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# Modeling Interdependent Network Systems for Identifying Cascade-Safe Operating Margins

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*Abstract*— **Infrastructure interdependency stems from the functional and logical relations among individual components in different distributed systems. To characterize the extent to which a contingency affecting an infrastructure is going to weaken, and possibly disrupt, the safe operation of an interconnected system, it is necessary to model the relations established through the connections linking the multiple components of the involved infrastructures. In this work, the modeling of interdependencies among network systems and of their effects on failure propagation is carried out within the simulation framework of a failure cascade process. The sensitivity of the critical loading value (the lower bound of the cascading failure region) and of the average cascade size with respect to the coupling parameters defining the interdependency strength is investigated as a means to arrive at the definition and prescription of cascade-safe operating margins.**

*Index Terms*—**Critical infrastructures, interdependent systems, complex networks, failure cascade.**

## ACRONYMS

CI<sub>s</sub>     critical infrastructures  
CI       critical infrastructure

NSF National Science Foundation

## NOTATION

$D$	initial disturbance on the components in the two identically operating systems
$D_{j_i}$	initial disturbance on component $j_i$ in system $i$
$I$	amount of load transferred over the interdependency links
$K$	number of links in the two identical systems
$K_i$	number of links in system $i$
$L$	average initial load in the two identically operating systems
$L^i$	average initial load in system $i$
$L_{cr}$	critical load in the two identically operating systems
$L_{cr}^i$	critical load in system $i$
$L_{fail}$	limit of operation for components in the two identically operating systems
$L_{fail}^i$	limit of operation for components in system $i$
$L_{j_i}$	load of component $j_i$ in system $i$
$L_{max}^i$	maximum value of the initial load sampling interval in system $i$
$L_{min}^i$	minimum value of the initial load sampling interval in system $i$
$M$	number of interdependency links
$N$	number of nodes in the two identical systems
$N_i$	number of nodes in system $i$
$P$	load transferred to neighboring components after failure in the two identically operating systems
$P^i$	load transferred to neighboring components after failure in system $i$
$S$	average cascade size in the two identically operating systems
$S^i$	average cascade size in system $i$
$S_{cr}$	maximum allowable cascade size in the two identically operating systems
$S_{cr}^i$	maximum allowable cascade size in system $i$

## I. INTRODUCTION

Understanding, modeling, and assessing the normal functioning and the possible faulty conditions of critical infrastructures (CIs) is essential to safely design these complex systems, and to effectively operate the services they provide. Distributed infrastructures have been modeled as graphs to unravel their structure and dynamics, and to evaluate to what extent the structure may impact the dynamics [1]. However, it is evident that infrastructures do not

exist in isolation of one another, and the relations among them must be identified to perform realistic and applicable analyses [2]. The focus of the research on critical infrastructures must then be shifted from single, isolated systems to multiple, interconnected, mutually dependent systems. The aim becomes that of assessing the influences and limitations which interacting infrastructures impose on the individual system operating conditions, for avoiding fault propagation by designing redundancies and alternative modes of operations, and by detecting and recognizing threats [3].

In modern society, the linking among service infrastructures is required for optimal and economical operation. Yet, these interconnections introduce weaknesses in the systems due to the fact that failures may cascade from one system to other interdependent systems until, possibly, affecting the overall functioning, if proper protection of interdependencies is not considered [4]. The role of dependencies among infrastructures (so called interdependencies), and the intrinsic difficulties arising in their modeling, have been highlighted in empirical studies. For example, the database in [5] has been built from public reports of disruptions of CIs from open sources like newspapers and internet news. Events have been classified as “cascade initiating” (i.e., an event that causes an event in another CI), “cascade resulting” (i.e., an event that results from an event in another CI), and “independent” (i.e., an event that is neither a cascade initiating nor a cascade resulting event). The information in the database shows in particular that: *i*) “cascade resulting” events are more frequent than generally believed, and that “cascade initiators” are about half as frequent; *ii*) the dependencies are more focused and directional than often thought; and *iii*) energy and telecommunication are the main “cascading initiating” sectors.

Interdependencies among telecommunication systems, transportation systems, and power distribution grids played a negative role in the small telecommunication blackout which took place in the suburbs of Rome in January 2004 [6]. The chain of events originated in a major telecommunication service node when a metallic pipe carrying cooling water for the air conditioning broke. The resulting flood led to a communication blackout in the area, which in turn hit the country’s biggest printed news agency transmissions; stopped the check-in, ticketing and luggage acceptance at the International Fiumicino Airport; disturbed post offices and banks operations; and caused problems to the communication network connecting the main Italian research institutions. Furthermore, the telecommunication blackout had effects on the operation of the power distribution grid causing the disconnection of two control centers, and resulting in the loss of control on a number of remote substations in the area of Rome.

Similar occurrences [3] point to the need for identifying infrastructure interdependencies [7], and determining the design and operational conditions which can prevent the onset and propagation of cascading failures.

Due to the highlighted intrinsic complexities related to interdependencies among CIs, their modeling is far from trivial, and abstractions can be of help for preliminary investigation and broad understanding.

Many models and computer simulation techniques exist for analyzing the behavior of individual infrastructures (e.g., load flow and stability codes for electric power networks, connectivity and hydraulic analyses for pipeline systems, traffic management models for transportation networks), but modeling and simulation frameworks that allow the coupling of multiple interdependent infrastructures are needed to address infrastructure protection,

mitigation, response, and recovery issues. In developing these, it is important to know that simply linking together existing infrastructure models into a reduced model may fail to capture the emergent behavior arising in interdependent infrastructures [8].

In the last ten years, a number of researchers have focused on the study of interdependent infrastructures, and developed a variety of methodologies to be applied for ensuring the safe, reliable, continuous operation of interdependent infrastructures. According to [2], these modeling and simulation techniques can be grouped into six broad categories: (i) aggregate supply and demand tools, which evaluate the total demand for infrastructure services in a region and the ability to supply those services [9]–[15]; (ii) dynamic simulations, which examine infrastructure operations, the effects of disruptions, and the associated downstream consequences [16]–[22]; (iii) agent-based models, which allow the analysis of the operational characteristics and physical states of infrastructures [23]–[26]; (iv) physics-based models, which analyze physical aspects of infrastructures with standard engineering techniques (e.g., power flow and stability analyses for electric power grids or hydraulic analyses on pipeline systems) [27], [28]; (v) population mobility models, which examine the movement of entities through geographical regions [29]–[31]; and (vi) Leontief input-output models, which in the basic case provide a linear, aggregated, time-independent analysis of the generation, flow, and consumption of various commodities among infrastructure sectors [32]–[37].

The present work falls in the dynamic simulations category, and aims at developing a simulation framework which abstracts the physical details of the services provided by the infrastructures while at the same time capturing their essential operating features. Such type of simulation models is appealing because of its simplicity and feasibility of use, which allows identifying the factors which most contribute to the safe, efficient design and operation of interdependent infrastructures. This knowledge is achieved through the assessment of the extent to which the presence of interdependencies affects the performance of the individual infrastructures. In particular, this scheme of analysis can be used for a first evaluation of the operational margins of interdependent systems which ensure the services provided by the involved infrastructures with cascade-safe conditions and prevent the propagation of failures over large areas.

A model of cascading failures in interdependent network systems is developed as an extension of the work in [38] and [16], considering the local propagation of the overload originated from a failure to first-neighbors, and to the interdependent set of the failed component. The proposed model differs from similar studies such as [39] because it does not rely on the assumption that the load on a component is proportional to the number of shortest paths running through it, which do not always reflect the actual routes of the flow in a network; on the contrary, it shares similarity with the fiber bundle model in complex networks which has been applied to the blackout scenarios for cascading breakdown of power plants from overloading [40]. Similar to the fiber bundle model, if the load in a node exceeds its capacity, the node gets disconnected from the network, and the demand is transferred locally to neighboring nodes through the edges of the network. Moreover, it is assumed that the network connections cannot fail while the vertices are damaged and removed from the systems.

The current model is applied on a modified literature case study [41] with the aim of identifying the

interdependency features most critical for the cascade-safe operations of interdependent infrastructures, and defining related cascade-safe operational margins. The findings are in agreement with those of [17], in which a parallel study on cascading models in interdependent networks is presented, based on the forest fire model [42], and mean-field theory analysis [43]. A percolation threshold [44] is identified above which cascading failures of all sizes can propagate in the interdependent systems, and a parameter is found to characterize the cascading threshold in the coupled systems. Following [17], the present work adopts the loading level in the interdependent systems as a metric for practical application to characterize the percolation threshold in more realistic systems.

The paper is structured as follows. The research framework and the motivations for the proposed analysis are stated in Section I. The modeling assumptions of interdependencies among network systems are detailed in Section II. The resulting algorithm for modeling cascading failures in interdependent networks is illustrated in Section III. In Section IV, a crude sensitivity analysis is performed with respect to the main model parameters related to interdependency, and conclusions are drawn in Section V.

## II. INTERDEPENDENT NETWORK SYSTEMS

The modeling development carried out in this work considers two interdependent network systems, each one made by a number of components (nodes) connected by links (arcs) representing the physical and/or logical relations among them. The interdependencies are modeled as links connecting nodes of the two systems; these links are conceptually similar to those of the individual systems, and are bidirectional with respect to the “flow” between the two interdependent networks.

The number of interdependency links among interdependent components in the two systems,  $M$ , and the fixed amount of load transferred over the interdependency links upon a component failure,  $I$ , are essential features characterizing the “coupling energy” between the two interdependent systems.

In the following analysis, the interest is on systems with fixed “interdependency energy”, i.e., such that each node in system 1 could be interdependent with any other node in system 2, but  $M$  is fixed. Communication systems, in which each node in system 1 can interact with any other node in system 2 but there is a maximum amount of connecting energy between the two systems, could be an example of such energy-limited systems.

To account for the dynamics of changing connections between the two network systems under developing failure cascade processes, Monte Carlo simulations are performed in which  $M$ , and  $I$  are kept constant, but the interdependency connections among components are randomly rewired at each Monte Carlo trial. This approach allows evaluating the average systems responses to cascading failures because it accounts for multiple connectivity patterns among the  $M$  interdependent nodes in the two systems.

Fig. 1 shows the systems considered. They are two identical networks which are an abstract topological model of the IEEE Reliability Test System – 96 [41].  $M$  interdependency links are drawn between them as explained above (dashed lines in Fig. 1). In the proposed analysis, interest is on cascade onset and propagation over the bare topological structure of the test systems; no reference is made to the specific electrical properties which

characterizes these electrical infrastructures. The topology corresponds to the IEEE Reliability Test System – 96; however, no other aspect of this paper is related to the real network.

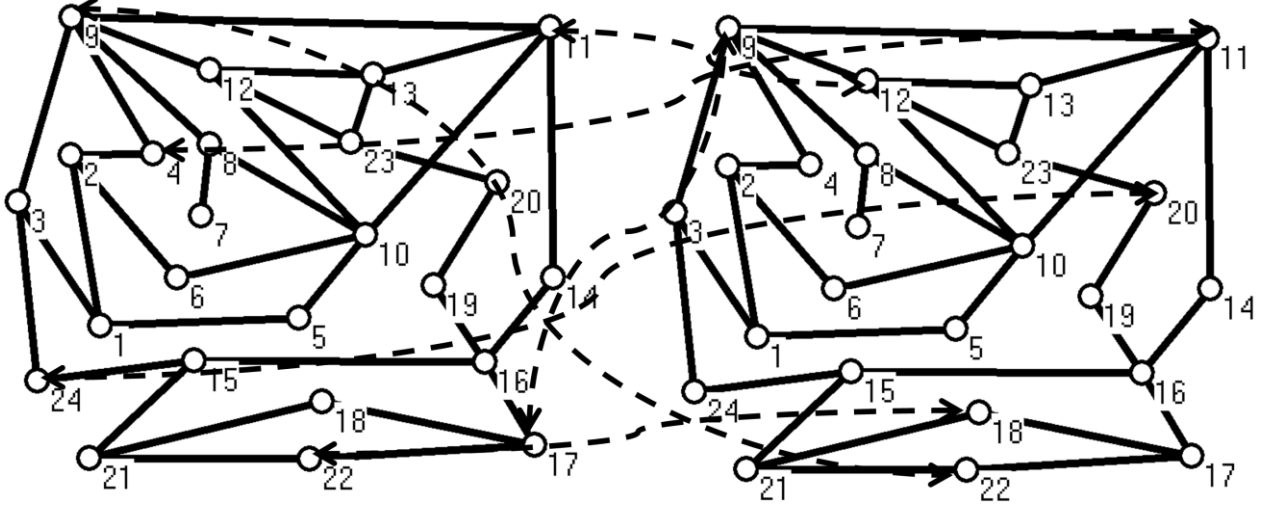


Fig. 1. The topological model of the IEEE Reliability Test System – 96 [41]. Each system  $i$  has  $N_i=24$  nodes (circles), and  $K_i=34$  links (solid lines),  $i=1, 2$ . For visual clarity, only six interdependency links connecting the two systems are shown (dashed lines).

### III. ABSTRACT MODELING OF CASCADING FAILURES IN INTERDEPENDENT NETWORKS

Consider two systems of  $N_1$  and  $N_2$  identical components ( $N_1 = N_2 = N = 24$  in this study) connected by  $K_1$  and  $K_2$  links ( $K_1 = K_2 = K = 34$  in this study) with random initial loads sampled uniformly between a minimum value  $L_{min}^i$  and a maximum value  $L_{max}^i$ ,  $i = 1, 2$ . The two systems are connected by  $M$  interdependency links ( $M=34$  in the study, except during the sensitivity analysis with respect to changing  $M$ ).

For simplicity, but with no loss of generality, all components in the  $i$ -th system are assumed to have the same limit of operation  $L_{fail}^i$ , beyond which they are failed ( $L_{fail}^1 = L_{fail}^2 = L_{fail} = 1$  in this study, upon normalization of all loads relative to the failure load value). In the reference cascading failure model of [16], when a component fails, a fixed and positive amount of load is transferred to each of the system's components; on the contrary, in the proposed extended model, the overload is propagated locally, to the first-neighbors of the failed node within the network structure it belongs to (a fixed and positive amount of load,  $P^i$ ,  $i = 1, 2$ ;  $P^1 = P^2 = P = 0.07$  in this study), and to the interdependent components which the failed component is connected to in the other network system (a fixed and positive amount of load,  $I$ ), if any. If there is no working node in the neighborhood of a failed component or among the interdependent nodes connected to it, the cascade spreading along that path is stopped. The case of two fully connected systems, where all nodes are first-neighbors and every component in system 1 is interdependent to every component in system 2, coincides with the original model proposed in [16].

The interdependency links between the two network systems are treated in the same way as the individual system links. They are bidirectional connections and upon the failure of a node in system 1 or 2, the fixed amount of load,  $I$ ,

is propagated locally to the nodes in the interdependent network system 2 or 1, if any interdependency is present for the failed node. This transfer accomplishes the coupling between the two systems.

To start the cascade, an initial disturbance imposes an additional load  $D_{j_i}$  on each component  $j_i$  of the two systems,  $j_i = 1, 2, \dots, N_i$ ,  $i = 1, 2$  ( $D_{j_1} = D_{j_2} = D = 0.02$  in this study). If the sum of the initial load  $L_{j_i}$  and the disturbance  $D_{j_i}$  of component  $j_i$  in system  $i = 1, 2$  is larger than the component load threshold  $L_{fail}^i$ , component  $j_i$  fails. This failure occurrence leads to the redistribution of additional loads,  $P^i$  on the neighboring nodes, and  $I$  on the interdependent nodes, which may, in turn, get overloaded, and thus fail in a cascade which follows the connection and interdependency patterns of the network systems. As the components become progressively more loaded, the cascade proceeds.

The algorithm for simulating the cascading failures proceeds in successive stages as follows:

- 0 At stage  $m = 0$ , all  $N_1 + N_2$  components  $j_i = 1, 2, \dots, N_i$ ,  $i = 1, 2$  in systems 1 and 2 are working under independent uniformly random initial loads  $L_{j_i} \in [L_{min}^i, L_{max}^i]$ , with  $L_{max}^i \leq L_{fail}^i$ ,  $j_i = 1, 2, \dots, N_i$ ,  $i = 1, 2$ .
- 1  $M$  interdependency links between system 1 and system 2 are generated, connecting two randomly selected components; multiple interdependency links are allowed for each component.
- 2 An initial disturbance  $D_{j_i}$ ,  $j_i = 1, 2, \dots, N_i$ ,  $i = 1, 2$ , is added to the load of each component in the two systems.
- 3 Each component that has not failed is tested for failure: for  $j_i = 1, \dots, N_i$ , if component  $j_i$  has not failed, and its load  $L_{j_i} > L_{fail}^i$ , then component  $j_i$  in system  $i$  fails,  $i = 1, 2$ .
- 4 The component loads are incremented, taking into account the network topology, i.e. the failed component neighborhood: for each failed node in system  $i$ , the load of its first-neighbors is incremented by an amount  $P^i$ ,  $i = 1, 2$ . If the working neighborhood set of the failed node is empty, the associated failure propagation into the system comes to an end.
- 5 The component loads are incremented, taking into account the interdependency pattern, i.e., the nodes interdependent to the failed component: for each failed node in system 1 or 2, the load of its interdependent nodes in system 2 or 1 is incremented by an amount  $I$ . If the interdependency set of the working components of the failed node is empty, the associated failure propagation to the interdependent system comes to an end.
- 6 The stage counter  $m$  is incremented by 1, and the algorithm is returned to step 3.

The algorithm stops when failures are no further propagated within or between the two systems.

Various initial system loading levels are evaluated, varying the uniform sampling ranges  $[L_{min}^i, L_{max}^i]$ ,  $i = 1, 2$ , whose midpoints,  $L^i$ , are indicators of the average initial systems loading levels. Large  $L^i$  values relate to operating conditions in which the systems are more stressed. In this study, when the systems are operating at varying average initial load,  $L^i$ , its range of variation is  $[0.5, 1]$  at steps of 0.005.



#### IV. SENSITIVITY ANALYSIS

The effects of the interdependencies between the two systems are shown in Fig.2 in terms of the average cascade size,  $S^i$ , i.e., the number of failed components in the  $i$ -th system at the end of the cascade spread, versus  $L^i$ , which represents the system operating level,  $i = 1, 2$ . For each value of  $L^i$ , varying in the range  $[0.5, 1]$  at steps of 0.005, 100,000 Monte Carlo simulations have been repeated. In these simulations,  $L_{fail}^i=1$ ,  $D_{j_i}=2\%$ ,  $P^i=I=7\%$ ,  $K_i=M=34$  for  $j_i = 1, 2, \dots, N_i$ ,  $i = 1, 2$ . Each simulation corresponds to a different sampled pattern of the  $M$  interdependency links.

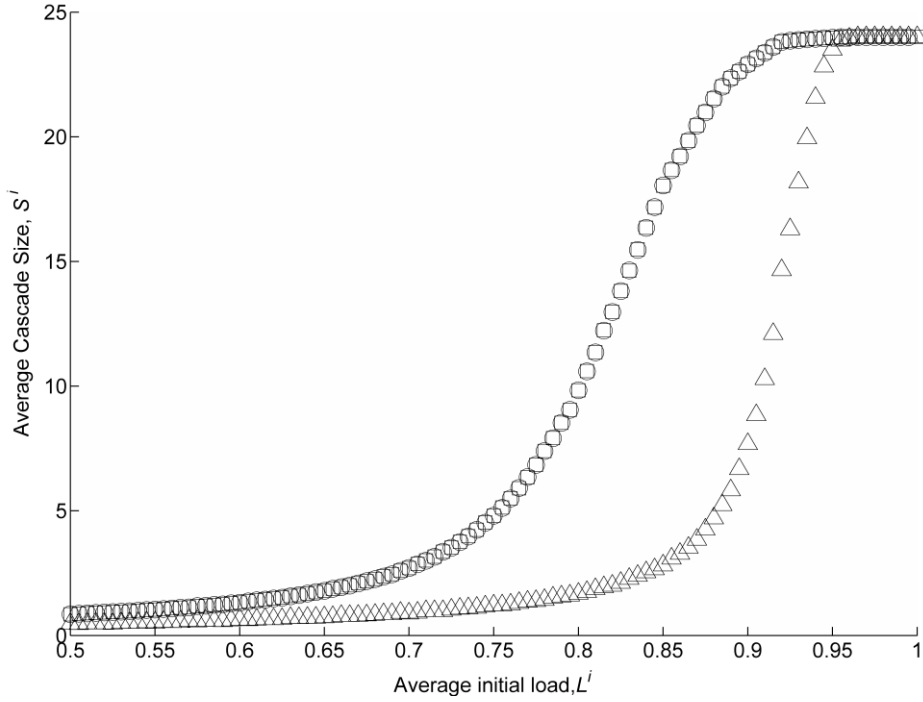


Fig. 2. The average cascade size vs. the average initial load. Triangles: isolated single system. Squares and circles: identical interdependent systems.

The triangles in Fig. 2 represent  $S^i$  in system  $i = 1, 2$ , as a function of  $L^i=L$ , for the isolated single system  $i = 1, 2$ , i.e., when no interdependency is present. The overlapping squares and circles represent the same quantity  $S^1 = S^2$  for the identical and identically operating systems 1, and 2, respectively.

As expected, the interdependencies cause a shift to lower values of the loading threshold for which the cascading phenomenon starts appearing with significance (from approximately 0.9 for the individual isolated system to approximately 0.8 for the interdependent systems). As  $L^i=L$  increases, the systems are increasingly vulnerable to cascading failures. The transition, to the region where cascade of significant size appear, is smoother for the two interdependent networks than for the individual isolated system. This result is due to the fact that cascades appear at lower  $L^i=L$  in the interdependent networks, which thus are less stressed, and prone to their propagation. As found in [17], the interdependencies between the two networks make access to the critical point easier, which means that the systems when coupled are more susceptible to large-scale failures, and a failure in one system can cause a similar

failure in the coupled system.

To quantitatively assess the effects of the interdependency in Fig. 2, a threshold representing the maximum allowable cascade size,  $S_{cr}^i$ , in system  $i = 1, 2$ , can be set. The intersection with the horizontal line,  $S^i = S_{cr}^i$ , identifies the critical load,  $L_{cr}^i$ , beyond which the threshold is exceeded in system  $i = 1, 2$ .  $S_{cr}^i$  is interpreted as the maximum number of components which can be lost in system  $i$  without affecting the global service provided by the infrastructure. This threshold can vary from system to system, and is a distinguishing feature of the provided service. In the following, for simplicity but with no loss of generality,  $S_{cr}^1 = S_{cr}^2 = S_{cr} = 15\%$  is assumed, which identifies  $L_{cr}^1 = L_{cr}^2 = L_{cr} = 0.8662$  for the individual systems in isolated conditions and  $L_{cr}^1 = L_{cr}^2 = L_{cr} = 0.7266$ , for the two interdependent systems (Fig. 2).

$L_{cr}^i$  is a relevant feature of a network system because it identifies, together with the continuous change in gradient, a type-two transition [45] between the cascade-safe region and the onset of disrupting cascades in terms of the loading conditions,  $L^i$ . Along with  $S^i$ , it gives essential information on the system vulnerability towards cascading failures, and it can help identify safety margins of system operation. On the other hand, see from Fig. 2 that  $S^i$  has small values until it reaches the proximity of  $L_{cr}^i$ , beyond which the transition to cascade region occurs. Thus, its use as a vulnerability indicator needs to be gauged against the sensitivity of detecting small changes in its value. Then, appropriate countermeasures of reducing the working load for limiting the failure propagation can be taken as the operation conditions are approaching  $L_{cr}^i$ . From Fig. 2, observe that the transition from cascade-safe to cascade regime is smoother for interdependent networks, allowing more time for reacting against the cascading failure onset.

Fig. 3 shows the probability distribution function for  $S^1$  in the interdependent system 1, when  $L^1 = L_{cr}^1$ . The point is characterized by the  $L_{cr}^1$  value at which the percolation transition occurs [44], and cascades of all sizes can propagate in the interdependent systems. The power-law tail for large  $S^1$  values developed at the critical point  $L_{cr}^1$  with an exponent of approximately -0.4 is consistent with the forest tree model analyzed in [17]. It is important to stress that even at the critical point  $L_{cr}^1$ , which marks the upper bound of the cascade-safe region, cascades of significant size ( $S^1=22$ , i.e. 92% of the system size  $N_1=24$ ) appear, even if their frequency is extremely low. Once more, this finding points to the fact that the use of  $L_{cr}^i$  as a vulnerability indicator needs to be gauged against the sensitivity of detecting small changes in its value. Then, appropriate countermeasures of reducing the working load for limiting the failure propagation can be taken as the operation conditions are approaching  $L_{cr}^i$ .

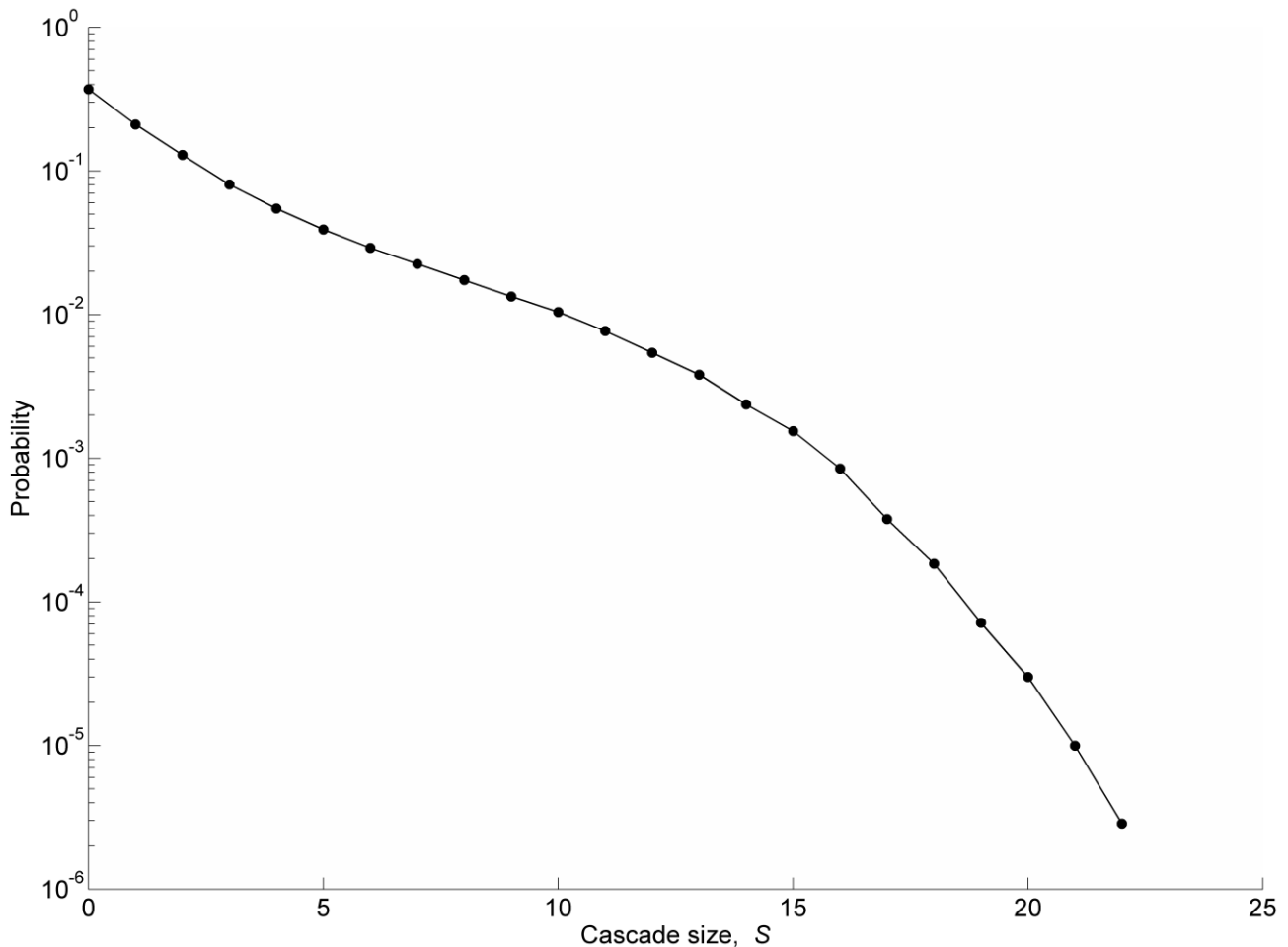


Fig. 3. The probability distribution of the cascade size  $S^i$  at the critical load  $L^i_{cr}$ . In these simulations  $L^i_{fail}=1$ ,  $D_{ji}=2\%$ ,  $P^i=I=7\%$ ,  $K_i=M=34$  for  $i=1, 2$ .

To understand the effects on the cascade process of the parameters characterizing the interdependency between the two systems, we conduct a further sensitivity analysis in several operating conditions which reflect real system operations.

#### A. Sensitivity with respect to the load of interdependent system 2

The following analysis aims at assessing the extent to which an interdependent system working at different, fixed load levels influences the coupled network system with respect to its vulnerability towards cascading failures. To this aim, the variation of  $L^1_{cr}$  is assessed while system 2 is working at fixed constant loads; the analysis is performed crudely for fixed values of average initial loads of system 2 ranging between  $L^2=0.5$  and  $L^2=1$ , in steps of 0.05.

Fig. 4 shows the results of this analysis. As expected, the coupling between the two systems is such to weaken the resistance of system 1 to failure cascade, forcing it to be operated at increasingly lower levels as  $L^2$  increases. The emerging functional dependence, however, could not be easily anticipated.  $L^1_{cr}$  decreases linearly for system 2 loading levels below  $L^2=0.75$ , and suddenly drops when system 2 loading levels rise above  $L^2=0.75$ , until it saturates

for system 2 average initial loads beyond  $L^2=0.9$ . This behavior indicates that, when  $L^2 \geq 0.9$ , system 1 experiences unbearable cascades irrespective of its loading level. There is no cascade-safe region for system 1 when system 2 is operating beyond 90% of  $L^2_{fail}$ .

The trend found indicates that the coupling between the two interdependent systems is such that under given loading conditions and beyond certain thresholds, the effects of the system nonlinearities become relevant, and an emergent behavior arises in the interdependent systems [7]. This result is in agreement with [17], in which it was found that symmetric coupling of network systems actually decreases nonlinearly the percolation threshold above which cascading failures of all sizes can propagate in the interdependent systems. In the present model, the percolation threshold for the propagation of cascading failures is given by  $L^1_{cr}$ , which has a nonlinear relation with the load of the interdependent system,  $L^2$ .

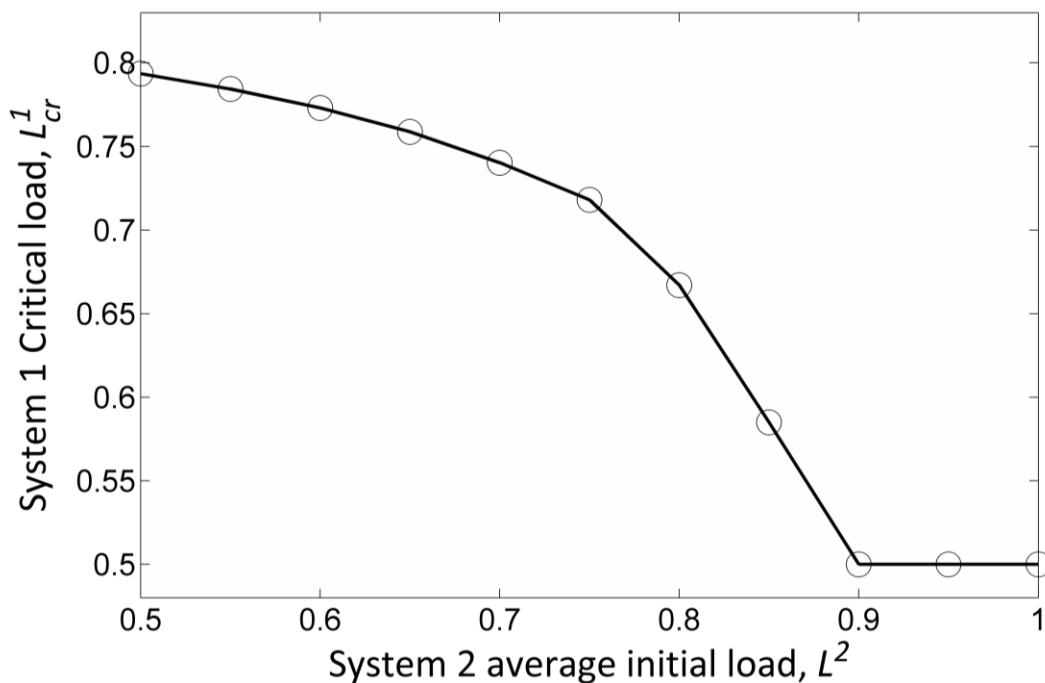


Fig. 4. Critical load,  $L^1_{cr}$ , in system 1 for constant average initial load levels,  $L^2$ , of system 2.

This result points to the fact that the loading level of the interdependent system plays a role of paramount importance in the failure behavior of its companion coupled system because it affects the cascading failure behavior of the latter in an unpredictable way. When designing and operating interdependent infrastructures, it is then necessary to control the operating levels of the systems, and assess the values beyond which nonlinearities start governing the system cascading failure behavior.

### B. Sensitivity with respect to the number of interdependency links, $M$

In the following analysis, the effects of  $M$  on the vulnerability to cascading failures are assessed in two different system operating conditions.

In the first case, both systems are operating at the same varying  $L^1 = L^2 = L \in [0.5, 1]$  at steps of 0.005, and the variation of  $L_{cr}^1$  is analyzed with respect to  $M$  (Fig. 5).  $L_{cr}^1$  is assessed starting the cascade simulation from  $L=0.5$ , and increasing this value until the onset of the cascade regime is encountered. Because both systems are identical and operate at the same loading conditions, they will show identical trends of  $L_{cr}^1 = L_{cr}^2 = L_{cr}$ , similarly to what is observed in Fig. 2. See from Fig. 5 that there is an approximately linear functional relationship between  $L_{cr}$  and  $M$ , up to the value  $M=70$  for which the system cascade-safe region disappears. If  $M \geq 70$ , the systems are going to experience unbearable cascades irrespective of the loading level. There is no cascade-safe region when more than  $M=70$  interdependency links are present between the two systems. Thus, if one were to try to protect the interdependent systems from cascade failure by controlling  $M$ , it appears that nonlinearities do not play a significant role. A linear decrease of the cascade-safe region is to be characterized with respect to the addition of interdependency links between the two systems instead.

The characterization of this relationship is relevant in the definition of cascade-safe operating regimes for the interdependent systems: for a fixed  $M$ , a critical loading level can be identified below which the systems can be safely operated. In the present example, it turns out that there is no safety margin when more than  $M=70$  interdependency links are present between the two systems, which is more than twice the number of links in each system,  $K_i=34$ ,  $i=1, 2$ . This means that beyond  $M=70$ , cascades of significant size spread in the network irrespective of  $L$ .

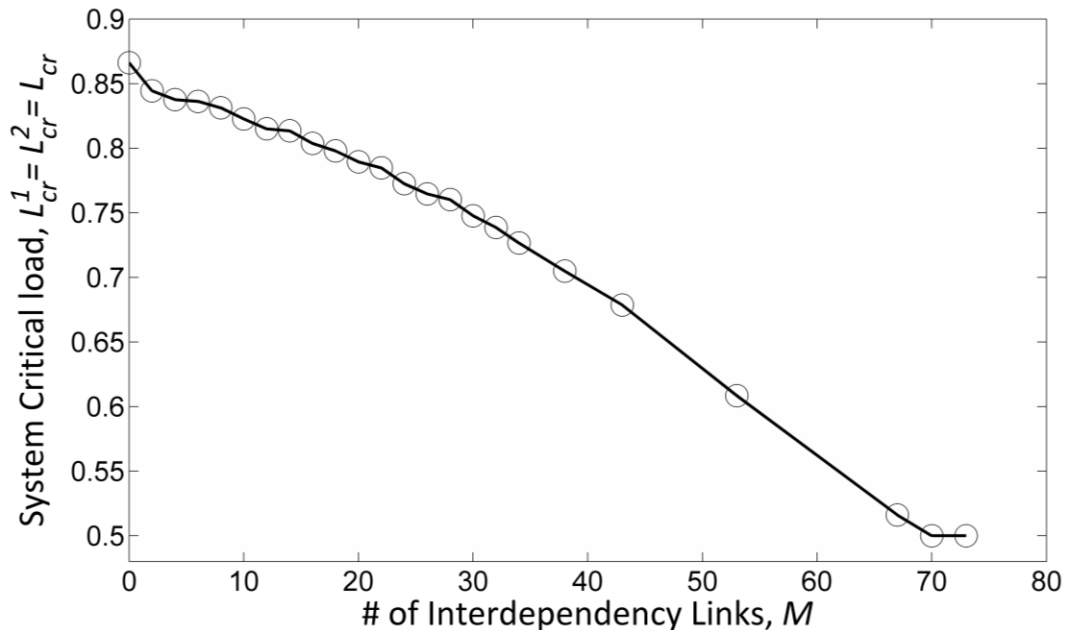


Fig. 5. Critical load,  $L_{cr}^1 = L_{cr}^2 = L_{cr}$ , in systems 1 and 2 vs.  $M$ . Both systems are working at the same varying average initial loads,  $L^1 = L^2 = L$ . In these simulations  $I=0.07$ .

In the second case, both systems are operating at the same constant  $L^1 = L^2 = L = 75\%$  of  $L_{fail}^1 = L_{fail}^2 = L_{fail}$ , and  $S^1 = S^2 = S$  is assessed with respect to the variation of  $M$  (Fig. 6).

As expected,  $S^1 = S^2 = S$  increases as  $M$  increases until a saturation value is reached, which is a function of the load transferred over the interdependency links,  $I$ , and the constant  $L$ .

Cascade-safe operating regimes can be identified with respect to  $S$ ; once the operating level is known ( $L = 75\%$  in this case), the systems can be operated or designed to limit the maximum  $S$ . As an example, from Fig. 6 see that to have cascades involving less than 15% of the system components, no more than  $M=29$  interdependency links can be operated between the two systems.

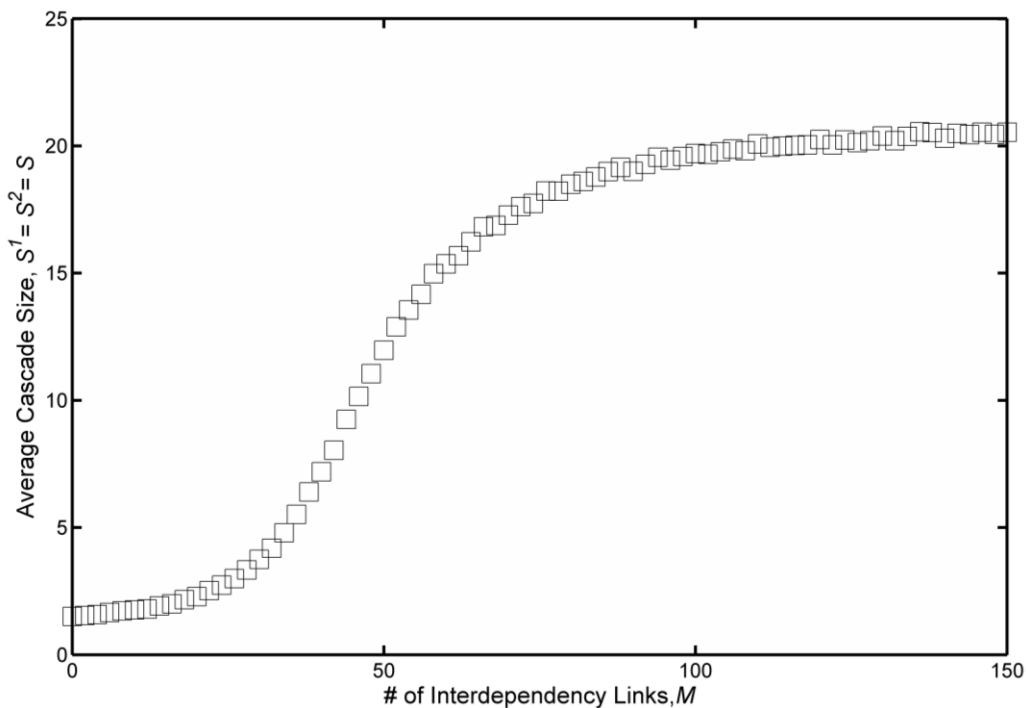


Fig. 6. Average cascade size,  $S^1 = S^2 = S$ , in systems 1 and 2 vs.  $M$ , while both systems operate at the same constant working load,  $L^1 = L^2 = L = 0.75$ . In these simulations,  $I=0.07$ .

### C. Sensitivity with respect to load transferred over the interdependency links, $I$

In the following analysis, the effects of the load transferred upon failure over the interdependency links of the failed component, are assessed in the same two system operating conditions introduced in Section B.

In the first case of both systems operating at the same varying average initial load, the variation of  $L_{cr}$  is investigated as a function of  $I$  (Fig. 7). Similar to the findings of the previous analysis, an approximately linear functional relationship emerges. The system cascade-safe region is dropping slowly, and only when  $I=25\%$  of  $L_{fail}$  is transferred over each interdependency link, cascades of significant size spread in the network irrespective of  $L$ . This slow drop points to the fact that  $I$  is a less critical parameter in designing and operating interdependent network

systems against cascading failures.

As before, cascade-safe operating regimes for the systems can be identified with respect to  $L_{cr}$ : for a fixed value of  $I$ , a critical loading level can be determined below which the systems can be safely operated. In the present example (Fig. 7), if  $I=9\%$ , operating the systems beyond  $L_{cr}=70\%$  of  $L_{fail}$ , could result in cascading sizes larger than the maximum allowable size (15% of the system size,  $N=24$ ).

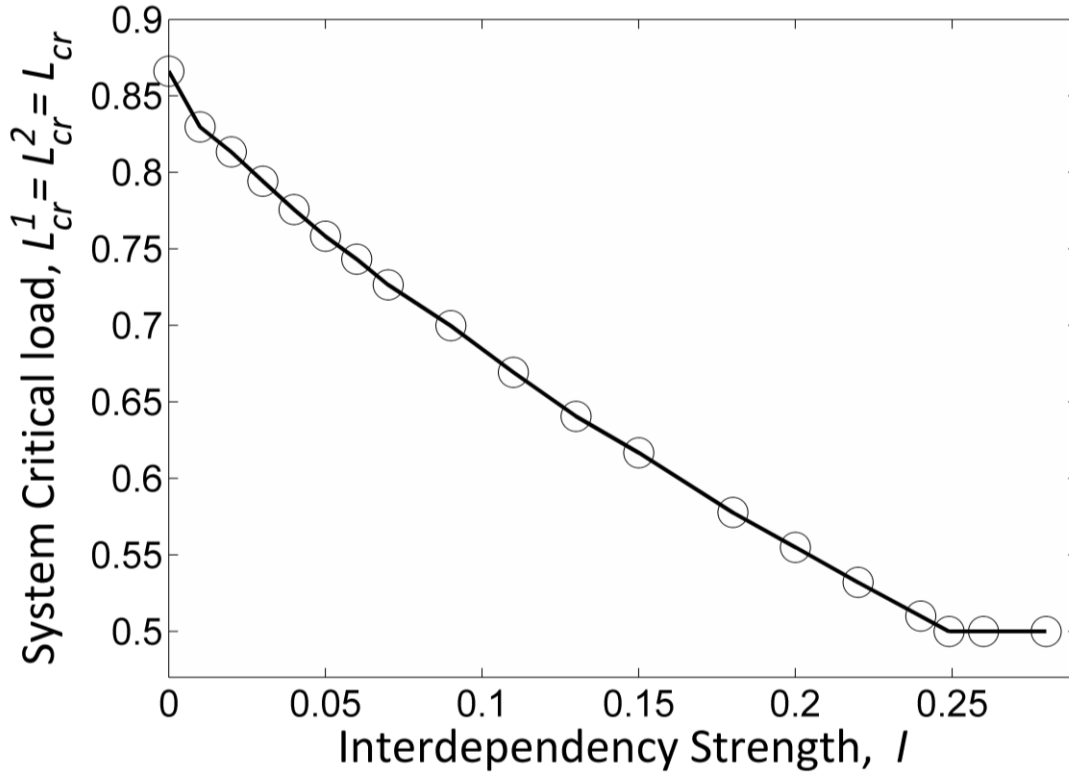


Fig. 7. Critical load,  $L_{cr}^1 = L_{cr}^2 = L_{cr}$ , in systems 1 and 2 vs.  $I$ , while both systems work at the same varying average initial load,  $L^1 = L^2 = L$ . In these simulations,  $M=34$ .

In the second case of both systems operating at the same constant  $L^1 = L^2 = L = 75\%$  of  $L_{fail}^1 = L_{fail}^2 = L_{fail} = 1$ ,  $S^1 = S^2 = S$  is assessed with respect to the variation of  $I$  (Fig. 8).

As expected,  $S$  increases as  $I$  increases until a saturation value is reached, which is a function of  $M$  and the constant  $L$ . Cascade-safe operating regimes can be identified with respect to  $S$ : given the operating level ( $L = 75\%$  in this case), the systems can be operated or designed to limit the maximum  $S$ . As an example, from Fig. 8 it can be understood that to have cascades involving less than 15% of the system components, no more than  $I=5.4\%$  of  $L_{fail}$  should be allowed to flow over each interdependency link connecting the two systems.

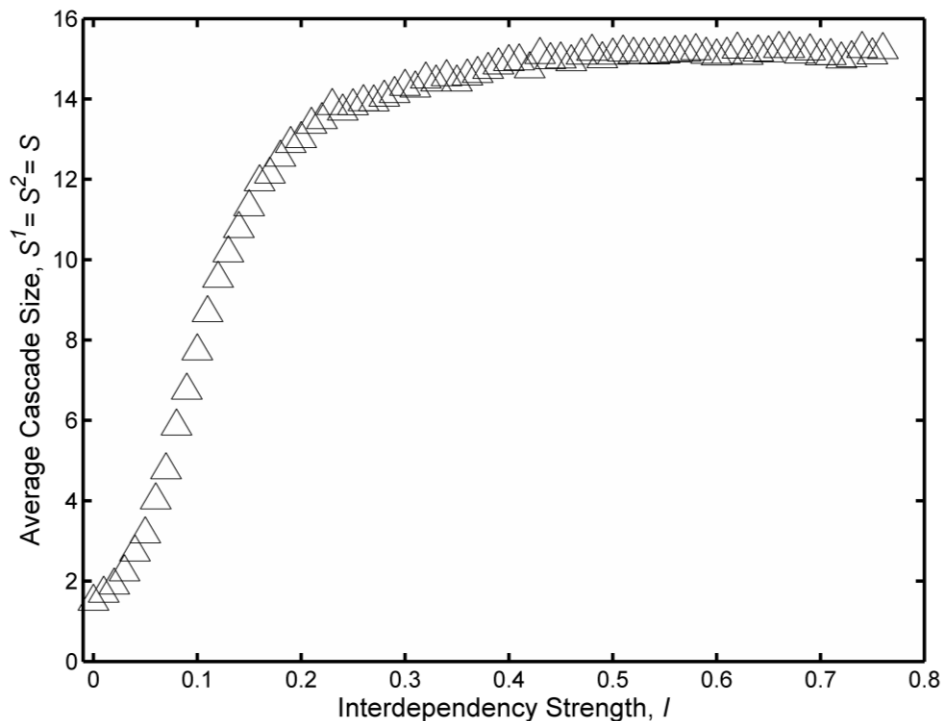


Fig. 8. Average cascade size,  $S^1 = S^2 = S$ , in systems 1 and 2 vs.  $I$ , while both systems operate at the same constant working load,  $L^1 = L^2 = L = 0.75$ . In these simulations  $M=34$ .

## V. CONCLUSION

An abstract model of cascading failures in interdependent network systems has been developed. Its application on a modified literature case study network structure has shown the usefulness of this kind of models for identifying the relevant factors affecting the failure cascade process, and for capturing their relationships with the effects of the cascade.

The simulation results show that the critical load,  $L_{cr}$ , and the average cascade size,  $S$ , give essential information on the system vulnerability towards cascading failures, and help identify cascade-safe regions for the system operations.

To investigate the effects of the parameters characterizing the interdependency between the two systems, a sensitivity analysis has been performed in several operating conditions which reflect real system operations.

The analysis has revealed that the interdependent system loading level is a vital feature to control because it affects the interdependent systems cascading failure behavior in an unpredictable way. In particular, when designing and operating interdependent infrastructures, it is important to determine the threshold level of loading beyond which nonlinearities emerge in the system cascading failure behavior.

The results of analyses of this kind allow defining cascade-safe operating regimes with respect to  $L_{cr}$  for the network systems: given the number of interdependency links in the system,  $M$ , and the load transferred over the interdependency links,  $I$ , loading levels can be identified below which the systems can be safely operated.



Furthermore, cascade-safe operating regimes can be identified with respect to  $S$ . Once the operating level is known, the systems can be designed or operated, tweaking  $M$  and/or  $I$ , to limit the maximum  $S$ .

An interesting direction currently under investigation is the extension to coupled infrastructures of an analytical model for computing thresholds of criticality. The thresholds are functions of the average degree of the network, and of the amount of load transferred in the uniform network base case.

As a concluding remark, a recent NSF workshop report points at the fact that uncertainty is pervasive in complex systems [46]. Thus, developing reliable predictions about system behavior in the face of the large numbers of uncertain parameters in models of actual complex systems is a major challenge. Quantifying this uncertainty, and determining how it propagates throughout the system is a key aspect of reliable prediction and control of cascading failure in critical infrastructures. Indeed, the choice of the model parameters can be critical for the system response, and for the full representation of the variability associated with the system response, including that due to the uncertainty in the model parameters (e.g.  $I$ ). Therefore, the evaluation of the cascade-safe operating margins will be complemented with a full quantification of uncertainty in a following study. This future study will add a further level of confidence to the findings of the sensitivity analysis.

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