# Implicit cooperation in distributed energy-efficient networks

Mael Le Treust, Samson Lasaulce, Mérouane Debbah

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***Abstract*—We consider the problem of cooperation in distributed wireless networks of selfish and free transmitters aiming at maximizing their energy-efficiency. The strategy of each transmitter consists in choosing his power control (PC) policy. Two scenarios are considered: the case where transmitters can update their power levels within time intervals less than the channel coherence time (fast PC) and the case where it is updated only once per time interval (slow PC). One of our objectives is to show how cooperation can be stimulated without assuming cooperation links between the transmitters but only by repeating the corresponding PC game and by signals from the receiver. In order to design efficient PC policies, standard and stochastic repeated games are respectively exploited to analyze the fast and slow PC problems. In the first case a cooperation plan between transmitters, that is both efficient and relies on mild information assumptions, is proposed. In the second case, the region of equilibrium utilities is derived from very recent and powerful results in game theory.**

## I. Introduction

In the wireless literature, when it is referred to cooperative networks, this generally means that some nodes in the network act as relays in order to help other nodes (the sources or transmitters) to better communicate with their respective destination or receiver nodes. This idea has been formalized in information theory in [1][2] for the relay channel, for the cooperative multiple access channel (MAC) [3], and for other types of cooperative channels during the last decade ([4][5] etc). The vast majority of these papers address centralized networks and cooperation between nodes is based on the existence of physical links between some nodes. In the present paper we consider the case of decentralized or distributed networks with implicit cooperation. By decentralized/distributed, we mean that the nodes are assumed to be free decision makers who decide by themselves what is good for them and can ignore possible recommandations from central nodes (namely the power control policy in our case). By implicit, we mean that nodes cooperate without using dedicated cooperation channels between nodes. To be more concrete, we consider multiple access channels where no link between the transmitters is assumed and transmitters are modeled by selfish players aiming at maximizing the energy-efficiency of their communication. A very simple and pragmatic way of knowing to what extent a communication is energy-efficient has been proposed by [6][7]. The authors of these articles define energy-efficiency as the net number of information bits that are transmitted without error per time unit (goodput) to the transmit power level. More specifically, the authors analyze the problem of distributed power control (PC) in flat fading multiple access channels. The problem is formulated as a non-cooperative one-shot game where the players are the transmitters, the strategy of a given player is his transmit power for a given channel realization, and his payoff/reward/utility function is energy-efficiency of his communication with the receiver. Unfortunately, the Nash equilibrium (NE) resulting from this game is generally inefficient.

In the papers on energy-efficient power control cited above and related papers (e.g., [8][9]), the used game-theoretic framework is the one of static or one-shot games for which transmitters are assumed to interact once per block and from block to block in an independent manner; the block duration is assumed to be less than the channel coherence time. In practice, there will be some scenarios where transmitters can update their power level several times within a block or/and are active over several and possibly many blocks. In game theory, it is well known that this feature can change the behavior of the players and incite them to cooperate while staying selfish [10]. The corresponding game-theoretic framework is then the one of dynamic games. In this paper we propose to model the distributed energy-efficient power control problem by exploiting two types of repeated games (RG), namely the standard and stochastic RG, which are special cases of dynamic games. In standard RG [11], the same game is repeated a certain number of times. In stochastic RG [12], players' utilities depend on a certain state (or parameters) which vary over time according to a stochastic process. We use standard RG to analyze scenarios where transmitters can update their powers several times within each block (the same game is therefore repeated within a block) and stochastic RG for scenarios where transmitters update their powers once per block (the game is therefore parameterized by the channel state and is repeated from block to block). We will respectively refer to these scenarios as fast and slow power control (FPC, SPC).

The contributions of this paper are as follows: 1. The framework of repeated games is applied for the first time to the distributed energy-efficient power control problem; 2. In the case of FPC, we derive equilibrium PC strategies which are based on a cooperation plan between the transmitters, Pareto-efficient, and only require individual channel state information (CSI) at the transmitters and a public signal to be implemented; 3. In the case of SPC, which is much more difficult to treat properly, only the set of possible equilibrium utilities of the stochastic RG (which can be seen as a counterpart of a capacity region of a distributed channel when Shannon transmission

M. Le Treust, S. Lasaulce are with LSS-CNRS-Supelec-Université de Paris 11 ; 3, rue Joliot-Curie, 91190 Gif-sur-Yvette, France and M. Debbah is with Chaire Alcatel-Supelec ; 3, rue Joliot-Curie, 91190 Gif-sur-Yvette, France

rate is considered for the utilities) is derived by exploiting a very recent result in game theory derived by Hörner et al. [13] and Fudenberg and Yamamoto [14]. To achieve this utility region, global CSI and a public signal are assumed at the transmitters. The determination of the equilibrium strategies is left as a non-trivial extension of this work.

## II. Signal model

We consider a distributed MAC with a finite number of users, which is denoted by $K$. The network is said to be distributed in the sense that the receiver (e.g., a base station) does not dictate to the transmitters (e.g., mobile stations) their PC policy. Rather, all the transmitters chooses their policy by themselves and want to selfishly maximize their energy-efficiency; in particular they can ignore some specified centralized policies. We assume that the users transmit their data over block flat fading channels. The equivalent baseband signal received by the base station can be written as

$$y = \sum_{i=1}^{K} g_i x_i + z \tag{1}$$

with $i \in \mathcal{K}$, $\mathcal{K} = \{1, ..., K\}$, $\mathbb{E}|x_i|^2 = p_i$, $z \sim \mathbb{C}\mathcal{N}(0, \sigma^2)$. Each channel gain $g_i$ varies over time following a Markov chain and is assumed to be constant over each block. For each transmitter $i$, the channel gain $g_i$ is assumed to lie in a discrete set (e.g., because channel chains are quantized). The notation $\eta_i = |g_i|^2$, with $\eta_i \in \Gamma_i$, $|\Gamma_i| < +\infty$, will be used. For the transmit power levels $p_i$ they will be assumed to lie in a compact set $\mathcal{P}_i = [0, P_i^{\max}]$ in Sec. III and in a discrete set $\mathcal{P}_i = \left\{P_i^1, ..., P_i^M\right\}$, with $P_i^M = P_i^{\max}$, in Sec. IV. The discrete set assumption is suited to exploit the results of [13][14] without introducing additional technicalities we wanted to avoid in this (relatively short) paper. At last, the receiver is assumed to implement single-user decoding.

## III. Fast power control and standard repeated games

### A. Review of the one-shot power conrol game

Here we review a few key results from [7] concerning the static PC game. We denote by $R_i$ the transmission information rate (in bps) for user $i$ and $f$ an efficiency function representing the block success rate, which is assumed to be sigmoidal and identical for all the users. For a given block, the signal-to-interference plus noise ratio (SINR) at receiver $i \in \{1, ..., K\}$ is denoted by $\mathrm{SINR}_i$ and writes as:

$$\mathrm{SINR}_i = \frac{p_i \eta_i}{\sum_{j \in \mathcal{K} \setminus i} p_j \eta_j + \sigma^2} \tag{2}$$

where $p_i \in [0, P_i^{\max}]$. With these notations, the static PC game, denoted by $\mathcal{G}$, is defined in its normal form as follows.

*Definition 1 (Static PC game): The static PC game is a triplet $\mathcal{G} = (\mathcal{K}, \{\mathcal{P}_i\}_{i \in \mathcal{K}}, \{u_i\}_{i \in \mathcal{K}})$ where $\mathcal{K} = \{1, ..., K\}$ is the set of players, $\mathcal{P}_1, ..., \mathcal{P}_K$ are the corresponding discrete sets of strategies, $\mathcal{P}_i = \{0, \ldots, P_i^{\max}\}$, $P_i^{\max}$ is the maximum transmit power for player $i$, and $u_1, ..., u_k$ are the utilities of the different players which are defined by:*

$$u_i(p_1, ..., p_K) = \frac{R_i f(\mathrm{SINR}_i)}{p_i} \ [\mathrm{bit/J}]. \tag{3}$$

We suppose from now that the above description of the game is common knowledge and the players are rational (every player does the best for himself and knows the others do so and so on). An important game solution concept is the Nash equilibrium (i.e., a point from which no player has interest in unilaterally deviating). When it exists, the non-saturated Nash equilibrium of this game is given by

$$\forall i \in \{1, ..., K\}, \ p_i^* = \frac{\sigma^2}{\eta_i} \frac{\beta^*}{1 - (K-1)\beta^*} \tag{4}$$

where $\beta^*$ is the unique solution of the equation $xf'(x) - f(x) = 0$. By using the term "non-saturated NE" we mean that the maximum transmit power for each user, denoted by $P_i^{\max}$, is assumed to be sufficiently high for not being reached at the equilibrium i.e., each user maximizes his energy-efficiency for a value less than $P_i^{\max}$ (see [9] for more technical details about this assumption). An important property of the NE given by (4) is that transmitters only need to know their individual channel gain (i.e., $g_i$) to play their equilibrium strategy. One of the interesting results we want to prove is that it is possible to obtain a more efficient equilibrium point when transmitters can update their powers several times per block while keeping this key information property of individual CSI.

### B. The discounted repeated power control game

In this section, we assume that the transmitters can update their powers within time intervals less than the channel coherence time. The instants at which the transmitters update their powers are called game stages. Therefore, for each channel realization $\underline{g} = (g_1, ..., g_K)$ a given repeated game is played. As mentioned in Sec. I, the fact that the PC game is repeated induces new behaviors (namely cooperative behaviors) for the transmitters. Because of repetition, selfish but efficient agreements between transmitters are possible. In this work, we propose an operating point (OP) of the one-shot PC game which can serve as a part of a cooperation plan between the transmitters. Before defining the repeated power control game, let define the proposed OP.

By considering all the points $(p_1, ..., p_K)$ such that $p_i \in [0, P_i^{\max}]$, $i \in \mathcal{K}$, one obtains the feasible utility region. We consider a subset of points of this region for which the power profiles $(p_1, ..., p_K)$ verify $p_i|g_i|^2 = p_j|g_j|^2$ for all $(i, j) \in \mathcal{K}^2$. Such a subset is made of the following system of equations:

$$\forall (i, j) \in \mathcal{K}^2, \ \frac{\partial u_i}{\partial p_i}(\underline{p}) = 0 \ \text{ with } p_i|g_i|^2 = p_j|g_j|^2. \tag{5}$$

It turns out that, following the lines of the proof of SE uniqueness in [9], it is easy to show that a sufficient condition for ensuring both existence and uniqueness of the solution to this system of equations is that there exists $x_0 \in ]0, \frac{1}{K-1}[$ such that $\frac{f''(x)}{f'(x)} - \frac{2(K-1)}{1-(K-1)x}$ is strictly positive on $]0, x_0[$

and strictly negative on $]x_0, \frac{1}{K-1}[$. It is satisfied for the two efficiency functions the authors are aware of, which are: $f(x) = (1-e^{-x})^M$ [6] and $f(x) = e^{-\frac{c}{x}}$ [15] with $c = 2^R - 1$ ($R$ is the transmission rate). Under the aforementioned condition, the unique solution of (5) can be checked to be:

$$\forall i \in \mathcal{K},\ p_i^{\mathrm{OP}} = \frac{\sigma^2}{\eta_i} \frac{\gamma^*}{1-(K-1)\gamma^*} \tag{6}$$

where $\gamma^*$ is the unique solution of $x[1-(K-1)\cdot x]f'(x) - f(x) = 0$. The proposed OP, given by (6), is thus fair in the sense of the SINR since $\forall i \in \mathcal{K}$, $\mathrm{SINR}_i = \gamma^*$. We are going to exploit this point of the one-shot PC game to build equilibrium strategies of the DRG.

Let us define a strategy for the discounted repeated game. The transmitters are assumed to receive a public signal $s(t)$ after playing at game stage $t$ and keep this in memory. This public signal is linked to the actions of the transmitters by an observation function $\phi : \mathcal{P}_1 \times ... \times \mathcal{P}_K \longrightarrow \mathcal{S}$.

*Definition 2 (Players' strategies in the RG): A pure strategy for player $i \in \mathcal{K}$ is a sequence of causal functions $(\tau_{i,t})_{t\geq 1}$ with*

$$\tau_{i,t} : \left| \begin{array}{ccc} \mathcal{H}_t & \rightarrow & [0, P_i^{\max}] \\ \underline{h}_t & \mapsto & p_i(t) \end{array} \right. \tag{7}$$

*where $t$ is the game stage index, $\underline{h}_t = (s(1), ..., s(t-1))$ is the game history vector and $\mathcal{H}_t = \mathcal{S}^{t-1}$.*

The strategy of player $i$, which is a sequence of functions, will be denoted by $\tau_i$. The vector of strategies $\underline{\tau} = (\tau_1, ..., \tau_K)$ will be referred to a joint strategy. A joint strategy $\underline{\tau}$ induces in a natural way a unique action plan $(\underline{p}(t))_{t\geq 1}$. To each profile of powers $\underline{p}(t)$ corresponds a certain instantaneous utility $u_i(\underline{p}(t))$ for player $i$. In our setup, each player does not care about what he gets at a given stage but what he gets over the whole duration of the game. This is why we consider a utility function resulting from averaging over the instantaneous utility.

*Definition 3 (Players' utilities in the RG): Let $\underline{\tau} = (\tau_1, ..., \tau_K)$ be a joint strategy. The utility for player $i \in \mathcal{K}$ is defined by:*

$$v_i^{\lambda}(\underline{\tau}) = \sum_{t=1}^{\infty} \lambda(1-\lambda)^{t-1} u_i(\underline{p}(t)) \tag{8}$$

*where $\underline{p}(t)$ is the power profile of the action plan induced by the joint strategy $\underline{\tau}$ and $0 < \lambda < 1$ is a parameter of the DRG called the discount factor and is known to every player (since the game is with complete information).*

In the current available wireless literature on the problem under investigation discounted repeated games (DRG) are used as follows: in [16] the discount factor is used as a way of accounting for the delay sensitivity of the network; in [17] the discount factor is used to let the transmitters the possibility to value short-term and long-term gains differently. Interestingly, [11][12] offers another interpretation of this model. Indeed, the author sees the DRG as a finite RG where the number of game duration would be unknown to the players and considered as an integer-valued random variable, finite almost surely, whose law is known by the players. Otherwise said, $\lambda$ can be seen as the stopping probability at each game stage: the probability that the game stops at stage $t$ is thus $\lambda(1-\lambda)^{t-1}$. The function $v_i^{\lambda}$ would correspond to an expected utility given the law of the game duration. This shows that the discount factor is also useful to study wireless games where a player enters/leaves the game.

*Theorem 4 (Equilibrium strategies in the DRG): Assume that the following condition is met:*

$$\lambda \leq \frac{1-(K-1)\gamma^*}{(K-1)\gamma^*} \frac{f(\gamma^*)}{f(\beta^*)} - \frac{1-(K-1)\beta^*}{(K-1)\beta^*}. \tag{9}$$

*Then, for all $i \in \mathcal{K}$, the following action plan is a subgame perfect NE of the DRG for any distribution for the channel gains:*

$$\forall t \geq 1,\ \tau_{i,t} = \left| \begin{array}{ll} p_i^{\mathrm{OP}} & \textit{if the other players play } \tilde{p}_{-i} \\ p_i^* & \textit{otherwise} \end{array} \right. . \tag{10}$$

The proof of this theorem is not provided here; the main idea of the proof is to derive a sufficient condition on the discount factor such that the maximum gain induced by a unilateral deviation is less than the loss induced by the punishment procedure that the other transmitters apply by playing at the one-shot game NE. The proposed cooperation plan therefore consists in playing at the operating point if no transmitter deviates from this point. If one transmitter deviates from the OP, then all the other transmitters play the action corresponding to one-shot game NE. At this point it is possible to see very clearly the information assumptions needed to implement the proposed distributed power control policies. To play at $p_i^{\mathrm{OP}}$ or $p_i^*$ only the individual CSI ($\eta_i = |g_i|^2$) is needed by each transmitter. To detect the deviation of a transmitter we propose the following mechanism: the receiver broadcasts the public signal $s(t) = \sigma^2 + \sum_{i=1}^{K} \eta_i(t) p_i(t) \in \mathcal{S}$ (note that the knowledge of the individual SINR is a sufficient condition to re-construct this public signal). At the OP, this signal equals $\frac{2\sigma^2}{1-(K-1)\gamma^*}$. Thus, if one transmitter deviates all the other transmitters detect this unilateral deviation and can therefore stop cooperating and start playing the one-shot game NE. Interestingly, the proposed equilibrium strategies have been found to be Pareto-optimal for all simulations we have performed. As a result, the proposed PC policies are both efficient and rely on reasonable information assumptions. For comparison, the policies based on pricing [18] require global CSI.

## IV. Slow power control and stochastic discounted repeated games

From now on, we consider a more general scenario in which channel gains $\underline{\eta}$ can vary from game stage to game stage. The utility function at a given game stage therefore depends not only on the profile of actions $\underline{p}(t)$ played at stage $t$ but also on the vector of channel gains $\underline{g}(t) = (g_1(t), ..., g_K(t))$ and more precisely on $\underline{\eta}(t) = (\eta_1(t), ..., \eta_K(t)) \in \Gamma$, with $\Gamma = \Gamma_1 \times ... \times \Gamma_K$. The corresponding game-theoretic framework is the one of stochastic repeated games. Our objective is

to characterize the set of equilibrium utilities of the repeated game. This can be thought of as a counterpart of a capacity region in information theory. The corresponding result is called a Folk theorem. It turns out that no general Folk theorem is available for stochastic RG. It is only very recently that some authors [13][14] succeeded to derive a Folk theorem for stochastic RG with public information. To be able to exploit these very interesting results we assume that every transmitter knows the public signal $s \in \mathcal{S}$ as in the previous section and has global CSI $\underline{\eta}$ at each game instance or stage. An important condition which is assumed to be satisfied by the channel gain process is the irreducibility property.

*Definition 5: Let $\underline{\eta}$ and $\underline{\eta}'$ be two channel states and $\pi(\underline{\eta}'|\underline{\eta})$, the probability that the next state will be $\eta'$ knowing that the actual state is $\eta$. The transition probability $\pi$ is irreducible if for any channel states $\underline{\eta}$ and $\underline{\eta}'$, we have $\pi(\underline{\eta}'|\underline{\eta}) > 0$.*

The mobility in wireless communication impose that for a given channel realization, there is always a positive probability for each channel gain to be drawn in the next stage. In order to characterize the set of equilibrium payoff, we assume that the transition probability is irreducible. As in the previous section, we assume that the player does not observe perfectly the actions played by the other player in the past stages (imperfect monitoring) but have only access to the public signal $s(t) \in \mathcal{S}$.

### A. The game course

The game starts at stage $t = 1$ with an initial state $g(1)$ which is known by the players. The transmitters simultaneously choose a power level $\underline{p}(1) = (p_1(1), \ldots, p_K(1))$ and get a public signal $s(1) \in \mathcal{S}$ from $\phi(\underline{p}(1))$. The stage utility, denoted by $u_i(\underline{p}(1), \underline{\eta}(1))$ is not known by the player $i$. After the stage $t-1$, the channel states are drawn from the probability distribution $\pi(\cdot|\underline{\eta}(t-1)) \in \Delta(\Gamma)$ and the realization is publicly announced : $\underline{\eta}(t) = (\eta_1(t), \ldots, \eta_K(t))$. Taking into account the past history of the game, the players choose simultaneously their action $p_i(t)$ and get a public signal $s(t) \in \mathcal{S}$ from $\Phi(\underline{p}(t))$ and does not know their stage utility $u_i(\underline{p}(t), \underline{\eta}(t))$, and so on. We define the vector of private $\underline{\tilde{h}}_i(t)$ and public $\underline{\tilde{h}}(t)$ history of player $i$ :

$$\begin{aligned} \underline{h}_i(t) &= (p_i(1), s(1), \underline{\eta}(1), ..., p_i(t-1), s(t-1), \underline{\eta}(t-1), \underline{\eta}(t)) \\ \underline{h}(t) &= (s(1), \underline{\eta}(1), ..., s(t-1), \underline{\eta}(t-1), \underline{\eta}(t)) \end{aligned}$$

We define the public history of the game as the intersection of all private histories. Note that the private history contains the public one and the sequence of transmission power $(p_i(t))_{T-1 \geq t \geq 1}$ of player $i$. The vector $\underline{h}(t)$ lies in the set

$$\widetilde{\mathcal{H}}_t = (\mathcal{S} \times \Gamma)^{t-1} \times \Gamma \tag{11}$$

where the notation $(.)^{t-1}$ refer to the Cartesian product of sets. This vector (11) that is assumed to be known by each transmitters before playing for block $t$. The private and public histories are introduced in order to define the private and the public strategies. In the sequel we will restrict ourself only to the public strategies for which it is possible to characterize the set of equilibrium utilities. Note that this restriction does not affect the final result in terms of set of equilibrium utilities. In fact, we show that, in our framework, the players should not take into account their private history. A strategy is a sequence of functions from the history of the game onto a probability distribution over the set of power.

*Definition 6 (Players' strategies in the RG): A public strategy for player $i \in \mathcal{K}$ is a sequence of functions $(\tilde{\tau}_{i,t})_{t \geq 1}$ with*

$$\widetilde{\tau}_{i,t} : \left| \begin{array}{ccc} \widetilde{\mathcal{H}}_t & \rightarrow & \Delta(\mathcal{P}_i) \\ \underline{\tilde{h}}_t & \mapsto & p_i(t). \end{array} \right. \tag{12}$$

Where $\Delta(\mathcal{P}_i)$ denote the set of probability over $\mathcal{P}_i$.

The public strategy of player $i$ will therefore be denoted by $\tilde{\tau}_i$ while the vector of public strategies $\underline{\tilde{\tau}} = (\tilde{\tau}_1, ..., \tilde{\tau}_K)$ will be referred to a joint public strategy. A joint public strategy $\underline{\tilde{\tau}}$ induce in a natural way a unique probability $\mathbb{P}_{\underline{\tilde{\tau}},\pi}$ over the set of action plans $(\underline{p}(t))_{t \geq 1}$ and sequence of signals $(\underline{s}(t))_{t \geq 1}$. The averaged utility for player $i$ can then be defined as follows.

*Definition 7 (Players' utilities in the RG): Let $\underline{\tilde{\tau}} = (\tilde{\tau}_1, ..., \tilde{\tau}_K)$ be a joint mixed strategy. The utility for player $i \in \mathcal{K}$ if the initial channel state is $\eta(1)$, is defined by:*

$$\tilde{v}_i(\underline{\tilde{\tau}}, g) = \sum_{t \leq 1} \lambda(1-\lambda)^{t-1} \mathbb{E}_{\underline{\tilde{\tau}},\pi} \left[ u_i(\underline{p}(t), \underline{\eta}(t)) | \underline{\eta}(1) \right] \tag{13}$$

*where $(\underline{p}(t))_{t \geq 1}$ is the sequence of power profile induced by the joint strategy $\underline{\tilde{\tau}}$.*

We present now the proper definition of a stochastic repeated game.

*Definition 8 (Stochastic RG with Public Monitoring): A stochastic repeated game with public monitoring is defined as $\mathcal{G} = (\mathcal{K}, (\widetilde{\mathcal{T}}_i)_i, (\tilde{v}_i)_i, (\Gamma_i)_i, \pi, \mathcal{S}, \Phi)$, where $\mathcal{K}$ is the set of players, $\widetilde{\mathcal{T}}_i$ is the set of strategy of player $i$, $\tilde{v}_i$, her long-term utility function, $\pi$ is the transition probability over the set of channels gains $(\eta_i)_i$, $\Phi$ is the public observation function and $\mathcal{S}$ is the set of public signals.*

We suppose from now that the above description of the game is common knowledge and the players are rational (every player does the best for himself and knows the others do so and so on).

### B. Equilibrium concept

At this point, public Nash equilibrium strategies of the stochastic repeated game starting with the channel state $g$ can be defined.

*Definition 9 (Public Equilibrium Strategies of the RG): A public mixed strategy $\underline{\tilde{\tau}}$ supports an equilibrium of the stochastic repeated game with initial channel state $\underline{\eta}(1)$ if*

$$\forall i \in \mathcal{K}, \forall \tilde{\tau}_i', \ \tilde{v}_i(\underline{\tilde{\tau}}, \underline{\eta}(1)) \geq \tilde{v}_i(\tilde{\tau}_i', \underline{\tilde{\tau}}_{-i}, \underline{\eta}(1)) \tag{14}$$

where $-i$ is the standard notation to refer to the set $\mathcal{K} \backslash \{i\}$; here $\underline{\tilde{\tau}}_{-i} = (\tilde{\tau}_1, ..., \tilde{\tau}_{i-1}, \tilde{\tau}_{i+1}, ..., \tilde{\tau}_K)$.

The notion of Nash equilibrium in repeated game is refined by the sub-game perfection property, introduced by Selten for extensive games [19], [20]. For a sub-game perfect equilibrium, the incentives hold along the duration of the game.

*Definition 10 (Perfect Public Equilibrium Strat. of the RG): A public strategy profile $\underline{\tilde{\tau}}$ is a perfect public equilibrium if*

*for every $h^t \in \widetilde{\mathcal{H}}_t$, the continuation profile $\tilde{\underline{\tau}}_{|h^t}$ is a Nash equilibrium of the restricted stochastic repeated game starting with the channel state $g(t)$. We denote $E_\lambda(\eta(1))$ the set of Perfect public equilibrium of the game with initial state $\eta(1) \in \Gamma$ and discount factor $\lambda$.*

An important issue is precisely to characterize the set of possible equilibrium payoff or public perfect equilibrium payoff in the repeated game. This kind of result often appears as "Folk Theorem" (see e.g.,[10][11]). A huge part of the literature is dedicated to find the set of equilibria under different assumptions, but a general characterization is still unavailable. Our model is included in the framework of stochastic repeated game with imperfect public monitoring.

### C. Independence of the initial State

In classical models of stochastic repeated game, the initial state $\eta(1)$ could be determinant for characterizing the solutions of our problem. However, it is natural to think that the initial state of channel gain will not influence the future sequence of channel realization. We present some results of Dutta (1995) [21] that formalize the above statement. Because of the irreducibility property of the channel stochastic process, the limit set of feasible utilities, the set of perfect public equilibrium utilities and the minmax utilities are independent of the initial state.

*Theorem 11 (Independence of the Initial State):* Suppose that the stochastic repeated game is irreducible (5), it implies that :

- *The limit of the minmax is independent of the initial state i.e.* $\lim_{\lambda \longrightarrow 0} min_{\tau_{-i}} max_{\tau_i} \tilde{v}_i(\tilde{\tau}_i, \tilde{\underline{\tau}}_{-i}, \eta(1)) = \tilde{v}_i$ *for all* $\underline{\eta}(1)$ *and all* $i \in K$.
- *The limit set of feasible utilities is independent of the initial state i.e.* $\lim_{\lambda \longrightarrow 0} F_\lambda(\underline{\eta}(1)) = F$ *for all* $\eta(1)$.
- *The limit set of public perfect equilibrium utilities is independent of the initial state i.e.* $\lim_{\lambda \longrightarrow 0} E_\lambda(\eta(1)) = E$ *for all* $\eta(1)$.

The following definition is fundamental for characterize the set of public perfect equilibrium payoff of our repeated game.

*Definition 12: We define the set of asymptotic feasible and individually rational payoff by*:

$$F^* = \{x \in F | x_i \geq \tilde{v}_i, \ \forall i \in K\} \tag{15}$$

The set $F^*$ is defined as the set of energy-efficiency utilities the players can get such that each of them has more than his minmax utility.

### D. Main Result : Folk Theorem

The following theorem state that only a condition over the discount factor $\lambda$ is sufficient to have a sub-game perfect equilibrium property for a utility vector $\underline{u}$ in $F^*$.

*Theorem 13: For each utility vector $\underline{u} \in F^*$, there exists a $\lambda_0$ such that for all $\lambda < \lambda_0$, there exists is a perfect public equilibrium strategy of our stochastic repeated power control game, such that the long-term utility equals $\underline{u} \in F^*$.*

The proof is based on Hörner, Sugaya, Takahashi and Vieille (2009), [13] ; Kandori and Matsushima (1998) [22].

## V. Numerical Illustration of Optimal Equilibrium Utilities

The above result implies that each Pareto-optimal utility vector that is individually rational can be sustained by a public perfect equilibrium strategy for a discount factor sufficiently small. In practice, we have to focus on a particular Pareto-optimal point which is individually rational. For example, denote by $\tilde{p}$ the solution of the maximization problem : $\max_{p \in P \text{ s.t. } u_i \geq v_i \forall i \in K} \sum_{i \in K} \alpha_i u_i(p)$ and $\tilde{u}$ it's corresponding utility vector. The above theorem states that $\tilde{u}$ is a public perfect equilibrium utility of the $\lambda-$discounted repeated game for a sufficiently small discount factor if $\tilde{u}$ Pareto-dominates the Minmax utilities. In the whole section we consider the same type of scenarios as [8][9] namely random code division multiple access systems with a spreading factor equal to $N$ and the efficiency function $f(x) = (1 - e^{-x})^M$, $M$ being the block length.

□ We consider a simple stochastic process with two channel states: $(\eta_1, \eta_2) \in \{(7,1),(1,7)\}$. The transition probability is constant over the channel states: $\pi(\cdot) = (\frac{1}{2}, \frac{1}{2})$ and its invariant measure is $\mu = (\frac{1}{2}, \frac{1}{2})$. Consider the scenario $(K, M, N) = (2, 2, 2)$. Fig. 1 represents the achievable utility region for the two different channel state and the long-term expected utility region. The positive orthan denotes the set of expected individually rational utilities. Its intersection with the expected achievable utility region describes the set of public perfect equilibrium utility. Three important points are highlighted in the different scenario: the expected Nash equilibrium of the one-shot game studied in [7], the expected operating/cooperation point studied by Le Treust and Lasaulce 2010 [23], and the point where the expected social welfare (sum of utilities) is maximized (star). From this figure it can be seen that: a significant gain can be obtained by using a model of repeated games instead of the one-shot model. Moreover, significant improvement in term of expected utilities is a direct consequence of the full CSI instead of individual CSI.

□ As a second type of numerical results, the performance gain brought by the stochastic discounted repeated game (SDRG) formulation of the distributed PC problem is assessed. Considering a simple stochastic process where $\eta_i = 2$ and $\eta_j = 1$ for all $j \in \mathcal{K} \backslash \{i\}$ and the $i$'s player is drawn with uniform distribution over the $K$ players. We compute the expected social utility the players get at the social optimum $w_{SDRG}$. Denote by $w_{NE}$ (resp. $w_{DRG}$ and $w_{SDRG}$) the efficiency of the NE (resp. DRG and SDRG equilibrium) in terms of social welfare i.e. the sum of utilities of the players. Fig. 2 represents the quantity $\frac{w_{SDRG} - w_{NE}}{w_{NE}}$ and $\frac{w_{DRG} - w_{NE}}{w_{NE}}$ in percentage as a function of the spectral efficiency $\alpha = \frac{K}{N}$ with $N = 128$ and $2 \leq K < \frac{N}{\beta^*} + 1$. The asymptotes $\alpha_{max} = \frac{1}{\beta^*} + \frac{1}{N}$ are indicated by dotted lines for different values $M \in \{10, 100\}$. The improvement becomes very significant when the system load is close to $\frac{1}{N} + \frac{1}{\beta^*}$, this is because the power at the one-shot game NE becomes large when the system becomes more and more

loaded. As explained in [9] for the Stackelberg approach these gains are in fact limited by the maximum transmit power.

## VI. Conclusion

Repeating a power control game is a way of introducing cooperation between selfish transmitters. In this paper, we have shown that the corresponding cooperative power control policies can be implemented without using explicit cooperation channels between the transmitters. In the case of fast PC, only individual CSI and a realistic public signal are required to implement the proposed schemes. In the case of slow power control, only the feasible utility region has been derived, the equilibrium power control strategies to achieve the corresponding points still need to be found. Both in the cases of fast and slow power control, the cooperation gain induced by the underlying cooperation plans is shown to be significant. The repeated game formulation of the distributed power control problem therefore shows a way of reaching interesting trade-offs in terms of global network energy-efficiency and signalling.

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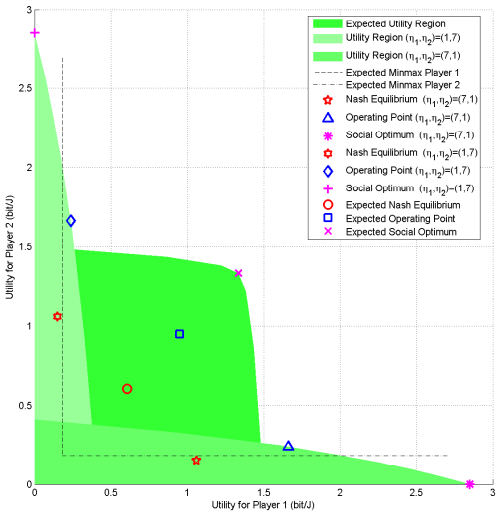


Fig. 1. Utility regions for $(K, M, N) = (2, 2, 2)$ considering different channels configurations and their expected utilities. Our procedure lead to the expected social optimum.

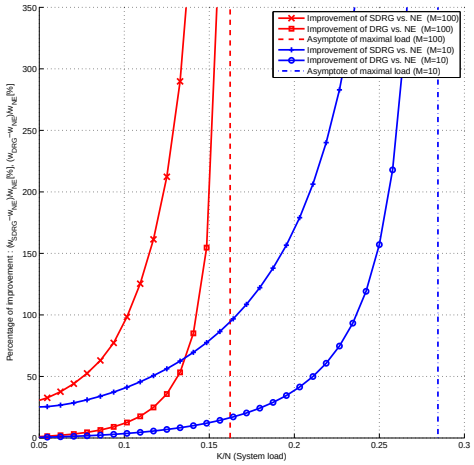


Fig. 2. The gain brought by the repeated-game based cooperation in terms of the sum of utilities (standard and stochastic repeated games) w.r.t. to the purely non-cooperative scenario (Nash Equilibrium).