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# When Network Coding and Dirty Paper Coding Meet in a Cooperative Ad Hoc Network

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**Abstract**—We develop and analyze new cooperative strategies for ad hoc networks that are more spectrally efficient than classical Decode & Forward (DF) protocols. Using analog network coding, our strategies preserve the practical half-duplex assumption but relax the orthogonality constraint. The introduction of interference due to non-orthogonality is mitigated thanks to precoding, in particular Dirty Paper coding. Combined with smart power allocation, our cooperation strategies allow to save time and lead to a more efficient use of the bandwidth and to improved network throughput with respect to classical Repetition-DF and Parallel-DF.

**Index Terms**—Cooperative communications, network coding, dirty paper coding, precoding, ad hoc network.

## I. INTRODUCTION

COOPERATIVE communications occur when distributed wireless nodes interact to jointly transmit information. Several radio terminals relaying signals for each other form a virtual antenna array and their cooperation enables the exploitation of spatial diversity in fading channels. Several relaying strategies already exist, the simplest and most famous ones being [1] Amplify and Forward (AF) and Decode and Forward (DF) with repetition coding (RDF) or parallel channel coding (PDF). Since radio terminals cannot transmit and receive simultaneously in the same frequency band, most cooperative strategies are based on the half-duplex mode. When considering a three-node cooperative network, with a source  $S$ , a relay  $R$  and a destination  $D$ , each transmission is divided into two blocks: in the first block,  $S$  transmits and  $R$  and  $D$  receive; in the second block  $R$  relays and  $D$  receives. In some strategies  $S$  also transmits in the second block.

Now let us consider the four-node network in fig. (1) with two sources  $S_1$  and  $S_2$  transmitting in a cooperative fashion to two destinations  $D_1$  and  $D_2$  as in [1]. The previous transmission scheme is repeated twice, first for the relay channel  $S_1 - S_2 - D_1$  and second for the relay channel  $S_2 - S_1 - D_2$  as described in fig. 2 (b), resulting in a four-block transmission. The use of orthogonal interference free channels for sources and relays transmissions simplifies receivers algorithms but results in a loss of bandwidth.

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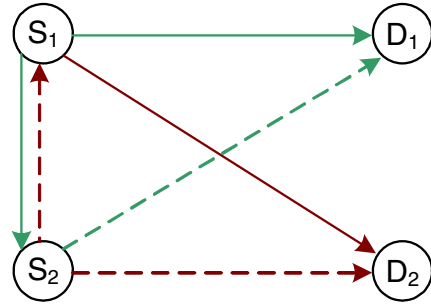


Fig. 1. A four node network with 2 cooperating sources and 2 destinations.

## A. The Idea in Brief

Loss of bandwidth issues have been tackled at higher layers thanks to network coding (NC). Packets arriving at a node on any edge of a network are put into a single buffer. At each transmission opportunity, an output packet is generated as a random linear combination of packets in the buffer within "current" generation [2]. Inspired by network coding, consider a four-node cooperative network using "network precoding" in a two-block transmission scheme, where in each single block one source simultaneously transmits and relays as in fig. 2 (c):

- **first block** :  $S_1$  sends a single signal  $f_1(s_1(n), s_2(n-1))$  which is a function of both its own message  $s_1(n)$  and a message  $s_2(n-1)$  received, decoded and re-encoded by  $S_1$  in the second block of previous transmission (repetition of the codeword - RDF - or use of an independent codeword -PDF), now relayed for  $S_2$ .  $S_2$ ,  $D_1$  and  $D_2$  receive. Since  $S_2$  knows the message in  $s_2(n-1)$ , it can extract  $s_1(n)$ , if it also knows the mixing function  $f_1$ .
- **second block** :  $S_2$  sends a single signal  $f_2(s_2(n), s_1(n))$  which is a function of both its own message  $s_2(n)$  and a message  $s_1(n)$  received, decoded and re-encoded by  $S_2$  in the first block of the current transmission, now relayed for  $S_1$ .  $S_1$ ,  $D_1$  and  $D_2$  receive. Since  $S_1$  knows the message in  $s_1(n)$ , it can extract  $s_2(n)$ , if it also knows  $f_2$ .

Functions  $f_1$  and  $f_2$  are the network precoding functions which help improving communication in terms of bandwidth. Knowing  $f_1$  and  $f_2$  allows sources  $S_2$  and  $S_1$  to easily cancel interference and extract the message they will have to relay in next block. But unfortunately, bandwidth usage improvements have a cost: the introduction of interference at destinations  $D_1$  and  $D_2$ . In the first block,  $s_2(n-1)$  is intended to  $D_2$  as relayed signal and acts as interference for  $D_1$ , which is only interested in  $s_1(n)$ ; reciprocally,  $s_1(n)$ , intended to  $D_1$ , generates interference for  $D_2$  interested in  $s_2(n-1)$ .

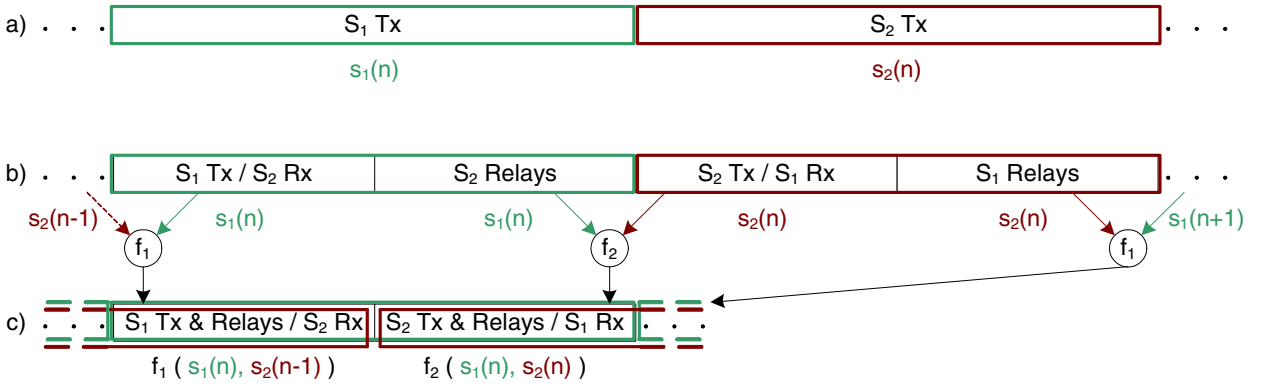


Fig. 2. Time division channel allocations for (a) orthogonal direct transmissions, (b) usual orthogonal cooperative transmissions (c) proposed scheme : analog network coding cooperative transmissions.

A similar interference problem occurs in the second block. Nevertheless, interference is known at transmitter, thus one can design the precoding functions to take into account this issue. In particular Dirty Paper Coding (DPC) [3], a well-known coding technique to mitigate interference known at transmitter, may help NC. We may expect DPC-like network precoding to help improving bandwidth efficiency in a cooperative network as well as mitigating interference, thus enhancing performance with respect to usual cooperative schemes.

### B. Related Work

Works in [4]–[6] proposed several cooperative strategies but considered a common destination and did not address interference mitigation issues arising in multi-source multi-destination cooperative ad hoc systems. DPC was also considered in relay networks, eg. in [7]–[10], as joint coding between cooperating pairs, or to mitigate interference at relay.

Analog network coding at the physical layer was proposed in [11] with power allocation, interference mitigation thanks to DPC and results on the total network throughput, nevertheless the full analysis is presented in this paper. Recently [12] studied AF with analog network coding and showed that joint relaying and network coding can enhance the network throughput.

Our main contribution is to bring network coding, in an analog way, at the physical layer, to provide novel cooperative protocols using analog network coding and to analyze their performance in terms of the network throughput and outage behavior. Thanks to analog Network Coding combined with Dirty Paper precoding, time is saved compared to classical DF protocols, interference resulting from non-orthogonality is mitigated, leading to a better use of resources and improved spectral efficiency. Analysis show that our cooperative strategies clearly outperform classical orthogonal DF protocols.

### C. Outline

The rest of the paper is organized as follows. In section II, notations and the system model are presented. In section III, cooperative precoding methods are described whereas the performance criteria are derived in section IV. Numerical results and comparison with other cooperative protocols are provided in section V and lead to the concluding section VI.

## II. SYSTEM MODEL

Given  $i \in \{1, 2\}$ ,  $\bar{i}$  denotes the complementary integer in the ensemble  $\{1, 2\}$ , e.g. if  $i = 1$ ,  $\bar{i} = 2$ . Matrices and vectors are represented by boldface uppercase.  $\mathbf{A}^T$ ,  $\mathbf{A}^*$ ,  $\mathbf{A}^H$  denote the transpose, the conjugate and the conjugate transpose of matrix  $\mathbf{A}$ .  $\text{tr}(\mathbf{A})$ ,  $\det(\mathbf{A})$  and  $\|\mathbf{A}\|_F = \sqrt{\text{tr}(\mathbf{A}\mathbf{A}^H)}$  stand for trace, determinant and Fröbenius norm of  $\mathbf{A}$ .  $\mathbf{I}_N$  is the identity matrix of size  $N$ .

To capture the gain resulting from the NC approach, we consider that all terminals are equipped with a single antenna. Consider the four node network in fig. 1. Each source  $S_i$ ,  $i \in \{1, 2\}$  generates a sequence  $s_i(n)$ ,  $n \in \{1, \dots, N\}$ . These symbols are modeled by independent identically distributed (i.i.d.) circularly-symmetric complex gaussian random variables, with zero mean and variance  $\varepsilon_s = \mathbb{E}[|s_i(n)|^2]$ . With a transmission bandwidth  $W$ , there are  $W$  complex symbols per second. At time  $t = k/W$ ,  $k \in \mathbb{N}$ , the signal transmitted by  $S_i$  is denoted  $x_i(k)$  whereas  $y_{S_i}(k)$  and  $y_{D_j}(k)$  represent the signals received by source  $S_i$  and destination  $D_j$  respectively, with  $i, j \in \{1, 2\}$ . Finally  $f_i$  represents the network coding function performed at  $S_i$ . Those functions can be of any kind, not necessarily linear. Nevertheless, in this paper developing a network coding approach for cooperative ad hoc networks, we focus first on functions performing a linear operation on symbols  $s_1$  and  $s_2$ , to simplify analysis and detection at destinations. Then a DPC approach is considered and shown to outperform the other strategies.

As described in section I and figure 2 (c), NC cooperative communication divides each transmission into two blocks.

- **First block** at even time indexes  $k = 2n$ , signals transmitted by  $S_1$  and received by other terminals are:

$$\begin{aligned} x_1(2n) &= f_1(s_1(n), s_2(n-1)) \\ y_{S_2}(2n) &= h_{S_2 S_1} x_1(2n) + z_{S_2}(2n) \\ y_{D_j}(2n) &= h_{D_j S_1} x_1(2n) + z_{D_j}(2n), j \in \{1, 2\} \end{aligned}$$

- **Second block** at odd time indexes  $k = 2n+1$ , signals transmitted by  $S_2$  and received by other terminals are:

$$\begin{aligned} x_2(2n+1) &= f_2(s_1(n), s_2(n)) \\ y_{S_1}(2n+1) &= h_{S_1 S_2} x_2(2n+1) + z_{S_1}(2n+1) \\ y_{D_j}(2n+1) &= h_{D_j S_2} x_2(2n+1) + z_{D_j}(2n+1), j \in \{1, 2\} \end{aligned}$$

The channel between transmitter  $u \in \{S_1, S_2\}$  and receiver  $v \in \{S_1, S_2, D_1, D_2\}$  is represented by  $h_{vu}$  which includes the effects of path-loss, shadowing and slow flat fading. These channel coefficients are modeled by independent circularly-symmetric complex gaussian random variables with zero mean and variance  $\sigma_{vu}^2$ , i.e. Rayleigh fading.  $z_v(k)$  are i.i.d circularly-symmetric complex gaussian noises at receivers, with variance  $\sigma^2$ . Each source has a power constraint in the continuous time-channel of P Joules/s and transmits only half of the time, both in orthogonal interference-free cooperation schemes and in the proposed NC cooperation schemes. Thus the power constraint translates into  $P_i = \mathbb{E}[|x_i(n)|^2] \leq \frac{2P}{W}$ . Since a source transmits only part of time, it can increase its transmit power in its transmission block and remain within its average power constraint for the whole transmission. Finally, each destination is assumed to have perfect CSI of its two incoming channels from sources, whereas sources are assumed to have knowledge of the amplitudes of all channels and perfect CSI of the source-source channel.

### III. PRECODING METHOD

#### A. Linear Precoding

In Linear Network Coding for RDF,  $S_1$  detects  $s_2(n-1)$  in the signal transmitted by  $S_2$  and re-encodes it using the same codeword. Then  $S_1$  forms its transmitted signal  $x_1(n)$  as a linear combination of its own codeword  $s_1(n)$  and the repeated  $s_2(n-1)$ . The same process happens at  $S_2$ . Therefore function  $f_i$  can be represented by a matrix  $\mathbf{F}_i$  of size  $N_t \times N_s$ , i.e. (number of transmit antennas at source) times (number of symbols on which  $f_i$  acts). In the single antenna scenario,  $\mathbf{F}_i = [f_{i1}, f_{i2}]$  is a row of size 2. Transmitted signals are thus:

$$\begin{aligned} x_1(2n) &= \mathbf{F}_1 [s_1(n), s_2(n-1)]^T = f_{11}s_1(n) + f_{12}s_2(n-1) \\ x_2(2n+1) &= \mathbf{F}_2 [s_1(n), s_2(n)]^T = f_{21}s_1(n) + f_{22}s_2(n) \end{aligned}$$

In Linear NC cooperation scheme, the power constraint becomes  $P_i = \varepsilon_s \|\mathbf{F}_i\|_F^2 \leq \frac{2P}{W}$ . We will consider precoding functions such that  $\|\mathbf{F}_i\|_F^2 = 1$ , i.e.  $f_i$  does not increase the power transmitted by source  $S_i$  but shares it between the source message and the relayed message.

**Remark :** orthogonal TDMA transmissions without relaying can be seen as a particular case of network coding where  $\mathbf{F}_1 = [1, 0]$  and  $\mathbf{F}_2 = [0, 1]$ . Orthogonal interference-free cooperation [1] is also a particular case of our scheme where  $\mathbf{F}_1 = [1, 0]$  and  $\mathbf{F}_2 = [1, 0]$  during two blocks, and then  $\mathbf{F}_2 = [0, 1]$  and  $\mathbf{F}_1 = [0, 1]$  during the next two blocks.

#### B. Dirty Paper Precoding

Since interference resulting from NC approach is known at the transmitter, more advanced NC functions can include decoding and re-encoding with DPC of messages intended to different destinations [13]. In Dirty Paper NC for PDF,  $S_1$  decodes the message carried by  $s_2(n-1)$  and re-encodes it using an independent Gaussian codebook. More precisely, in order to use dirty paper coding,  $S_1$  first orders destinations based on channel knowledge. Then  $S_1$  picks a codeword for the first destination, before choosing a codeword for the second destination, with full non-causal knowledge of

the codeword intended to first destination. Thus the second destination does not see interference due to the codeword for the first destination, whereas the first destination will see the signal intended to the second destination as interference. The signal transmitted by  $S_1$  is the sum of the two codewords, with power sharing across the two codewords taking into account channel knowledge.  $S_2$  will proceed the same way in the following block. The ordering of destinations chosen at each source affects performances. Transmitted signals thus become:

$$\begin{aligned} x_1(2n) &= f_{11}s_1(n) + f_{12}s'_2(n-1) \\ x_2(2n+1) &= f_{21}s'_1(n) + f_{22}s_2(n) \end{aligned}$$

where  $f_{ij}^2$  stands for the power allocated by source  $S_i$  to the codeword intended to destination  $D_j$ , and  $s'_j$  is the independent codeword produced by a source acting as relay after decoding the message carried by  $s_j$ . Destinations are assumed to know the orderings (each source can with a single bit indicate the ordering it selected).

### IV. PERFORMANCE ANALYSIS

Average rate, per user and network throughputs as well as outage behavior are analyzed in slow fading channels.

#### A. Orthogonal interference-free RDF and PDF

For cooperative channels in fig. 2 (b), using RDF the mutual information between input  $s_1$  and output  $y_{D_1}$  at  $D_1$  is [1]:

$$I_{RDF}(s_1; y_{D_1}) = \frac{1}{2} \min \{ \log(1 + \rho |h_{S_2 S_1}|^2), \log(1 + \rho |h_{D_1 S_1}|^2 + \rho |h_{D_1 S_2}|^2) \} \quad (1)$$

where the input SNR is  $\rho = \varepsilon_s / \sigma^2 = 2P / (W\sigma^2)$ , and the factor 1/2 is due to the two channel-uses to send a message. Mutual information  $I_{RDF}(s_2; y_{D_2})$  between input  $s_2$  and output  $y_{D_2}$  at  $D_2$  is given similarly. Half the degrees of freedom are allocated for transmission to a destination - each destination is passive half of the time when the signals transmitted do not contain information intended to that destination- therefore the throughput of the first user is  $\frac{1}{2} I_{RDF}(s_1; y_{D_1})$  and the total network throughput using RDF is:

$$C_{RDF} = \frac{1}{2} I_{RDF}(s_1; y_{D_1}) + \frac{1}{2} I_{RDF}(s_2; y_{D_2}) \quad (2)$$

The outage probability is defined as in [1]:

$$P_{RDF}^{out}(\rho, R) = \Pr[I_{RDF} < R], \text{ with } R = \frac{r}{W/2} \text{ in b/s/Hz} \quad (3)$$

where the spectral efficiency  $R$  is by definition the ratio between rate  $r$  in bits per second and the number of degrees of freedom utilized by each terminal [1].

Using PDF, mutual information between  $s_1$  and  $y_{D_1}$  is [14]:

$$I_{PDF}(s_1; y_{D_1}) = \frac{1}{2} \min \{ \log(1 + \rho |h_{S_2 S_1}|^2), \log(1 + \rho |h_{D_1 S_1}|^2) + \log(1 + \rho |h_{D_1 S_2}|^2) \} \quad (4)$$

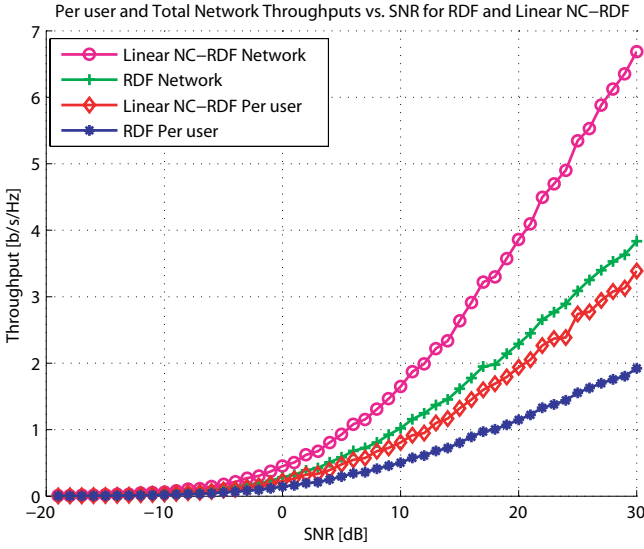


Fig. 3. Comparison of Per user and Network Throughputs of classical RDF and LNC cooperative methods.

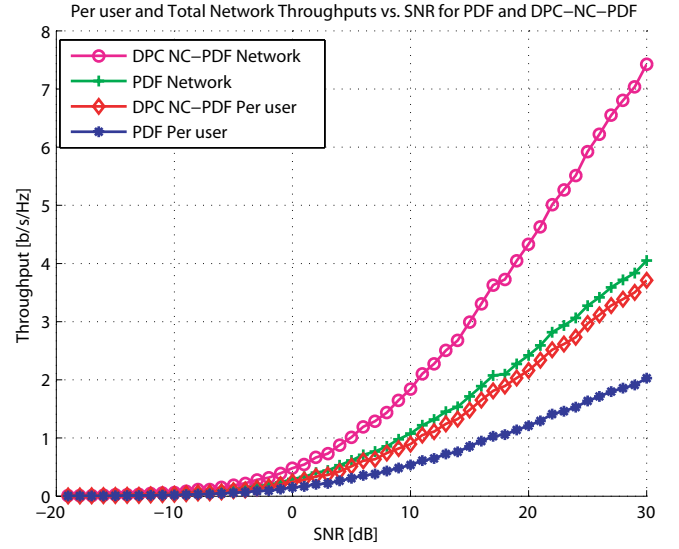


Fig. 4. Comparison of Per user and Network Throughputs of classical PDF and NC-DPC cooperative methods.

Mutual information  $I_{PDF}(s_2; y_{D_2})$  at  $D_2$  is also given by a similar formula [14]. The total network throughput of PDF is given by:

$$C_{PDF} = \frac{1}{2} I_{PDF}(s_1; y_{D_1}) + \frac{1}{2} I_{PDF}(s_2; y_{D_2}) \quad (5)$$

and the outage probability for a source-destination pair is:

$$P_{PDF}^{out}(\rho, R) = Pr[I_{PDF} < R] \quad (6)$$

### B. Linear NC RDF

For our proposed network coding cooperative scheme in figure 2 (c), when the network coding functions are linear transformations, mutual information between input  $s_1$  and output  $y_{D_1}$  at destination  $D_1$  can be shown to be:

$$I_{LNC}(s_1; y_{D_1}) = \frac{1}{2} \min \left\{ \log(1 + \rho |h_{S_2 S_1} f_{11}|^2), \log \left( 1 + \rho \frac{|h_{D_1 S_1} f_{11}|^2}{1 + \rho |h_{D_1 S_1} f_{12}|^2} + \rho \frac{|h_{D_1 S_2} f_{21}|^2}{1 + \rho |h_{D_1 S_2} f_{22}|^2} \right) \right\} \quad (7)$$

In the minimum in equation (7), the first term represents the maximum rate at which relay  $S_2$  can decode the source message  $s_1$  after canceling the interference known at the relay (interference is due to the symbol  $s_2$  the relay emitted previously), whereas the second term represents the maximum rate at which destination  $D_1$  can decode given the transmissions from source  $S_1$  and relay  $S_2$ .

A similar formula gives the mutual information between input  $s_2$  and output  $y_{D_2}$  at destination  $D_2$ , with appropriate changes:

$$I_{LNC}(s_2; y_{D_2}) = \frac{1}{2} \min \left\{ \log(1 + \rho |h_{S_1 S_2} f_{22}|^2), \log \left( 1 + \rho \frac{|h_{D_2 S_2} f_{22}|^2}{1 + \rho |h_{D_2 S_2} f_{21}|^2} + \rho \frac{|h_{D_2 S_1} f_{12}|^2}{1 + \rho |h_{D_2 S_1} f_{11}|^2} \right) \right\} \quad (8)$$

With Network Coding, all degrees of freedom are used for transmission to each destination. No time is wasted from the destination point of view, thus the throughput of the first user is  $I_{LNC}(s_1; y_{D_1})$  and the total network throughput for this strategy is :

$$C_{LNC} = \max_{\{f_{ij}\}_{i,j \in \{1,2\}}} I_{LNC}(s_1; y_{D_1}) + I_{LNC}(s_2; y_{D_2}) \quad (9)$$

$$\begin{aligned} |f_{11}|^2 + |f_{12}|^2 &\leq 1 \\ |f_{21}|^2 + |f_{22}|^2 &\leq 1 \end{aligned}$$

The optimization problem turns out to be a non-convex problem, both for LNC and for DPC in next section, so that classical convex optimization techniques cannot be used to find a closed-form expression of the power allocation scheme. Moreover, because of limitations due to the quality of the source-relay link, MAC-BC duality [15] cannot be used to solve the optimization problem as in non-cooperative systems. Finding the optimal power allocation scheme between transmitted and relayed signals at each source is different from BC power allocation problem, because power terms  $f_{11}^2$  and  $f_{22}^2$  appear in the capacity of the links between the two sources, first terms in the minimums in formulas (7), (8), (11), so that the power allocation scheme maximizing the sum-rates of the two BC channels between a source and the two destinations may not be the same as the one maximizing the sum-rate of the cooperative system.

Since all degrees of freedom are used by each terminal, the outage probability for a pair is:

$$P_{LNC}^{out}(\rho, R') = Pr[I_{LNC} < R'] , \text{ with } R' = \frac{r}{W} \text{ in b/s/Hz} \quad (10)$$

### C. DPC NC PDF

The mutual information between a source message and the received signals at the intended destination depends on the two orderings  $\Pi_1, \Pi_2$  of destinations for DPC chosen by both sources. Knowing all channel amplitudes, each source can compute alone the DPC orderings maximizing the network



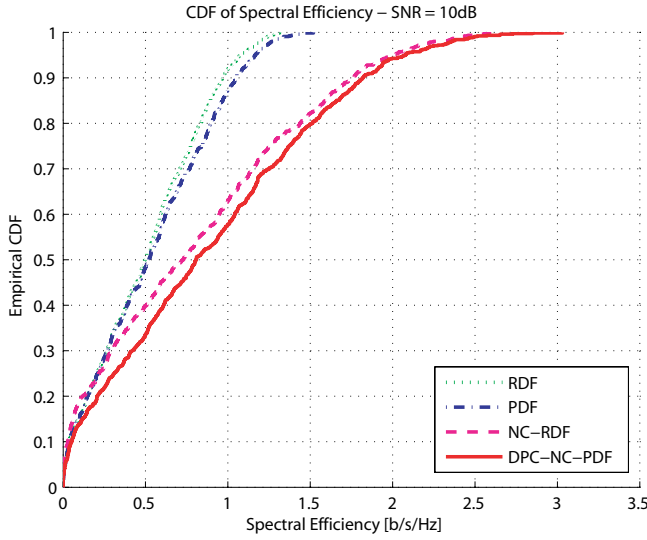


Fig. 5. CDF of Spectral Efficiency - SNR = 10 dB.

throughput. Since a relay uses an independent codeword to re-encode the signal it received from the previous source, the total network throughput for this cooperation scheme belonging to the family of PDF can be written :

$$C_{DPC} = \max_{\Pi_1, \Pi_2, \{f_{ij}\}_{i,j \in \{1,2\}} \atop |f_{11}|^2 + |f_{12}|^2 \leq 1 \atop |f_{21}|^2 + |f_{22}|^2 \leq 1} I_{DPC}(s_1; y_{D_1}) + I_{DPC}(s_2; y_{D_2})$$

with :

$$I_{DPC}(s_1; y_{D_1}) = \frac{1}{2} \min \left\{ \log(1 + \rho |h_{S_2 S_1} f_{11}|^2), \log(1 + SINR_{11}) + \log(1 + SINR_{21}) \right\}$$

$$I_{DPC}(s_2; y_{D_2}) = \frac{1}{2} \min \left\{ \log(1 + \rho |h_{S_1 S_2} f_{22}|^2), \log(1 + SINR_{12}) + \log(1 + SINR_{22}) \right\} \quad (11)$$

where  $SINR_{ij}$  is the Signal-to-Interference plus Noise ratio resulting from the signal transmitted by  $S_i$  at  $D_j$ :

$$SINR_{ij} = \begin{cases} \rho |h_{D_j S_i} f_{ij}|^2, & \text{if } S_i \text{ does DPC in favor of } D_j \\ \frac{\rho |h_{D_j S_i} f_{ij}|^2}{1 + \rho |h_{D_j S_i} f_{i\bar{j}}|^2}, & \text{if } S_i \text{ does DPC in favor of } D_{\bar{j}} \end{cases}$$

The outage probability is defined as

$$P_{DPC}^{out}(\rho, R') = Pr[I_{DPC} < R'] \quad (12)$$

## V. NUMERICAL RESULTS

In this section, numerical results are presented to compare the different cooperation strategies. Fig. (3) and (4) illustrate average per user throughput and total network throughput obtained through Monte Carlo Simulations (1000 channel realizations), in the case of symmetric networks, i.e. where the fading variances are identical  $\sigma_{vu}^2 = 1$ . Optimal power allocations and orderings  $\Pi_i$  were obtained numerically by exhaustive search. Average individual throughput and outage probability are the same for both users, since they are assumed to have the same power constraints and the network is symmetric. Fig. (5) and (6) show the outage behavior of the different strategies.

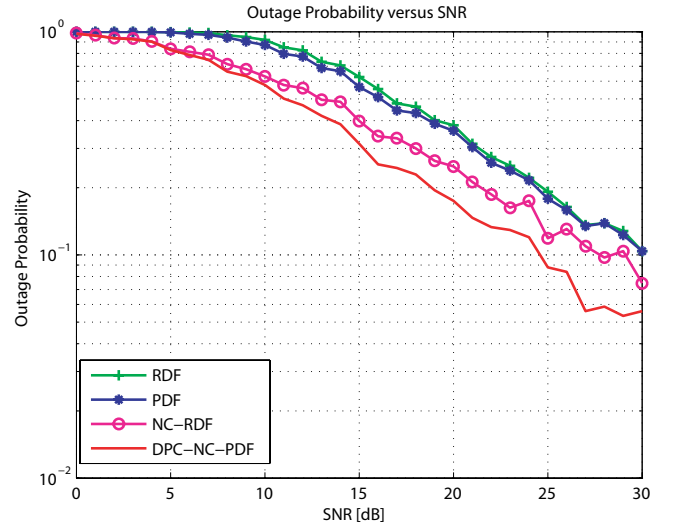


Fig. 6. Outage Probabilities versus SNR.

### A. Average Throughputs

Fig. (3) compares RDF [1] and LNC for RDF that we propose, and shows that our technique based on Linear Network coding performs much better in terms of per user throughput, thanks to a more efficient use of spectral resources as well as power resources. Fig. (4) plots the per user throughputs for PDF [1] and our DPC-NC for PDF. Once again, the NC based strategy enhances performance in terms of individual throughput.

Finally fig. (3) and (4) also allow to compare the total network throughput of all techniques, and show neat improvements in the network performance thanks to NC methods. Thanks to smart power sharing between own and relayed signals, even with repetition coding, and increased spectral efficiency, Linear NC enhances considerably performance compared to classical RDF and PDF. Using a more advanced coding technique, DPC, to mitigate interferences generated at destination by the NC methods leads to even better results.

### B. Outage Behavior

Fig. (5) plots the cumulative distribution functions of the per user throughputs. Indeed

$$P_{RDF}^{out}(\rho, R) = Pr[I_{RDF} < R] = Pr[I_{RDF}/2 < R']$$

Recalling that  $I_{RDF}/2$  is the per user throughput, analyzing the outage behavior of the different strategies for a target rate  $r$  is equivalent to comparing the CDF of the per user throughputs for a rate value  $R'$ . A neat improvement in the outage probability is visible in fig. (5) when using network coding cooperation. Fig. 6 shows the outage probabilities (3), (6), (10) and (12), versus the SNR for the various strategies, and a target rate  $r = 1b/s$ . They illustrate in particular the large energy savings that NC based cooperative strategies allow to reach a target rate.

## VI. CONCLUSION

Inspired by network coding, we proposed new cooperative strategies for ad hoc networks, which improve spectral efficiency of the cooperative system by relaxing the orthogonality

constraint, though preserving the practical half-duplex constraint. The introduction of interferences between source and relayed messages, when considering non-orthogonal transmission scheme, is mitigated thanks to precoding at transmitter. We presented two precoding approaches, linear NC with RDF and Dirty-Paper NC with PDF, relevant technique since the transmitter knows the interference. Thanks to precoding, linear or Dirty Paper based, the cost of the NC approach - introduction of interference - is less than the resulting gain in terms of spectral efficiency and performance analysis shows great improvements in terms of throughput over classical RDF/PDF cooperative strategies. Future work may include solving the optimization in particular scenarios, development of a selective strategy to circumvent limitations due to link source-relay, extension to multiple-antenna terminals, in particular assessing how beamforming can improve performances, and last but not least extension to a large network with several source-destination pairs.

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