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Design of Rate Constrained Multi-user Receivers for Satellite Communications

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***Abstract*—In the realm of satellite communications, one of the great impairments to increase the spectral efficiency is multi-user interference in the reverse link (mobile to satellite). Standards techniques based on reuse factors and beam design, although interesting from an interference cancellation point of view, penalize dramatically the rate. In this contribution, we show how proper power allocation for successive interference cancellation can enhance the performance of satellite communications in the context of DVB-RCS systems with a high number of users. For a given user target rate, we provide a suitable algorithm based on Minimum Mean Square Error successive interference cancellation and assess the performance through various simulations.**

I. Introduction

The[1] ever growing demand in satellite wireless communications requires new ways of tackling the problem of multiple access communications in the reverse link. Traditional approaches based on orthogonal access such as TDMA or based on reuse factors are not interference limited schemes but rather noise limited. In this setting, where orthogonalization is achieved by re-use partitioning, the spectral efficiency is reduced dramatically as it penalizes the pre-log factor of the system's capacity. Moreover, system interference mitigation techniques do not provide adequate gains as multi-beam interference is rather limited (due to the reuse factor). Hence, if one completely abolishes frequency re-use (which is the approach proposed in this paper), on can gain a factor of M (if M is the re-use factor) in the bandwidth. Of course, the factor of M does not automatically translate into a similar factor in terms of spectral efficiency but the admittedly larger interference level can be abolished by appropriate techniques of multiuser detection with adequate power allocation as shown in [1]. The main reason for this important gain in terms of spectral efficiency comes from the fact that capacity scales linearly with bandwidth and only logarithmically with signal to noise ratio. As a consequence, all the efforts must be focused to reduce interference and the important gain factor leaves also room, as shown hereafter, for suboptimum multi-user detection algorithms, channel estimation, phase noise, non-linearities, synchronization issues,... Hence, in this contribution, a suitable algorithm in the context of DVB-RCS systems is provided taking into account each user's target rate. The algorithm [2], based on successive interference cancellation, optimizes the power allocation of all the users according to their rates and different channels. Our studies provide **the first implementation and proof of concept** of such an algorithm to our knowledge. The paper is organized as follows: In Section II, the system model is defined and suitable channel estimation methods are provided in section III. Multi-user detection is described in section IV with a special focus on the MMSE and the MMSE SIC receiver. Section V depicts the power allocation design for multi-user detection algorithms when rates are given as a priori inputs. Finally, some practical results, using DVB-RCS turbo codes, are shown in section VI.

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II. System Model

A. System description

The system under consideration consists of a satellite with M antennas, a gateway and K users to be covered by the satellite. K users are supposed to transmit simultaneously to the satellite and each user k sends a signal at a requested rate R_k depending on its needs. In this case, the received signal has the following Multiple Input Multiple Output (MIMO) form:

$$\mathbf{y} = \mathbf{H}\mathbf{P}^{\frac{1}{2}}\mathbf{s} + \mathbf{n} \tag{1}$$

where $\mathbf{y}^T = [y_1, y_2 \ldots, y_M]$ is the $M\times 1$ received signal, $\mathbf{H} = \{h_{i,j}\}_{j=1\ldots K}^{i=1\ldots M}$ is a $M\times K$ channel matrix where $h_{i,j}$ represents the channel from user j to antenna i, $\mathbf{P}^{\frac{1}{2}} = diag\left(p_1^{\frac{1}{2}}, p_2^{\frac{1}{2}}, \ldots, p_K^{\frac{1}{2}}\right)$ is a $K\times K$ diagonal matrix of transmitted powers and $\mathbf{s}^T = [s_1, s_2 \ldots, s_K]$ and $\mathbf{n}^T = [n_1, n_2 \ldots, n_M]$ are respectively the $K \times 1$ transmitted signal and $M \times 1$ additive white Gaussian noise of variance σ^2. We suppose that users can have different power allocations in order to satisfy their different (or equal) requested rates.

B. Definitions

In all the rest of the paper, we will suppose that:

$$tr(\mathbf{H}\ \mathbf{H}^H) \quad = \quad M \tag{2}$$

This fact stems from the definition of the SNR where

$$SNR = C\,\frac{E_b}{N_0} = \frac{1}{\sigma^2} \tag{3}$$

C is the user capacity. With the normalization of equation (2), we have

$$SNR = \frac{\mathbb{E}[\mathbf{s}^H\mathbf{H}^H\mathbf{H}\ \mathbf{s}]}{\mathbb{E}[\mathbf{n}^H\mathbf{n}]} = \frac{tr(\mathbf{H}^H\mathbf{H})}{M\sigma^2} = \frac{1}{\sigma^2} \tag{4}$$

As a consequence, in the following, any matrix considered, *i.e.* $\widehat{\mathbf{H}}$, will be normalized such that:

$$\mathbf{H} \quad = \quad \sqrt{\frac{M}{tr(\widehat{\mathbf{H}}\ \widehat{\mathbf{H}}^H)}}\ \ \widehat{\mathbf{H}} \tag{5}$$

III. Channel Estimation

In all the following, perfect CSI stands for perfect Channel State Information, in other words the channel matrix $\mathbf{H}$ is supposed to be perfectly known at the receiver. However, in practice, this is not the case. The channel needs to be estimated by sending a sequence of *known data* called *training sequence*. Hence, the frame contains two parts, one being the training sequence, whose length influences the performance of the channel estimation, and the other part contains the useful information part. The transmission scheme can be therefore written according to:

$$\mathbf{y} = \mathbf{H}\mathbf{P}^{\frac{1}{2}} \left[\begin{array}{c|c} \mathbf{c}^1 & data^1 \\ \mathbf{c}^2 & data^2 \\ \vdots & \vdots \\ \mathbf{c}^K & data^K \end{array} \right] + \mathbf{n} \tag{6}$$

with $\mathbf{P}^{\frac{1}{2}}$ is the diagonal power matrix assumed to be equal to 1 (in the training phase). $\mathbf{c}^i$ is the *training sequence* of length T associated to the user i, and $\mathbf{n}$ represents an $M \times N$ additive White Gaussian noise matrix of variance σ^2. The total length of the sent sequence is N, so the data has length $(N - T)$.

The channel estimation is done during the *training* phase. During that phase, the received signal can be rewritten:

$$\mathbf{y} = \mathbf{H}\mathbf{C} + \mathbf{n} \tag{7}$$

where $\mathbf{C}$ is the $K \times T$ matrix of the codes $\mathbf{c}^i$.
In this section, we present two different cases for the choice of the *training sequence* considering synchronized and unsynchronized transmissions.

A. Synchronized transmission

In this case, one can use orthogonal codes based on Walsh-Hadamard sequences which are constructed in the following iterative manner:

$$\mathbf{C}_T = \left[\begin{array}{cc} \mathbf{C}_{\frac{T}{2}} & \mathbf{C}_{\frac{T}{2}} \\ \mathbf{C}_{\frac{T}{2}} & -\mathbf{C}_{\frac{T}{2}} \end{array} \right] \qquad \forall T \in \mathbb{N} \qquad \text{with} \quad \mathbf{C}_0 = 1$$

To construct $\mathbf{C} = \mathbf{C}_T$ when $K \leq T$, one extracts from a $T \times T$ Walsh-hadamard matrix, K codes. The estimated channel matrix $\mathbf{H}_{\text{est}}$ is then given by:

$$\mathbf{H}_{\text{est}} = \frac{1}{T}\mathbf{y}\mathbf{C}^H = \mathbf{H} + \frac{1}{T}\mathbf{n}\mathbf{C}^H \tag{8}$$

in which $\mathbf{C}\mathbf{C}^H = T\mathbf{I}$. In this case, the noise variance normalized to the number of beam is given by:

$$\frac{1}{M} trace\left(\mathbb{E}\left[\frac{1}{T^2}\mathbf{n}\mathbf{C}^H\mathbf{C}\mathbf{n}^H\right]\right) = \frac{1}{M} trace(\frac{1}{T^2}\mathbf{C}^H\mathbf{C}\ \mathbb{E}\left[\mathbf{n}^H\mathbf{n}\right]) = \sigma^2\frac{M}{T} \tag{9}$$

From the expression (9), the dependance of the channel estimation with respect to the training length stands out clearly: as T increases, channel estimation becomes more accurate. However, the price to be paid is the reduction in terms of spectral efficiency because of the penalty factor $(N - T)$ between data and training.

B. Unsynchronized case

Due to the lack of synchronization of the users, the orthogonality criteria can not be always met. This can be modelled by a random sequences taking the alphabet $(\pm\frac{1}{\sqrt{2}} \pm i\frac{1}{\sqrt{2}})$. In this case, the received signal can be rewritten :

$$\mathbf{y} = \mathbf{H}\mathbf{C_R} + \mathbf{n} \tag{10}$$

where $\mathbf{C_R}$, in this case, is $K \times T$ a matrix of codes generated randomly. The estimated channel matrix $\mathbf{H}_{\text{est}}$ is therefore given by:

$$\mathbf{H}_{\text{est}} = \mathbf{y}\mathbf{C_R}^H(\mathbf{C_R}\mathbf{C_R}^H)^{-1} = \mathbf{H} + \mathbf{n}\mathbf{C_R}^H(\mathbf{C_R}\mathbf{C_R}^H)^{-1} \tag{11}$$

where $(\mathbf{C_R}\mathbf{C_R}^H)^{-1}$ is the pseudo-inverse of matrix $\mathbf{C_R}$. In this case, the noise variance normalized to the number of beams is given by:

$$\begin{aligned} \frac{1}{M} trace\ \mathbb{E}\left[\mathbf{n}\mathbf{C_R}^H(\mathbf{C_R}\mathbf{C_R}^H)^{-2}\mathbf{C_R}\mathbf{n}^H\right] &= \frac{1}{M} trace\left((\mathbf{C_R}\mathbf{C_R}^H)^{-1}M\sigma^2\right) \\ &= \sigma^2\int_0^\infty \frac{1}{\lambda} f(\lambda)\, d\lambda \qquad (12) \\ &= \sigma^2\frac{M}{T}\frac{1}{1-\frac{M}{T}} \qquad (13) \end{aligned}$$

where $f(\lambda)$ is the eigenvalue distribution of $(\mathbf{C_R}\mathbf{C_R}^H)^{-1}$ is given by the Marchenko-Pastur law [3]. Interestingly, this result based on random matrix theory and not proved before, shows that random and orthogonal training have the same performance up to the scalar factor: $\frac{1}{1-\frac{M}{T}}$. In the case $M = 8$ and $T = 64$, this factor is equal to $\frac{8}{7}$ and shows that there is nearly no loss in terms of performance between the two cases.

IV. Multi-user Detection

A. MMSE Receiver

The *Minimum Mean Square Error* (MMSE) receiver has several attributes which makes it appealing for use. The MMSE receiver is known to generate a soft decision output which maximizes the output SINR (see [4]) (whereas in a AWGN channel with no interference, the matched filter maximizes the output SNR). In this section, we suppose that $\mathbf{P} = \mathbf{I}$. As far as the MMSE SINR is concerned and considering model (1), the output of the MMSE detector $\hat{\mathbf{s}} = [\hat{s}_1, \ldots, \hat{s}_K]^T$ is given by

$$\begin{aligned} \hat{\mathbf{s}} &= \mathbb{E}\left[\mathbf{s}\mathbf{y}^H\right]\left(\mathbb{E}[\mathbf{y}\mathbf{y}^H]\right)^{-1}\mathbf{y} \qquad (14) \\ &= \mathbf{H}^H\left(\mathbf{H}\mathbf{H}^H + \sigma^2\mathbf{I}_M\right)^{-1}\mathbf{y} \qquad (15) \\ &= \mathbf{H}^H(\mathbf{A})^{-1}\mathbf{y}\ . \qquad (16) \end{aligned}$$

with $\mathbf{A} = \mathbf{H}\mathbf{H}^H + \sigma^2\mathbf{I}_M$ and $\mathbf{H}$ satisfies the relation (2). Each component $\hat{s}_k$ of $\hat{\mathbf{s}}$ is corrupted by the effect of both the thermal noise and by the "multi-user interference" due to the contributions of the other symbols $\{s_l\}_{l\neq k}$.
Let us now derive the expression of the SINR at one of the K outputs of the MMSE detector. Let $\mathbf{h}_k$ be the column of $\mathbf{H}$ associated to element s_k, and $\mathbf{U}$ the $M \times (K-1)$ matrix which remains after extracting $\mathbf{h}_k$ from $\mathbf{H}$. The component $\hat{s}_k$ after MMSE equalization has the following form:

$$\hat{s}_k = \eta_{\mathbf{h}_k} s_k + \tau_k$$

where

$$\eta_{\mathbf{h}_k} = \mathbf{h}_k^H(\mathbf{A})^{-1}\mathbf{h}_k\ . \tag{17}$$

and

$$\tau_k = \mathbf{h}_k^H(\mathbf{A})^{-1}\mathbf{H}[s_1, \ldots, s_{k-1}, 0, s_{k+1}, \ldots, s_K]^T + \mathbf{h}_k^H(\mathbf{A})^{-1}\mathbf{n}$$

The variance of τ_k is given by: $V = \mathbb{E}[|\ \tau_k\ |^2|\ \mathbf{H}]$. Knowing that $\mathbf{U}\mathbf{U}^H = \mathbf{H}\mathbf{H}^H - \mathbf{h}_k\mathbf{h}_k^H$, we get:

$$\begin{aligned} V &= \mathbf{h}_k^H(\mathbf{A})^{-1}\mathbf{U}\mathbf{U}^H(\mathbf{A})^{-1}\mathbf{h}_k + \mathbf{h}_k^H(\mathbf{A})^{-1}(\mathbf{A})^{-1}\mathbf{h}_k \\ &= \left(\mathbf{h}_k^H(\mathbf{A})^{-1}[\mathbf{H}\mathbf{H}^H - \mathbf{h}_k\mathbf{h}_k^H + \mathbf{I}_K](\mathbf{A})^{-1}\mathbf{h}_k\right) \\ &= \left(\eta_{\mathbf{h}_k} - \eta_{\mathbf{h}_k}^2\right) \\ &= \eta_{\mathbf{h}_k}(1 - \eta_{\mathbf{h}_k}) \end{aligned}$$

The Signal to Interference plus Noise Ratio SINR_k at the output k of the MMSE detector can thus be expressed as:

$$\begin{aligned}\mathrm{SINR}_k &= \frac{\mathbb{E}[\mid \eta_{\mathbf{h}_k} x_k \mid^2 \mid \mathbf{H}]}{\mathbb{E}[\mid \tau_k \mid^2 \mid \mathbf{H}]} \\ &= \frac{(\eta_{\mathbf{h}_k})^2}{\eta_{\mathbf{h}_k}(1-\eta_{\mathbf{h}_k})} \\ &= \frac{\eta_{\mathbf{h}_k}}{1-\eta_{\mathbf{h}_k}}\end{aligned}$$

Writing $\mathbf{H}\mathbf{H}^H = \mathbf{U}\mathbf{U}^H + \mathbf{h}_k\mathbf{h}_k^H$ and invoking the matrix inversion lemma[2], we get after some simple algebra another useful expression for this SINR :

$$\mathrm{SINR}_k = \mathbf{h}_k^H \left(\mathbf{U}\mathbf{U}^H + \sigma^2 \mathbf{I}_M\right)^{-1} \mathbf{h}_k \ . \tag{18}$$

Note that in practical coding schemes and after applying the MMSE receiver, one will minimize $\mid \hat{s}_k - \eta_{\mathbf{h}_k} s_k \mid^2$ with respect to the alphabet in use s_k.

B. SIC Receiver

The MMSE Receiver has the advantage of a very low complexity implementation. This feature (due in part to its linearity) has triggered the search for other MMSE based receivers such as the MMSE SIC (Minimum Mean Square Successive Interference Cancellation). The MMSE SIC [5]–[7] is based on successive interference cancellation where each layer is decoded, re-encoded and subtracted from the transmitted signal. This approach has triggered a lot of implementation research schemes as it was shown in [8] to be optimal[3].

The algorithm relies on a sequential detection of the received block [9]. We also suppose in this section that $\mathbf{P} = \mathbf{I}$. Recall that $\mathbf{y} = \mathbf{H}\mathbf{s} + \mathbf{n}$ At the first step of the method, a MMSE equalization of matrix $\mathbf{T}_{N,K} = \mathbf{H}$ is performed by a multiplication of $\mathbf{y}$ by matrix $\mathbf{F}_1 = \mathbf{T}_{N,K}^H(\mathbf{T}_{N,K}\mathbf{T}_{N,K}^H + \sigma^2\mathbf{I})^{-1}$. In the case of finite systems, the SINR of each symbol is different. Therefore, only the symbol with the best SINR is detected first. There is an optimum ordering in the detection process (depending on the channel attenuations) which makes the analysis extremely difficult for finite dimensions. As a consequence, the users are not detected with respected to their SINR ordering but to their channel strength ordering. The matrix $\mathbf{H}$ is therefore ordered according to the channel strenght with $\mid \mathbf{h}_K \mid^2 \geq \mid \mathbf{h}_{K-1} \mid^2 \ldots \geq \mid \mathbf{h}_1 \mid^2$, where $\{\mathbf{h}_k\}_{k=1\ldots K}$ are the strength of $\mathbf{H}$ as defined in IV-A. As a consequence, suppose that the algorithm starts by decoding symbol s_K. The estimated symbol goes through a turbo-decoder chain in order to improve the reliability of the detection process. Assuming a perfect decision (this is possible if the information s_K has been encoded at a rate of $\log_2(1+\mathrm{SINR}_K)$), the resulting estimated symbol $\hat{s}_K$ is subtracted from the vector of received samples in the following manner: $\mathbf{r}_2 = \mathbf{r}_1 - \hat{s}_K\mathbf{t}_K$ ($\mathbf{t}_i$ represents the ith column of $\mathbf{T}_{M,K}$ and vector $\mathbf{r}_1 = \mathbf{y}$). This introduces one degree of freedom for the next canceling vector choice which enables to reduce the noise plus interference influence and yields an increase in the decision process reliability.
The second step can be virtually represented by a completely new system of $K-1$ symbols $(s_1, \ldots, s_{K-1})$ transmitted by an $N \times (K-1)$ matrix $\mathbf{T}_{M,K-1}$ on the same fading channel. Equalizing with matrix $\mathbf{F}_2 = \mathbf{T}_{M,K-1}^H(\mathbf{T}_{M,K-1}\mathbf{T}_{M,K-1}^H + \sigma^2\mathbf{I})^{-1}$, one can retrieve symbol s_{K-1} which has been encoded at a rate $\log_2(1+\mathrm{SINR}_{K-1})$, one can reiterate the same process describe at the beginning. The advantage of such a scheme is that $\mathrm{SINR}_{\mathrm{SIC}(K-1)} \geq \mathrm{SINR}_{\mathrm{MMSE}(K-1)}$: one is able therefore to convey much more information on the second symbol (since the SINR increases) than with MMSE equalization.

[2]The matrix inversion lemma states that for any invertible matrix $\mathbf{F}$ and $\mathbf{E}$: $(\mathbf{D}^{-1} + \mathbf{F}\mathbf{E}^{-1}\mathbf{F}^H)^{-1} = \mathbf{D} - \mathbf{D}\mathbf{F}(\mathbf{E} + \mathbf{F}^H\mathbf{D}\mathbf{F})^{-1}\mathbf{F}^H\mathbf{D}^H$

[3]The optimality follows in fact directly by a simple determinant identity.

SIC Algorithm

```
new signal = received signal;
for        i = K down 1 do

           1. Decode the user of which |h_i|^2 = max_{j=1...i} |h_j|^2;

           2. Remove the decoded data from the new signal
           and use it as the new signal;
end for;
```

V. Power Allocation

In the case of the MMSE SIC receiver, users may require different achievable rates which depend mainly on their channel energy strength as well as their decoding order. In practical systems, users request a target rate whatever the channel conditions may be. In this case, the gateway has to allocate the proper power distribution to the users in order to satisfy the users requirements as well as making the systems decodable. In its full generality, the problem is still an open issue and has not been solved [2], [10]–[16] . However, for a fixed decoding order (approach followed in this paper and which for intuitive reasons is linked to the channel strength of the users), the optimal power allocation can be found for a fixed target rate (in practice, this corresponds to a fixed target Frame Error Rate).
Suppose in the following that all the K users request a target rate R for which the target SINR γ is thus given by (supposing Gaussian interference at the output of the receiver): $\gamma = 2^R - 1$. In the case of QPSK constellation, γ is related to the coding rate R_c (and not the target rate $R = 2 * R_c$) by:

$$2 * R_c = 2\left(1 - \int_{-\infty}^{\infty} \log_2(1 + e^{-2\gamma - 2\sqrt{\gamma}u}) \frac{e^{-u^2/2}}{\sqrt{2\pi}} du\right) \tag{19}$$

For the sake of space limitation, the details of equation (19) are not provided (see in [17], where BPSK input is considered).
Hence, for a given coding rate R_c, one can easily determine the target SINR γ. The power allocation is obtained in the following decreasing order: suppose that all the $K-1$ users have been successively decoded using the SIC approach. In this case, at the last iteration, we have:

$$\mathbf{r}_K = \mathbf{h}_\mathbf{K} p_K^{\frac{1}{2}} s_K + \mathbf{n} \tag{20}$$

The SINR at the output of the MMSE filter is given by:

$$\mathrm{SINR} = \gamma = \frac{\mathbf{h}_K^H \mathbf{h}_K p_K}{\sigma^2}$$

and therefore,

$$p_K = \frac{\gamma\sigma^2}{\mathbf{h}_K^H \mathbf{h}_K}.$$

This analysis can be extended to the ith iteration for which the SINR at step i is given by:

$$\mathrm{SINR}_i = \gamma = p_i \mathbf{h}_i^H \left(\sum_{l=i+1}^{K} p_l \mathbf{h}_l \mathbf{h}_l^H + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{h}_i$$

and

$$p_i = \frac{\gamma}{\mathbf{h}_i^H \left(\sum_{l=i+1}^{K} p_l \mathbf{h}_l \mathbf{h}_l^H + \sigma^2 \mathbf{I}\right)^{-1} \mathbf{h}_i} \tag{21}$$

The algorithm derives therefore the power of users $K, K-1 \ldots 1$. The classical successive interference cancellation algorithm works then with $\mathbf{T}_{M,K} = \mathbf{H}\mathbf{P}$. Usually, one needs to use different interleavers for

the users as shown in [18], [19]. Although the algorithm is appealing, expression (21) is not simple to implement as it requires many matrix inversions. In order to avoid this, the following result is useful **Result.** Let $\Gamma_K = \sigma^2 I$. For any given SINR γ_k requirement of user k, the power is given by:

$$p_k = \frac{\gamma_k}{\mathbf{h}_k^H \mathbf{\Gamma}_k^{-1} \mathbf{h}_k} \tag{22}$$

$$\mathbf{\Gamma}_{k-1}^{-1} = \mathbf{\Gamma}_k^{-1} - \frac{p_k \mathbf{\Gamma}_k^{-1} \mathbf{h}_k \mathbf{h}_k^H \mathbf{\Gamma}_k^{-1}}{1+\gamma_k} \tag{23}$$

For the sake of space limitation the lecture should refer to [1], [2], [20] for calculation details of equation (22) and (23).

Theoretically speaking, when the turbo-code works close to the channel capacity and without channel estimation mismatches, the previous algorithm provides the adequate power allocation for a SINR requirement (19). However, (19) does not provide for practical coding scheme (for finite block length coding) the relationship between target rate and target SNR. This can be however deduced from standard tables connectiong the the target FER (10^{-4}) and the value of the E_b/N_0 that permits to reach that FER (*Frame Error Rate*) as shown in table I.

Table I
REFERENCE SNR ACCORDING TO E_b/N_0

Rates	$\frac{E_b}{N_0}(dB)$	SNR
$\frac{1}{3}$	11	8.3928
$\frac{2}{5}$	15	25.2982

VI. SIMULATION RESULTS

A. Parameters description

The simulations are performed according to the DVB-RCS standard [21] as far as the encoding and decoding schemes are concerned. The following parameters were considered:

- Each user is supposed to transmit 1.000.000 frames of 96 symbols.
- Due to space limitations, only the rate 1/3 of the turbo coder is depicted.
- QPSK modulation is used.
- Number of antennas at the satellite is equal to 8.
- Number of users $K = 16$.

The results are obtained with the data provided by a realistic simulator of *Space Engineering* and contains 1000 realizations of the satellite channel. Note that we consider an overloaded system, i.e. $K > M$.

B. Channel estimation considerations

In this section, simulations were carried out to compare the performance of non-perfect CSI with respect to the length of the training sequence. The simulations were performed for the MMSE and the MMSE SIC receiver with orthogonal sequences (as the gap with non-orthogonal has been shown theoretically to be negligible). We use the Shannon's definition of capacity. For a given user, the mean capacity in [bits/s/Hz] is expressed by the formula (24)

$$\mathscr{C} = \frac{1}{KL} \sum_{k=1}^{K} \sum_{n=1}^{L} C_n(k) \tag{24}$$

where $C_n(k)$ is the capacity of user k of a given realization of the channel n (L is the total number of realization and is equal to 1000). In particular, we have:

$$C_n(k) = \log_2(1 + \mathrm{SINR}_k^n) \tag{25}$$

where SINR_k^n is the SINR of user k at realization n, which is a function of the *training sequence* length T (in the case of non perfect channel estimation).

We notice from figure (1), that the capacity for non perfect CSI with T=256 is very close to that of Perfect CSI. This is not the case with T=16. We remark also that the performance for the MMSE SIC receiver is much better then the MMSE receiver.

For future simulations, we consider only a training sequence of length T=64 as they were shown to obtain the same result as T=256.

C. Simulations without target FER requirements

1) MMSE simulations: The MMSE simulations are depicted in figure (2) for perfect CSI whereas in figure (3) we show the FER with non perfect CSI. The simulations show that the training sequence of length T=64 is sufficient to reach the same results as with a perfect CSI. We notice also that not all the users are well decoded even for large values of E_b/N_0, which in some sense, requires more advanced receivers.

2) MMSE SIC Simulations: The results of the MMSE SIC are depicted in figure (4) for perfect CSI whereas non-perfect CSI is considered in (5). Remarkably, for a training sequence $T = 64$, the results are quite similar to the perfect CSI case (see figure (5)).

D. Simulations with target FER requirements

For a given target FER, the powers are allocated to each user according to the results in Section V. Figure (6) and figure (8) show the allocated power for each user. We have simulated different lengths of training sequence in the case of perfect and non perfect CSI. The target FER was set to 10^{-4}. We remark that at low E_b/N_0, the FER is very bad when the CSI is estimated, but at high values of E_b/N_0 and with power allocation the results are better than the case without power allocation. Indeed, almost all the users can be decoded which is not the case with the simple MMSE. This result is very interesting because as it agrees with our theoretical claims.

VII. CONCLUSION

In this paper, a successive interference cancellation schemes with adequate power allocation is proposed to increase the spectral efficiency of satellite communications. Moreover, a full implementation of the algorithm with DVB-RCS compliant turbo-codes has been performed. Interestingly, by reducing the reuse factor, multi-user interference can be tackled by a very simple algorithm compatible with the DVB-RCS system, even in overloaded systems. Further studies will consider theoretical assessment in the choice of the decoding order as well as deriving a soft decision framework for the users data. The authors would like to thank G. Gennaro from Space Engineering Sp.A for useful comments and for providing the satellite simulator.

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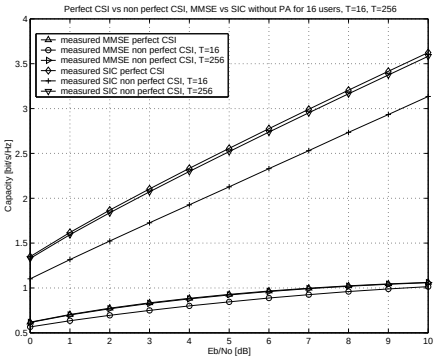


Figure 1. Capacity Comparison: Perfect CSI vs non Perfect CSI for 16 users with T=16, 256.

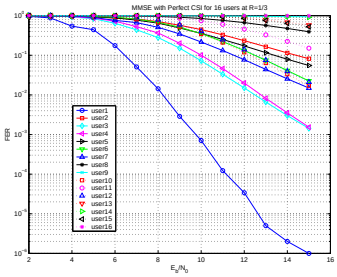


Figure 2. MMSE with perfect CSI.

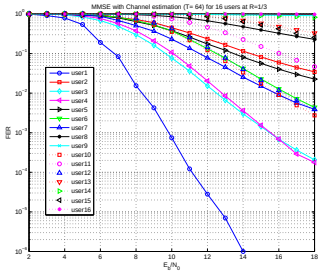


Figure 3. MMSE with non perfect CSI.

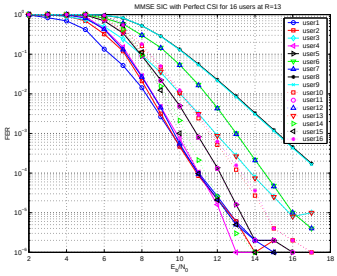


Figure 4. MMSE SIC with perfect CSI.

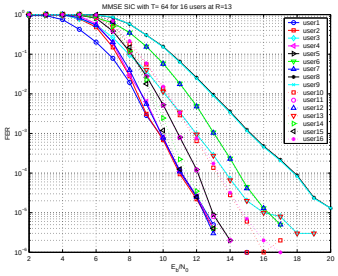


Figure 5. MMSE SIC with non perfect CSI.

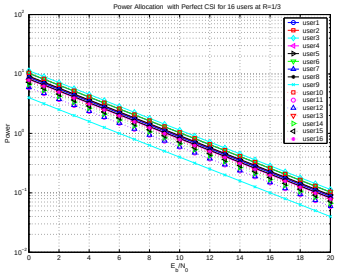


Figure 6. Power allocation with perfect CSI.

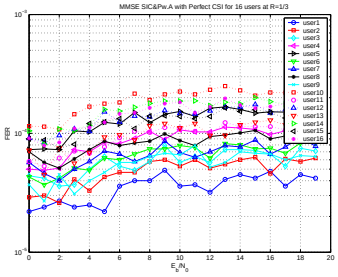
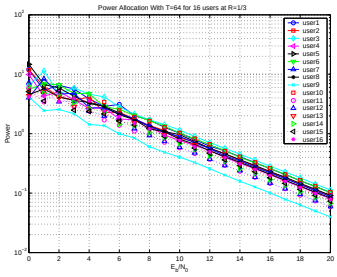


Figure 7. FER rate with Power allocation for MMSE SIC and perfect CSI.



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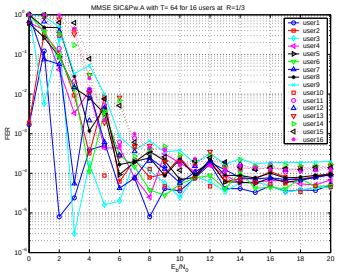

Figure 9. FER rate with Power allocation for MMSE SIC and non perfect CSI.