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Energy-Efficient Cooperative Routing in BER Constrained Multihop Networks

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***Abstract*— Due to the limited energy supplies of nodes, in many applications like wireless sensor networks energy-efficiency is crucial for extending the lifetime of these networks. We study the routing problem for multihop wireless ad hoc networks based on cooperative transmission. The source node wants to transmit messages to a single destination. Other nodes in the network may operate as relay nodes. In this paper, we propose a cooperative multihop routing for the purpose of power savings, constrained on a required bit error rate (BER) at the destination. We derive analytical results for line network topology. It is shown that energy savings of 100% are achievable in line networks with a large number of nodes for BER $= 10^{-4}$ constraint at the destination.**

I. Introduction

Energy consumption in multihop wireless networks is a crucial issue that needs to be addressed at all the layers of communication system, from the hardware up to the application. In this paper, we focus on energy savings in routing problem in which messages may be transmitted via multiple radio hops. After substantial research efforts in the last several years, routing for multihop wireless networks becomes a well-understood and broadly investigated problem [1], [2]. Nevertheless, with the emergence of new multiple antennas technology, existing routing solutions in the traditional radio transmission model are not efficient anymore. For instance, it is feasible to coordinate the multiple transmissions from multiple transmitters to one receiver simultaneously. As a result, transmitting signals with the same channel from several different nodes to the same receiver simultaneously are not considered collision but instead could be combined at the receiver to obtain stronger signal strength. In [3], the concept of multihop diversity is introduced where the benefits of spatial diversity are achieved from the concurrent reception of signals that have been transmitted by multiple previous terminals along the single primary route. This scheme exploits the broadcast nature of wireless networks where the communications channel is shared among multiple terminals. On the other hand, the routing problem in the cooperative radio transmission model is studied in [4], where it is allowed that multiple nodes along a path coordinate together to transmit a message to the next hop as long as the combined signal at the receiver satisfies a given SNR threshold value.

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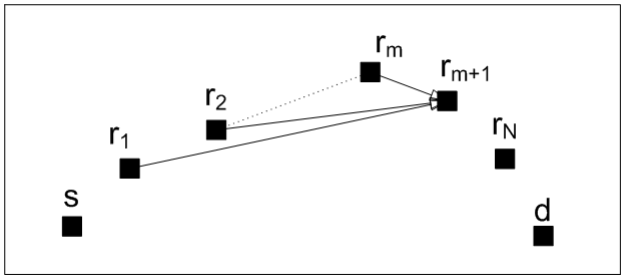

Fig. 1. Wireless multihop network under m-cooperation.

In this paper, a cooperative multihop routing is proposed for Rayleigh fading channels. The investigated system can achieve considerable power savings compared to non-cooperative multihop transmission, when there is a bit error rate (BER) QoS requirement at the destination node. We derive a simple closed-form solution for power allocation among the transmitting nodes at each phase. Simulation results show that, using the proposed power allocation strategies, considerable gains are obtained comparing to the non-cooperative multihop transmission.

II. System Model and Protocol Description

We consider an arbitrary N-relay wireless network, where information is to be transmitted from a source to a destination. Due to the broadcast nature of the wireless channel, some relays can overhear the transmitted information, and thus, can cooperate with the source to send its data. The wireless link between any two nodes in the network is modeled as a Rayleigh fading narrowband channel. The channel fades for different links are assumed to be statistically independent. The additive noise at all receiving terminals is modeled as zero-mean complex Gaussian random variables with variance $\mathcal{N}_0$. For medium access, the relays are assumed to transmit over orthogonal channels, thus no interrelay interference is considered in the signal model.

Following [4], we also assume that each transmission is either a broadcast transmission where a single node is transmitting the information, and the information is received by multiple nodes, or a cooperative transmission where multiple nodes simultaneously send the information to a single receiver. Various scenarios for the cooperation among the relays can be implemented. A general cooperation scenario, m-cooperation,

$(1 \leq m \leq N)$, can be implemented in which each relay combines the signals received from the previous relays and along with that received from the source.

For a general scheme m-cooperation, $(1 \leq m \leq N)$, each receiving node decodes the information after combining the signals received from the previous m transmitting nodes. Fig. 1 shows a wireless multihop network consisting of a source node s, N relays, and a destination node d, which is operating under m-cooperation scenario. The cooperation protocol has $N+1$ phases. In Phase 1, the source transmits the information, and the received signal at the destination and the ith relay can be modeled, respectively, as

$$y_0 = \sqrt{P_0} f_0 s + w_0, \tag{1}$$

$$y_{0,i} = \sqrt{P_0} f_{0,i} s + v_i, \tag{2}$$

where P_0 is the average total transmitted symbol energy of the source, since we assume the information bearing symbols s's have zero-mean and unit variance, w_0 and v_i are complex zero-mean white Gaussian noise, and $f_{i,j}$, $i = 0, 1, \ldots, N$, $j = 1, 2 \ldots, N+1$, are complex Gaussian random variables with zero-mean and variances $\sigma_{i,j}^2$, respectively. In Phase 2, relay nodes are sorted based on their received received SNR, such that relay 1 has the highest received SNR. Generally, in Phase n, $2 \leq n \leq N+1$, the previous $\min\{m, n\}$ nodes are transmitting their signal toward the next node. Similar to [4], we assume that transmitters are able to adjust their phases in such a way that the received signal at the nth receiving node in Phase n is

$$y_n = \sqrt{P_0} |f_{0,n}| \, u(m-n) \, s + \sum_{i=\max(1,n-m)}^{n-1} \sqrt{P_i} \, |f_{i,n}| \, \hat{s}_i + v_n, \tag{3}$$

where the function $u(x) = 1$, when $x \geq 0$, and otherwise is zero and the symbol $\hat{s}_i$ is re-encoded symbol at the ith relay.

III. BER-Based Link Cost Formulation

In this section, our objective is to find the optimal power allocation required for successful transmission from a set of transmitting nodes to a set of receivers. In order to derive explicit expressions for the link costs, we consider three distinct cases described as follows.

A. Point-to-Point Link Cost

The simplest case is the case where only one node is transmitting within a time slot to a single target node. For decoding the message reliably, the BER must be less than the threshold value $\mathrm{BER}_{\max}$.

Assuming a Rayleigh fading link with variance of σ_0^2 in the network, M-PSK or M-QAM modulations, and coherent detection, the average probability of error can be obtain as [5, Eq. (6)],

$$P_e^b = \frac{c}{\pi} \left(1 - \sqrt{\frac{g P_0 \sigma_0^2}{2\mathcal{N}_0 + g P_0 \sigma_0^2}} \right), \tag{4}$$

where the parameters c and g are dependent on the modulation type. Using (4), the minimum required power, and hence, the point-to-point link cost is given by

$$\mathcal{C}(\mathrm{tx}_1, \mathrm{rx}_1) = P_0 = \frac{2\mathcal{N}_0}{g\,\sigma_0^2} \frac{1}{\frac{1}{\left(1 - \frac{2\,\mathrm{BER}_{\max}}{c}\right)^2} - 1}, \tag{5}$$

where tx_1 and rx_1 denote the transmitter and receiver nodes, respectively. Since $\mathrm{BER}_{\max} \ll 1$, the link cost in (5) can be approximated as

$$\mathcal{C}(\mathrm{tx}_1, \mathrm{rx}_1) \approx \frac{\mathcal{N}_0}{g\,\sigma_0^2} \frac{c}{2\,\mathrm{BER}_{\max}}. \tag{6}$$

B. Point-to-Multipoint Link Cost

In this case, we assume a transmitter node tx_1 broadcast its information toward a set of receiving nodes $\mathrm{Rx} = \{\mathrm{rx}_1, \mathrm{rx}_2, \ldots, \mathrm{rx}_m\}$. Assuming that omnidirectional antennas are used, the signal transmitted by the node tx_1 is received by all nodes within a transmission radius proportional to the transmission power. Hence, a broadcast link can be treated as a set of point-to-point links, and the cost of reaching a set of nodes is the maximum of the costs for reaching each of the nodes in the target set. Thus, the minimum power required for the broadcast transmission, denoted by $\mathcal{C}(\mathrm{tx}_1, \mathrm{Rx})$, is given by

$$\mathcal{C}(\mathrm{tx}_1, \mathrm{Rx}) = \max\{\mathcal{C}(\mathrm{tx}_1, \mathrm{rx}_1), \mathcal{C}(\mathrm{tx}_1, \mathrm{rx}_2), \ldots, \mathcal{C}(\mathrm{tx}_1, \mathrm{rx}_m)\}, \tag{7}$$

where $\mathcal{C}(\mathrm{tx}_1, \mathrm{rx}_i)$ is found from (5).

C. Multipoint-to-Point Cooperative Link Cost

In this case, a set of multiple nodes $\mathrm{Tx} = \{\mathrm{tx}_1, \mathrm{tx}_2, \ldots, \mathrm{tx}_m\}$ cooperate to transmit the same information to a single receiver node rx_1. Assuming coherent detection at the receiving node, the signals simply add up at the receiver, and acceptable decoding is possible as long as the received BER becomes less than $\mathrm{BER}_{\max}$.

Now, we are going to derive a tractable BER formula at the receiving node rx_1, which leads to a closed-form power allocation strategy among the cooperative nodes. Therefore, we use the approach proposed in [6] to derive the BER expressions for the high SNR regime. That is [6, Eq. (10)]

$$P_e^b \simeq \frac{c \prod_{i=1}^{t+1} (2i-1)}{2(t+1) g^{t+1} t!} \frac{\partial^t p_\gamma(0)}{\partial \gamma^t}, \tag{8}$$

where $\frac{\partial^t p_\gamma(0)}{\partial \gamma^t}$ is the tth order derivative of the pdf of the equivalent channel, and the derivatives of $p_\gamma(\gamma)$ up to order $(t-1)$ are supposed to be zero. Using (3), the received SNR at the receiving node can be written as $\gamma = \sum_{i=1}^m \gamma_i$, where $\gamma_i = \frac{P_i |f_i|^2}{\mathcal{N}_0}$ with f_i denotes the channel between the ith transmitter and the receiving node.

In [6], the following proposition is proposed, which can be used to calculate the BER expression in (8).

Proposition *1:* Consider a finite set of nonnegative random variables $\{\gamma_1, \gamma_2, \ldots, \gamma_m\}$ whose pdfs $p_1, p_2, \ldots, p_m$ have nonzero values at zero, and denote these values as

$p_1(0), p_2(0), \ldots, p_m(0)$. If $\gamma = \sum_{i=1}^{m} \gamma_i$, then all the derivatives of $p_\gamma(\gamma)$ evaluated at zero up to order $m-2$ are zero, while the $(m-1)$th order derivative is given by

$$\frac{\partial^{m-1} p_\gamma(0)}{\partial \gamma^{m-1}} = \prod_{i=1}^{m} p_i(0). \tag{9}$$

Using Proposition 1 and (8), we get

$$P_e^b \approx \frac{c \prod_{i=1}^{m}(2i-1)}{2g^m m!} \prod_{i=1}^{m} p_i(0). \tag{10}$$

Hence, using (10) and the fact that the value of an exponential distribution with mean $\frac{P_i \sigma_i^2}{\mathcal{N}_0}$ at zero is $\frac{\mathcal{N}_0}{P_i \sigma_i^2}$, the average BER expression can be approximated as

$$P_e^b \approx \frac{c \prod_{i=1}^{m}(2i-1)}{2g^m m!} \prod_{i=1}^{m} \frac{\mathcal{N}_0}{P_i \sigma_i^2}. \tag{11}$$

The total transmitted power for the multipoint-to-point case is $\sum_{i=1}^{m} P_i$. Therefore, the power allocation problem, which has a required BER constraint on the receiving node, can be formulated as

$$\begin{aligned} &\min \sum_{i=1}^{m} P_i, \\ &\text{s.t. } \frac{c}{2g^m m!} \prod_{i=1}^{m} \frac{(2i-1)\mathcal{N}_0}{P_i \sigma_i^2} \leq \text{BER}_{\max}, \\ &\quad P_i \geq 0, \text{ for } i = 1, \ldots, m. \end{aligned} \tag{12}$$

Before deriving the optimal solution for the problem given in (12), the following theorem is needed.

Theorem *1:* The optimum power allocation $P_1^*, \ldots, P_m^*$ in the optimization problem stated in (12) is unique.

Proof: The objective function in (12) is a linear function of the power allocation parameters, and thus, it is a convex function. Hence, it is enough to prove that the first constraint in (12), i.e.,

$$f(P_1, \ldots, P_m) = \frac{c}{2g^m m!} \prod_{i=1}^{m} \frac{(2i-1)\mathcal{N}_0}{P_i \sigma_i^2} - \text{BER}_{\max}, \tag{13}$$

with $D_f = \{P_i \in (0, \infty), i \in \{1, \ldots, m\} \mid f(P_1, \ldots, P_m) \leq 0\}$, $f : D_f \longrightarrow \mathbb{R}$, is a convex function. From [7], it can be verified that $f(P_1, \ldots, P_m)$ is a *posynomial* function, which is a convex function. ■

The optimal power allocation strategy for high SNRs is found in the following. However, since the approximate BER expression derived in (11) is an upper-bound on BER, this result can be used reliably.

Proposition *2:* For the set of m transmitters, which send a common signal toward the destination, the optimum transmit power coefficients in (12) satisfy the following equations

$$P_i = \frac{\Psi(m)}{\text{BER}_{\max}} \frac{\mathcal{N}_0}{\sigma_i^2} \prod_{\substack{k=1 \\ k \neq i}}^{m} \frac{\mathcal{N}_0}{P_k \sigma_k^2}, \quad i = 1, \ldots, m, \tag{14}$$

where

$$\Psi(m) = \frac{c \prod_{i=1}^{m}(2i-1)}{2g^m m!}, \tag{15}$$

Proof: The Lagrangian of the problem stated in (12) is

$$L(P_1, \ldots, P_m) = \sum_{i=1}^{m} P_i + \lambda f(P_1, \ldots, P_m). \tag{16}$$

For nodes $i = 1, \ldots, m$ with nonzero transmitter powers, the Kuhn-Tucker conditions are

$$\frac{\partial}{\partial P_i} L(P_1, \ldots, P_m) = 1 + \lambda \frac{\partial}{\partial P_i} f(P_1, \ldots, P_m) = 0, \tag{17}$$

where

$$\frac{\partial}{\partial P_i} f(P_1, \ldots, P_m) = -\Psi(m) \frac{\mathcal{N}_0}{P_i^2 \sigma_i^2} \prod_{\substack{k=1 \\ k \neq i}}^{m} \frac{\mathcal{N}_0}{P_k \sigma_k^2}. \tag{18}$$

Using (17) and (18), we have

$$P_i^2 = \lambda \Psi(m) \frac{\mathcal{N}_0}{P_i^2 \sigma_i^2} \prod_{\substack{k=1 \\ k \neq i}}^{m} \frac{\mathcal{N}_0}{P_k \sigma_k^2}, \tag{19}$$

for $i = 1, \ldots, m$. Since the strong duality condition [7, Eq. (5.48)] holds for convex optimization problems, we have $\lambda f(P_1, \ldots, P_m) = 0$ for the optimum point. If we assume Lagrange multiplier has a positive value, we have $f(P_1, \ldots, P_m) = 0$, which is equivalent to

$$\text{BER}_{\max} = \Psi(m) \prod_{k=1}^{m} \frac{\mathcal{N}_0}{P_k \sigma_k^2}. \tag{20}$$

Dividing both sides of equalities (19) and (20), we can find the Lagrange multiplier as

$$\lambda = \frac{P_i}{\text{BER}_{\max}}. \tag{21}$$

Substituting λ from (21) into (19) we get (14). Moreover, since P_i in (14) are positive, the second set of constraints in (12) are satisfied. ■

Theorem *2:* The optimum power allocation $P_1^*, \ldots, P_m^*$ in the optimization problem stated in (12) are equal and is expressed as

$$P_i^* = \left(\frac{\Psi(m)}{\text{BER}_{\max}} \prod_{k=1}^{m} \frac{\mathcal{N}_0}{\sigma_k^2} \right)^{\frac{1}{m}}. \tag{22}$$

Proof: In Theorem 1, we have shown, this problem has a unique solution. Now, using Proposition 2, by the fact that the problem in (12) should have a unique solution, we put initial values $P_1^*, \ldots, P_m^*$ in (14), and we observe that the closed-form solution as (22) is achieved, which satisfies the set of equations in (14). ■

An interesting property of P_i^* derived in (22) is that it is just dependent on the product of all path-loss coefficients of links. Therefore, P_i^*s can be calculated in a decentralized manner by broadcasting the product term from the receiving node toward the transmitting nodes. Using Theorem 2, the resulting cooperative link cost $\mathcal{C}(\text{Tx}, \text{rx}_1)$, defined as the optimal total power, is given by

$$\mathcal{C}(\text{Tx}, \text{rx}_1) = \sum_{i=1}^{m} P_i^* = m \left(\frac{\Psi(m)}{\text{BER}_{\max}} \prod_{k=1}^{m} \frac{\mathcal{N}_0}{\sigma_k^2} \right)^{\frac{1}{m}}. \tag{23}$$

IV. Energy Savings via Cooperative Routing

The problem of finding the optimal cooperative route from the source node to the destination node can be mapped to a Dynamic Programming (DP) problem [4]. As the network nodes are allowed only to either fully cooperate or broadcast, finding the best cooperative path from the source node to the destination has a special layered structure. In [4], it is shown that in a network with $N+1$ nodes, which has 2^N nodes in the cooperation graph, standard shortest path algorithms have a complexity of $O(2^N)$. Hence, finding the optimal cooperative route in an arbitrary network becomes computationally intractable for larger networks. For this reason, we restrict the cooperation to nodes along the optimal noncooperative route. That is, at each transmission slot, all nodes that have received the information cooperate to send the information to the next node along the minimum energy noncooperative route [4]. Therefore, with the help of the link cost expressed in Subsections III-A and III-B, the minimum-energy non-cooperative route is first selected, which has N intermediate relays. Then, nodes along the optimal non-cooperative route cooperate to transmit the source information toward the destination. That is, at each transmission slot, all nodes that have received the information cooperate to send the information to the next node along the minimum energy non-cooperative route. In the nth transmission slot, the reliable set is $\text{Tx}_n = \{s, r_1, \ldots, r_{n-1}\}$, which is including the source node and the previous relays r_i, $i = 1, \ldots, n-1$. The link cost associated with the nodes in Tx_n, which cooperate to send the information to the next node n, follows from (23), and is given by

$$\mathcal{C}(\text{Tx}_n, n) = n \left(\frac{\Psi(n)}{\text{BER}_{\max}} \prod_{k=0}^{n-1} \frac{\mathcal{N}_0}{\sigma_{k,n}^2} \right)^{\frac{1}{n}}. \tag{24}$$

Note that the nth node denotes the nth relay when $n \leq N$, and the destination node when $n = N+1$. Therefore, the total transmission power for the cooperative multihop system is

$$P_T(\text{coop}) = \sum_{n=1}^{N+1} \mathcal{C}(\text{Tx}_n, n) = \sum_{n=1}^{N+1} n \left(\frac{\Psi(n)}{\text{BER}_{\max}} \prod_{k=0}^{n-1} \frac{\mathcal{N}_0}{\sigma_{k,n}^2} \right)^{\frac{1}{n}}. \tag{25}$$

For the case of m-cooperation scheme, in which just previous closest nodes cooperate to transmit along the non-cooperative route, $P_T(\text{cooperative})$ in (25) can be modified to

$$\begin{aligned} P_T(m\text{-coop}) &= \sum_{n=1}^{N+1} \mathcal{C}_m(\text{Tx}_n, n) \\ &= \sum_{n=1}^{m} n \left(\frac{\Psi(n)}{\text{BER}_{\max}} \prod_{k=0}^{n-1} \frac{\mathcal{N}_0}{\sigma_{k,n}^2} \right)^{\frac{1}{n}} \\ &\quad + \sum_{n=m+1}^{N+1} m \left(\frac{\Psi(m)}{\text{BER}_{\max}} \prod_{k=n-m}^{n-1} \frac{\mathcal{N}_0}{\sigma_{k,n}^2} \right)^{\frac{1}{m}}. \end{aligned} \tag{26}$$

The energy savings for a cooperative routing strategy relative to the optimal noncooperative strategy is defined as

$$\text{Energy Savings} = \frac{P_T(\text{noncoop}) - P_T(\text{coop})}{P_T(\text{noncoop})}, \tag{27}$$

where $P_T(\text{coop})$ is computed in (25) and (26) for the case of full-cooperation and m-cooperation routings, respectively. $P_T(\text{noncoop})$ denotes the total transmission power for the non-cooperative multihop strategy. Using (6), $P_T(\text{noncoop})$ can be calculated as

$$P_T(\text{noncoop}) = \frac{c}{2\,\text{BER}_{\max}} \sum_{n=0}^{N} \frac{\mathcal{N}_0}{g\,\sigma_{n,n+1}^2}. \tag{28}$$

For each of these topologies, we derive the optimal non-cooperative route and obtain a lower bound on the optimal energy savings achievable by cooperative routing. The bound is obtained by deriving analytical expressions for energy savings for a sub-optimal cooperative route, where cooperation is restricted to nodes along the optimal non-cooperative route. That is, at each transmission slot, all nodes that have received the information cooperate to send the information to the next node along the minimum energy non-cooperative route.

A. BER Upper-Bounds at the Destination Node

In this subsection, we will view the system from our end-to-end equivalent BER perspective. That is, we represent $\text{BER}_{\max}$ in each step in terms of the required BER at the destination.

In the case of non-cooperative multihop system in which N relays are in cascade, when BPSK is used, the BER P_n^b at nth node is affected by all previous $n-1$ hops and can be iteratively calculated according to the recursion [8]

$$P_n^b = (1 - P_{n-1}^b) P_{n-1,n}^b + P_{n-1}^b (1 - P_{n-1,n}^b), \tag{29}$$

with $P_0^b = 0$, where $P_{n-1,n}^b$ is the BER from the $(n-1)$th node to the nth node. The end-to-end BER at the destination is given by using $n = N+1$ in (29). Since the BER at the destination should be less than the required BER QoS, it is enough to consider the upper-bound for the BER. Thus, for any general constellation, the BER can be bounded as $P_n^b \leq (1 - P_{n-1}^b) P_{n-1,n}^b + P_{n-1}^b$. Assuming the power allocation strategies derive in Section III, P_n^b bound can be written as $P_n^b \leq 1 - (1 - \text{BER}_{\max})^n$.

For the case of cooperative routing, the following upper-bound can be obtained in the nth node

$$P_n^b \leq 1 - \left[(1 - P_{n-1,n}^b) \prod_{i=\max\{1,n-m\}}^{n-1} (1 - P_i^b) \right]. \tag{30}$$

If the power allocation strategy derived in (22) is used, (30) can be rewritten as

$$P_n^b \leq 1 - \left[(1 - \text{BER}_{\max}) \prod_{i=\max\{1,n-m\}}^{n-1} (1 - P_i^b) \right]. \tag{31}$$

To get an insight into the relationship between the end-to-end BER P_{N+1}^b and $\text{BER}_{\max}$, the upper-bound on P_{N+1}^b when full cooperation is used can be represented as $P_{N+1}^b \leq 1 - (1 - \text{BER}_{\max})^{2^{n-1}}$.

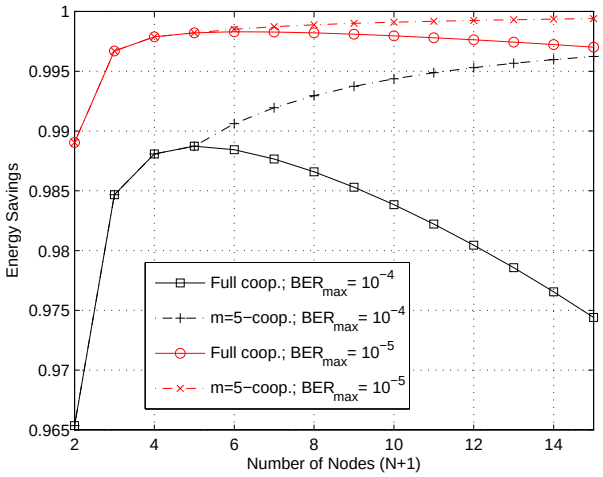

Fig. 2. The average energy savings curves versus the number of transmitting nodes, $(N+1)$, employing full-cooperation and m-cooperation with two different $\mathrm{BER}_{\max}$ constraints.

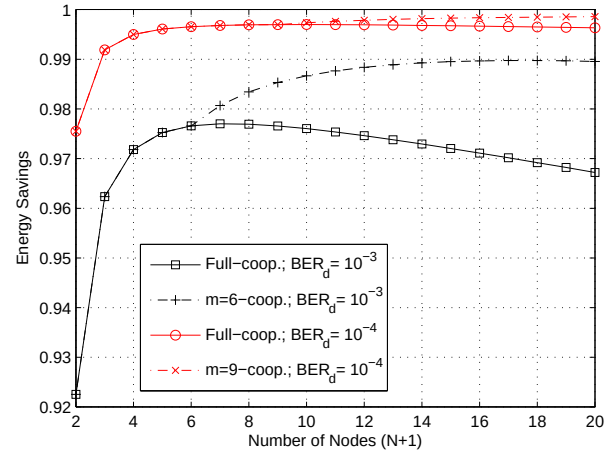

Fig. 3. The average energy savings curves versus the number of transmitting nodes $(N+1)$ relay networks employing full-cooperation and m-cooperation with two different BER_d constraints.

V. Simulation Results

In this section, we present some simulation results to quantify the energy savings due to the proposed cooperative routing scheme. We consider a regular line topology where nodes are located at unit distance from each other on a straight line. The optimal non-cooperative routing in this network is to always send the information to the next nearest node in the direction of the destination. From (6), (28), and by assuming that $\sigma_{i,j}^2$ is proportional to the inverse of the distance squared, the total power required for non-cooperative transmission can be calculated as

$$P_T(\text{noncoop}) = (N+1)\frac{c\mathcal{N}_0}{2\,g\,\mathrm{BER}_{\max}}. \tag{32}$$

Since we restrict the cooperation to nodes along the optimal non-cooperative route, the total transmitted power for full- cooperation and m-cooperation in line networks can be obtained from (25) and (26), respectively. Thus, by replacing $\Psi(n)$ from (28) and $\sigma_{i,j}^2 = 1/|i-j|^2$, we have

$$P_T(\text{cooperative}) = \sum_{n=1}^{N+1} n\left(\frac{c\prod_{i=1}^{n}(2i-1)}{\mathrm{BER}_{\max}}\frac{\mathcal{N}_0^n\, n!}{2g^n}\right)^{\frac{1}{n}}, \tag{33}$$

$$P_T(m\text{-coop}) = \sum_{n=1}^{m} n\left(\frac{c\prod_{i=1}^{n}(2i-1)}{\mathrm{BER}_{\max}}\frac{\mathcal{N}_0^n\, n!}{2g^n}\right)^{\frac{1}{n}} + (N-m+1)m\left(\frac{c\prod_{i=1}^{m}(2i-1)}{\mathrm{BER}_{\max}}\frac{\mathcal{N}_0^m\, m!}{2g^m}\right)^{\frac{1}{m}}, \tag{34}$$

In Fig. 2, we compare the achieved energy savings of the proposed cooperative routing with respect to the non-cooperative multihop scenario, in which satisfying the $\mathrm{BER}_{\max}$ at each step is used as a performance criteria. For the $\mathrm{BER}_{\max} = 10^{-4}$, it can be observed that using the full cooperation scheme around 99% saving in energy is achieved when 4 relays are employed. Since the corresponding curve has an optimum performance when $N = 4$, we consider the $m = 5$ cooperation as an appropriate scheme. As it can be observed from Fig. 2, increasing the number of nodes in the network, 100% savings in energy is achievable. For the case of $\mathrm{BER}_{\max} = 10^{-4}$, the same characteristics can be seen.

Fig. 3 demonstrates a lower-bound on the obtainable energy saving in line networks, when the required BER at the destination, i.e., BER_d, should be satisfied. We use (31) to get a reliable power allocation at transmitting nodes to fulfil the required BER QoS at the destination node. For two cases of $\mathrm{BER}_d = 10^{-3}$ and $\mathrm{BER}_d = 10^{-4}$, vast amount of energy savings are obtainable. Since the maximum values of the curves corresponding to the full-cooperation routing occur when 5 and 8 relays are used, the 6-cooperation and 9-cooperation are used for $\mathrm{BER}_d = 10^{-3}$ and $\mathrm{BER}_d = 10^{-4}$ cases, respectively.

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