HAL
open science

Random Matrix Theory for Small Cell Networks

Jakob Hoydis, Mari Kobayashi, Merouane Debbah

▶ **To cite this version:**

Jakob Hoydis, Mari Kobayashi, Merouane Debbah. Random Matrix Theory for Small Cell Networks. 25th Meeting of the Wireless World Research Forum, Nov 2010, London, United Kingdom. ⟨hal-00554958⟩

HAL Id: hal-00554958

https://centralesupelec.hal.science/hal-00554958v1

Submitted on 11 Jan 2011

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

Autorisation HAL

HAL Authorization

Random Matrix Theory for Small Cell Networks

Jakob Hoydis*†, Mari Kobayashi*, and Mérouane Debbah†
*Department of Telecommunications, Supélec, 91192 Gif-sur-Yvette, France
†Alcatel-Lucent Chair on Flexible Radio, Supélec, 91192 Gif-sur-Yvette, France
{jakob.hoydis, mari.kobayashi, merouane.debbah}@supelec.fr

***Abstract*—In this paper, we present several applications of recent results of large random matrix theory (RMT) to the performance analysis of small cell networks (SCNs). In a nutshell, SCNs are based on the idea of a very dense deployment of low-cost low-power base stations (BSs) that are substantially smaller than existing macro cell equipment. However, a massive network densification causes many new challenges to the optimal system design, such as interference and mobility management, self-organization, security, coverage and performance prediction. We focus especially on the last point and show how RMT can be used to provide tight and tractable approximations of key performance parameters, such as capacity and outage probability, and demonstrate how it can be applied to related optimization problems. Although the results are only tight in the large system limit, they yield close approximations for small systems with as little as three transmitters and receivers. Thus, we believe that RMT offers many yet unexplored applications to the study of SCNs and hope that this paper stimulates further research in this direction.**

***Index Terms*—Small cell networks (SCN), Coordinated Multi-Point (CoMP), imperfect CSI, random matrix theory.**

I. Introduction

The increasing demand for wireless multimedia services has lead to exploding mobile data traffic. Recently, a big telecommunications equipment manufacturer announced that wireless data traffic has surpassed voice globally and several market forecasts, e.g. [1], prognosticate exponential traffic growth during the next years. Traditionally, operators provide additional capacity to metropolitan hot-spots through the deployment of micro or pico cells. These devices are in essence scaled down macro cell base stations (BSs) and require the same amount of costly planning, management and maintenance as their bigger brothers. A further network capacity increase based on an even denser deployment of pico BSs would therefore incur prohibitive costs. Also from and ecological perspective, the increase in mobile data traffic has considerable consequences. The SMART 2020 report [2] predicts that the carbon emissions caused by information and communication technology (ICT) will grow by roughly 10% per year until 2020, when its contribution to the global emissions will amount to 2.5%. This gives rise to the question how mobile operators can satisfy the future traffic demands in an economical *and* ecologically worthwhile manner.

"Small cell networks" (SCNs) [3] is a novel and radically different network design concept which could provide a cost- and energy efficient solution to cope with the forecasted traffic growth. In short, the main idea behind SCNs is a very dense deployment of low-cost low-power BSs carrying most of the data traffic while macro cells are only needed to ensure area coverage. SCNs could share the backhaul infrastructure with already existing wireless or wireline access points (e.g. FTTN/B/H or VDSL street cabinets) or could be connected via power line carriers; small cell BSs could be integrated into the available street-furniture (e.g. lamp posts, bus stops, etc.) and made autonomous and independent from human maintenance through extensive self-organization functionalities. Thus, SCNs could eliminate the need for costly cell site acquisition, detailed network planning and regular maintenance, and reduce capital (CAPEX) as well as operational (OPEX) expenditures while ensuring high data-rates, uniformly delivered over the coverage area. Moreover, by bringing mobiles and BSs closer together, SCNs require less transmit power and have hence the potential to reduce the carbon footprint of cellular networks.

However, decreasing the cell size causes significant changes to the wireless link. While necessarily lower antennas heights lead to less favorable and predictable propagation conditions, user terminals (UTs) are more likely to be under line-of-sight (LOS) conditions to one or several BSs. Moreover, as the cell size decreases, hand-overs occur more often and BS-cooperation might not only be beneficial for interference reduction [4] but also necessary to handle user mobility. In a cooperative SCN, several BSs operate as a distributed antenna system which allows for joint signal processing and reduces the frequency of hard hand-overs between the small cells.

Motivated by the scenario detailed above, we consider in this paper a general class of channel models, well-suited for the study of cooperative SCNs. More precisely, we assume a Rician fading channel, composed of a random Rayleigh fading and a deterministic LOS component, where each complex channel gain between a transmitter and a receiver is allowed to have a different variance. The latter assumption is relevant to cooperative SCNs since a UT might be simultaneously served by multiple BSs to each of which it has a channel with a different path loss. In addition, we allow for arbitrary correlated noise at the receiving antennas, an assumption useful for the modeling of inter-cell/cluster interference.

For this general channel model, we review recent result of large random matrix theory (RMT) related to the study of key performance measures such as the mutual information and its fluctuations. These results, although of a rather mathematical flavor, have concrete applications in the performance analysis of cooperative SCNs. For example, we determine the optimal amount of channel training in a block fading uplink channel and derive a close approximation of the outage probability.

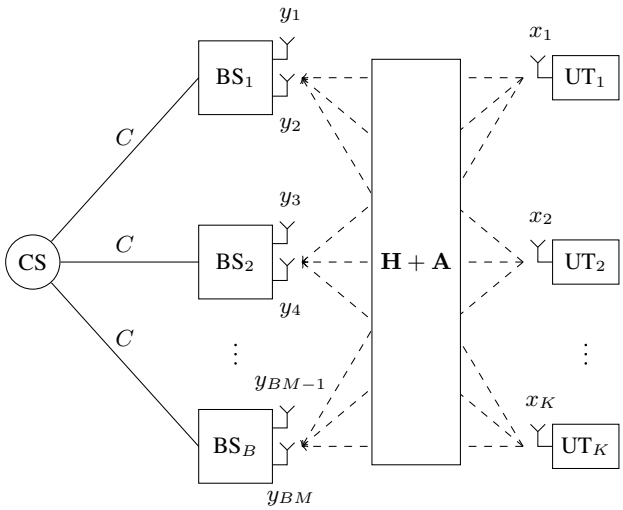

Fig. 1. Schematic system model for the case $M_b = 2\ \forall b$.

II. System model

We consider a general model for the wireless uplink channel from K single-antenna UTs to B BSs, equipped with M_b, $b = 1, \ldots, B$ antennas, respectively, as shown in Fig. 1. Denote $N = \sum_{b=1}^{B} M_b$ the total number of receive antennas. The BSs are assumed to be oblivious to the applied user codebooks and forward compressed versions of their received signals to a central station (CS) via error-, delay- and interference-free backhaul links of capacity C bits per channel use. The CS jointly decodes the messages of all users. At a given time instant, the received signal vector $\mathbf{y} \in \mathbb{C}^N$ at the CS reads

$$\mathbf{y} \;=\; \sqrt{\rho}\,(\mathbf{H}+\mathbf{A})\,\mathbf{x}+\mathbf{z} \tag{1}$$

where $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_K)$ is the vector of transmitted signals from all K UTs and $\rho > 0$ is the transmit power per UT. The noise vector $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \mathbf{Z})$ is used to model the contributions of several sources of complex Gaussian noise: (i) thermal noise at the BS-antennas normalized to unit variance, (ii) quantization noise due to signal compression with covariance matrix $\mathbf{Q}(C)$ and (iii) intercell-interference with covariance matrix $\mathbf{\Delta}$. Thus, the overall noise covariance matrix $\mathbf{Z} \in \mathbb{R}_+^{N\times N}$ is given as $\mathbf{Z} = \mathbf{I}_N + \mathbf{Q}(C) + \mathbf{\Delta}$. The quantization noise covariance matrix $\mathbf{Q}(C)$ depends in general on the backhaul capacity C and the applied compression scheme. For further information on this topic we refer the reader to [5] and references therein. We model the wireless channel by the random Rayleigh fading component $\mathbf{H} \in \mathbb{C}^{N\times K}$ and the deterministic LOS component $\mathbf{A} \in \mathbb{C}^{N\times K}$. The elements h_{ij} and a_{ij} of the matrices $\mathbf{H}$ and $\mathbf{A}$, respectively, are given as

$$h_{ij} \;=\; \sqrt{\frac{(1-\kappa_{ij})\sigma_{ij}^2}{K}}\,w_{ij} \tag{2}$$

$$a_{ij} \;=\; \sqrt{\frac{\kappa_{ij}\sigma_{ij}^2}{K}}\,e^{j\phi_{ij}} \tag{3}$$

where w_{ij} are i.i.d. standard complex Gaussian random variables, $\kappa_{ij} \in [0,1]$ is the Rician parameter, $\phi_{ij} \in [0, 2\pi)$ the phase of the LOS component and σ_{ij}^2 the inverse path loss of the channel between the jth UT and the ith receiving antenna. Under the assumption of perfect knowledge of $\mathbf{H}$ and $\mathbf{A}$ at the CS and Gaussian signaling of the UTs, the normalized ergodic mutual information of the channel in (1) is given by $I(\rho) = \mathbb{E}\left[\mathcal{I}(\rho)\right]$, where

$$\mathcal{I}(\rho) = \frac{1}{N}\log\left|\mathbf{I}_N + \rho\mathbf{Z}^{-1}\left(\mathbf{H}+\mathbf{A}\right)\left(\mathbf{H}+\mathbf{A}\right)^{\mathsf{H}}\right| \;. \tag{4}$$

III. Performance Analysis

Key performance measures of the wireless channel in (1) are the ergodic mutual information $I(\rho)$ and the outage probability $P_{\text{out}}(r) = \mathbb{P}\left(N\mathcal{I}(\rho) < r\right)$ for a given target rate $r > 0$. Unfortunately, closed-form expressions for both quantities can only be obtained for certain academic cases, such as $\mathbf{A} = \mathbf{0}$ and $\sigma_{ij}^2 = 1\ \forall i, j$ [6]. For more realistic models, performance analyses have been often carried out in the asymptotic regime [7], i.e., for $N, K \to \infty$ while

$$0 < \liminf \frac{N}{K} \le \limsup \frac{N}{K} < \infty \;. \tag{5}$$

Although this approach does not lead to closed-form expressions in general, it provides implicit equations whose solutions depend only on the statistical properties of the random matrix model at hand, e.g. [8], [9]. The evaluation of these equations is in general much less complex than Monte Carlo simulations and, most importantly, the asymptotic results provide tight approximations for even small values of N and K.

A. Deterministic Equivalents of the Mutual Information

In a recent work [10, Theorem 1], building upon the results of [8], we have derived an approximation $\bar{I}(\rho)$ of the ergodic mutual information $I(\rho)$ for the case of Rayleigh fading channels with arbitrary correlated noise, i.e., $\mathbf{A} = \mathbf{0}$, in the sense that

$$I(\rho) - \bar{I}(\rho) \xrightarrow[N,K\to\infty]{} 0 \;. \tag{6}$$

A similar result for Rician fading channels with white noise, i.e., $\mathbf{A} \neq \mathbf{0}$ and $\mathbf{Z}$ diagonal, has been established earlier in [8, Theorem 4.1]. Due to space reasons, we spare the rather lengthy expression of $\bar{I}(\rho)$ which can be found in [10].

Application 1: Optimal Channel Training: An important aspect of the performance analysis of SCNs is the question of how many resources should be used for channel estimation. A well-known model to study this problem, is the block fading channel model of [11], [12], where the channel $\mathbf{H}$ remains constant in time for a coherence block of T channel uses and then changes randomly from one block to the other. The matrix $\mathbf{A}$ is assumed to be known and deterministic. This is a reasonable assumption as the LOS components vary on a much slower time-scale than the fast fading channel fluctuations. A priori, neither the UTs, nor the BSs or the CS are aware of the realization of $\mathbf{H}$. The UTs broadcast orthogonal training sequences of length $\tau \ge K$ to the BSs. Based on these training sequences, the CS computes the minimum-mean-square-error (MMSE) estimate of all channel coefficients. This leads to the estimated channel matrix $\hat{\mathbf{H}}$ and the independent estimation

error $\tilde{\mathbf{H}}$, such that $\mathbf{H} = \hat{\mathbf{H}} + \tilde{\mathbf{H}}$. While increasing τ improves the channel estimate it also reduces the available time for data transmission. It is therefore natural to ask, what is the optimal training length τ^*? Similar to [11], [12], we tackle this problem by considering a lower bound $R(\tau)$ on the mutual information, given by

$$R(\tau) = \frac{1}{N}\mathbb{E}\left[\log\left|\mathbf{I}_N + \rho\tilde{\mathbf{Z}}^{-1}\left(\hat{\mathbf{H}} + \mathbf{A}\right)\left(\hat{\mathbf{H}} + \mathbf{A}\right)^{\mathsf{H}}\right|\right] \quad (7)$$

where $\tilde{\mathbf{Z}} = \mathbf{Z} + \rho E\left[\tilde{\mathbf{H}}\tilde{\mathbf{H}}^{\mathsf{H}}\right]$. As this expression does not account for the rate loss due to channel training, it is our goal to maximize the net ergodic achievable rate

$$R_{\text{net}}(\tau) = \left(1 - \frac{\tau}{T}\right) R(\tau) \ . \quad (8)$$

This optimization problem is intractable for finite dimension of the channel matrix $\mathbf{H}$. However, leveraging the results presented above, one can approximate $R(\tau)$ by its deterministic equivalent $\overline{R}(\tau)$ and maximize the new objective function $\left(1 - \frac{\tau}{T}\right)\overline{R}(\tau)$ instead. This leads to the approximative solution $\bar{\tau}^*$. We have recently shown in [13], that this approach is asymptotically optimal for Rayleigh fading channels, i.e., $\bar{\tau}^*$ satisfies $\tau^* - \bar{\tau}^* \xrightarrow[N,K\to\infty]{} 0$.

B. Fluctuations of the Mutual Information

The fluctuations of the random variable $N\mathcal{I}(\rho)$ have been also analyzed in several recent works [14]. One is generally interested in results of the form

$$\frac{N}{\Theta}\left(I(\rho) - \bar{I}(\rho)\right) \xrightarrow[N,K\to\infty]{\mathcal{D}} \mathcal{N}(0,1) \quad (9)$$

where $\mathcal{D}$ stands for convergence in distribution. Two recent publications [10], [15], provide explicit expressions for the asymptotic variance Θ^2 for the case of Rayleigh fading channels with colored noise, i.e., $\mathbf{A} = \mathbf{0}$, [10, Claim 1] and Rician fading with white noise, i.e., $\mathbf{A} \neq \mathbf{0}$ and $\mathbf{Z}$ diagonal, [15, Claim 1].

Application 2: Approximation of the Outage Probability: The results of the previous section have a direct application in the computation of the outage probability. Using the explicit expression of the asymptotic variance Θ^2, one can approximate the outage probability $P_{\text{out}}(r)$ by

$$P_{\text{out}}(r) \approx 1 - Q\left(\frac{r - N\bar{I}(\rho)}{\Theta}\right) \quad (10)$$

where $Q(x)$ is the Q-function.

IV. Numerical Results

In order to visualize the theoretical results of the previous sections, we consider a simple cellular system consisting of $B = 3$ BSs with $M = 2$ antennas and $K = \{3, 6, 9\}$ UTs, as shown in Fig. 2. The locations of the UTs are randomly chosen according to a uniform distribution. The inverse path loss factors σ_{ij}^2 are given as $\sigma_{ij}^2 = d_{ij}^{-3.6}$, where d_{ij} is the distance between between UT j and the ith receive antenna, normalized to the maximum distance within a cell. The UTs are assumed to have a Rician fading channel with parameters

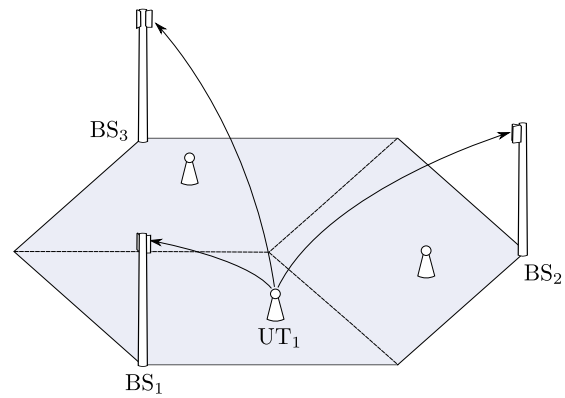

Fig. 2. Cellular example with $B = 3$ BSs and $K = 3$ UTs.

$\kappa_{ij} = \kappa$ to their closest BS while the channels to all other BSs are purely Rayleigh faded, i.e., $\kappa_{ij} = 0$. The phases ϕ_{ij} of the corresponding LOS components are drawn randomly from the interval $[0, 2\pi)$. The cell edge signal-to-noise-ratio is defined as SNR $= \rho$. We consider one random snapshot of user distributions, which defines the matrix $\mathbf{A}$ and the values of σ_{ij}^2, and average over many different realizations of $\mathbf{H}$.

Numerical Results for Application 1: We assume SNR $=$ 0 dB, channel coherence time $T = 100$ and backhaul capacity $C = 1$ bits per channel use. There is no inter-cell interference, i.e., $\mathbf{\Delta} = \mathbf{0}$. In Fig. 3, we depict the optimal training length τ^* as a function of the Rician parameter κ. We see a very good fit between the values of τ^* obtained by an exhaustive search based on extensive simulations and the approximative solution $\bar{\tau}^*$. With growing κ, the channel becomes more and more deterministic and less channel training is necessary.

Numerical Results for Application 2: We assume a noise covariance matrix $\mathbf{Z} = \mathbf{I}_N$. Before presenting the results for the outage probability approximation, we show in Fig. 4 the ergodic mutual information $I(\rho)$ versus SNR for three different numbers of UTs $K = \{3, 6, 9\}$. The markers are obtained through simulations while the solid lines correspond to the approximation by the deterministic equivalent $\bar{I}(\rho)$. We observe an almost perfect overlap between both results over the full range of SNR for all values of K. Fig. 5 shows the performance of the approximation of the outage probability $P_{\text{out}}(R)$ as given by (10) for a target rate of $r = K \times 3$ [nats/s/Hz]. The solid lines as calculated with the help of [15, Claim 1] are compared to simulation results, represented by dashed lines. Obviously the approximation gets better with a growing number of UTs but is still very accurate for a small system with three UTs and BSs.

V. Conclusion

In this paper, we have provided several applications of large random matrix theory to the performance analysis of cooperative small cell networks. We have considered a very general channel model for SCNs which accounts for several realistic aspects such as LOS components, path loss, correlated interference, quantization noise and imperfect CSI. For this general model, we have shown that RMT can provide tight and

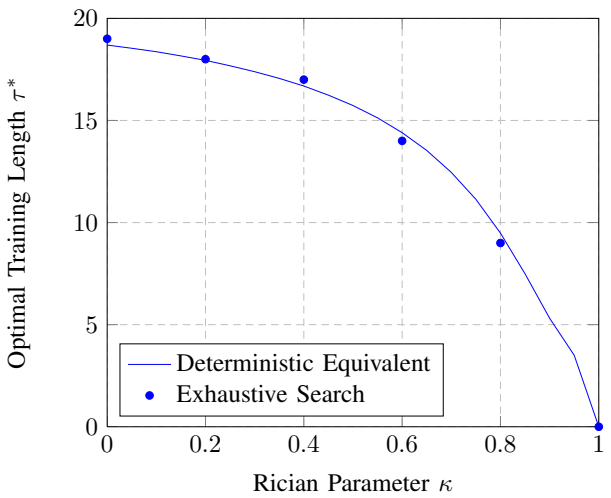

Fig. 3. Optimal training length τ^* vs Rician parameter κ.

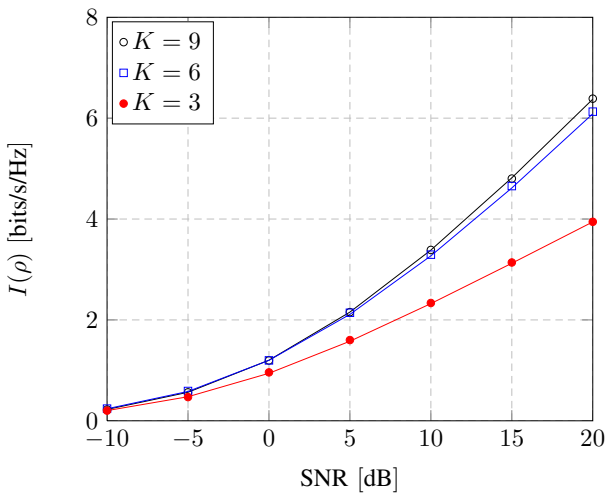

Fig. 4. Ergodic mutual information $I(\rho)$ versus SNR. Markers indicate simulation results, solid lines correspond to the deterministic equivalent approximation $\bar{I}(\rho)$.

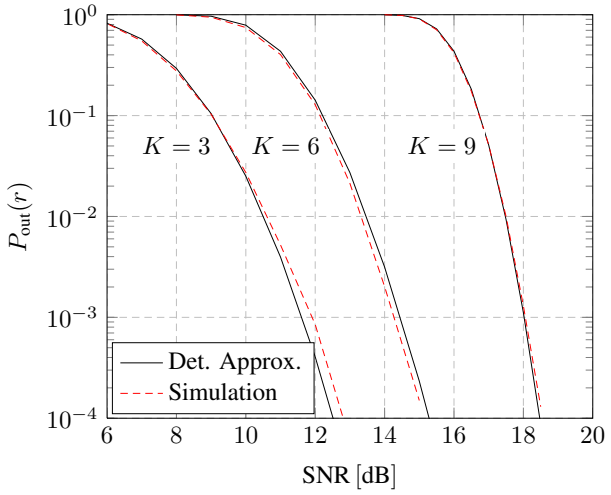

Fig. 5. $P_{\text{out}}(R)$ versus SNR for $K = \{3, 6, 9\}$ and target rate $r = K \times 3$ [nats/s/Hz].

analytically tractable approximations of information-theoretic quantities such as the mutual information and the outage probability. Moreover, we have demonstrated that RMT can be used to solve related optimization problems which were otherwise either intractable or needed to rely on massive Monte Carlo simulations. In summary, RMT provides a powerful tool for the study of information-theoretic performance limits of SCNs and allows, thus, to assess possible gains of this novel network architecture in contrast to other competing solutions.

References

[1] Morgan Stanley & Co. Inc., "The Mobile Internet Report," Dec. 2009. [Online]. Available: http://www.ms.com/techresearch

[2] The Climate Group and Global e-Sustainability Initiative (GeSI), "SMART 2020: Enabling the Low Carbon Economy in the Information Age," 2008. [Online]. Available: http://www.smart2020.org

[3] J. Hoydis and M. Debbah, "Green, Cost-effective, Flexible, Small Cell Networks," *IEEE ComSoc MMTC E-Letter Special Issue on "Multimedia Over Femto Cells"*, vol. 5, no. 5, pp. 23–26, Sep. 2010. [Online]. Available: http://committees.comsoc.org/mmc/e-news/E-Letter-September10.pdf

[4] D. Gesbert, S. Hanly, H. Huang, S. Shamai, O. Simeone, and W. Yu, "Multi-Cell MIMO Cooperative Networks: A New Look at Interference," *IEEE J. Sel. Areas Commun.*, 2010, to appear.

[5] A. Sanderovich, S. Shamai, and Y. Steinberg, "Distributed MIMO Receiver: Achievable Rates and Upper Bounds," *IEEE Trans. Inf. Theory*, vol. 55, no. 10, pp. 4419–4438, Oct. 2009.

[6] E. Telatar, "Capacity of Multi-antenna Gaussian Channels," *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585–596, 1999.

[7] A. Moustakas, S. Simon, and A. Sengupta, "MIMO Capacity Through Correlated Channels in the Presence of Correlated Interferers and Noise: A (Not So) Large N Analysis," *IEEE Trans. Inf. Theory*, vol. 49, no. 10, pp. 2545–2561, Oct. 2003.

[8] W. Hachem, P. Loubaton, and J. Najim, "Deterministic Equivalents for Certain Functionals of Large Random Matrices," *Annals of Applied Probability*, vol. 17, no. 3, pp. 875–930, 2007.

[9] R. Couillet, M. Debbah, and J. Silverstein, "A Deterministic Equivalent for the Capacity Analysis of Correlated Multi-User MIMO Channels," *IEEE Trans. Inf. Theory*, 2010, to appear.

[10] J. Hoydis, J. Najim, R. Couillet, and M. Debbah, "Fluctuations of the Mutual Information in Large Distributed Antenna Systems with Colored Noise," in *Forty-Eighth Annual Allerton Conference on Communication, Control, and Computing*, Illinois, US, Sep. 2010.

[11] P. Hassibi and B. M. Hochwald, "How Much Training is Needed in Multiple-Antenna Wireless Links?" *IEEE Trans. Inf. Theory*, vol. 49, no. 4, pp. 951–963, Apr. 2003.

[12] M. Godavarti and A. O. Hero, "Training in Multiple-Antenna Rician Fading Wireless Channels with Deterministic Specular Component," *IEEE Trans. Wireless Commun.*, vol. 6, no. 1, Jan. 2007.

[13] J. Hoydis, M. Kobayashi, and M. Debbah, "Optimal Channel Training in Uplink Network MIMO Systems," *IEEE Trans. Signal Process.*, 2010, submitted.

[14] W. Hachem, P. Loubaton, and J. Najim, "A CLT for Information-Theoretic Statistics of Gram Random Matrices with a Given Variance Profile," *Annals of Applied Probability*, vol. 18, no. 6, pp. 2071–2130, 2008.

[15] J. Hoydis, A. Kammoun, J. Najim, and M. Debbah, "Outage Performance of Cooperative Small Cell Systems Under Rician Fading Channels," in *IEEE International Conference on Communications (ICC'11)*, Kyoto, Japan, Jun. 2011, submitted.