no improvement in the performance. Figure 9 shows the aliasing terms related to TLS and LS optimization methods considering oversampling ratios 0 and 7%. TLS does not provide a lower aliasing level. In fact, TLS is anticipated to improve the performance for zero-residual problems [6]. The design of synthesis filter bank of HFB is not a zero-residual problem because there is no synthesis filter bank leading to a null aliasing contribution. Therefore, the TLS optimization method does not lead to a lower sensitivity to analog imperfection (fig.8 and 9).

For having an acceptable implementation of HFB-based A/D conversion, the realization errors have to be estimated and compensated in the design of FIR synthesis filter bank. The estimation method would be necessarily blind since the output of synthesis filter bank is the only available signal. Blind estimation techniques have been already offered for estimating the analog imperfections [7], [8]. However, these methods have to be more developed for including HFB structures where the nonlinear contributions of undersampling process (or the aliasing terms) are present.

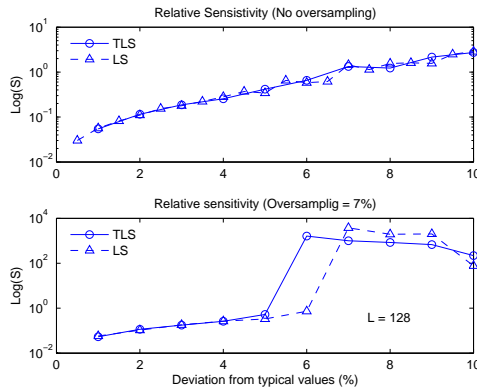


Fig. 8. Sensitivity S_F (logarithmic) versus the deviation from typical values (%) for the case of no oversampling (above) and with oversampling 7% (below) using TLS and LS optimization methods. The synthesis filters have 128 coefficients.

IV. CONCLUSION

The performance of two (original and two-stage) HFB structures versus different oversampling ratios have been studied in this paper. The oversampling process decreases the aliasing

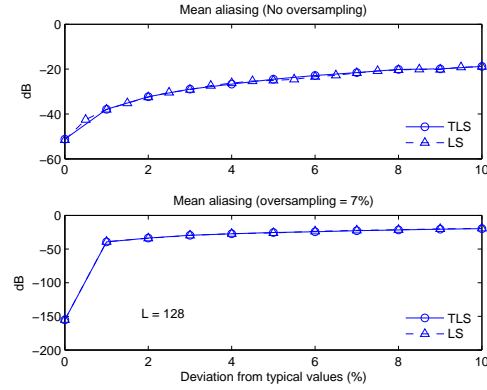


Fig. 9. Mean aliasing functions in dB versus the deviation from typical values (%) for TLS and LS optimization methods considering no oversampling (above) and the oversampling ratio 7% (below). FIR synthesis filters have 128 coefficients.

terms of HFB structures, but augments the sensitivity to analog imperfections. The optimum value of oversampling ratio for reducing the aliasing terms is about 7% for 8-channel HFB structure. The performance of HFB structures is studied in terms of sensitivity to analog imperfections. The original HFB structure is found a better candidate than the two-stage one. However it is still too sensitive to analog imperfections to be used for implementing the real A/D converters as well. Using TLS optimization, no change may be seen in the sensitivity to analog imperfections. It is shown that a blind estimation method is necessary to have an acceptable performance in the presence of analog imperfections for implementing the HFB-based A/D converters.

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