



Fast Imaging of Void Defects in Conductive Half-Space

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Outline

- **Introduction**
- **Origin**
- **Eddy-Current (Diffusive) MUSIC Imaging**
- **Numerical Validation of Asymptotics**
- **MUSIC Images and Discussion**
- **Recapitulation**
- **Future works**

Introduction

Main motivations

- **Eddy-Current (EC) Non-Destructive Testing (NDT)** of metal workpieces of interest in wide range of applications
- **Position & some shape of defect** (with distinction between volumetric and crack-like) of **real relevance**
- **Computation-heavy** imaging via **iterative optimization**
- **Quicker non-iterative method**, even at loss of accuracy

Present-day approach

- **MUSIC** (Multiple Signal Classification) alg. for **detection of small voids**
- To our knowledge, first employed in low-frequency EC-NDT

Question investigated herein

What to infer from Multi-Static Magnetic Response matrices for diffusive-like wavefields & eddy-currents?

Origin?

General asymptotic framework & 3D imaging methods

- **Ammari & Kang** *Reconstruction of Small Inclusions from Boundary Measurements* Springer (04) and following books
- **Ammari, Iakovleva, Lesselier, Perrusson** MUSIC-type electromagnetic imaging of a collection of small 3-D bounded inclusions *SIAM J. Scientific Comput.* (07)
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- **Iakovleva, Gdoura, Lesselier, Perrusson** Multi-static response matrix of a 3-D inclusion in half-space and MUSIC imaging *IEEE Trans. Antennas Propagat.* (07)
- **Iakovleva, Lesselier** On the back-propagation of singular fields and the multi-static response matrix *IEEE Trans. Antennas Propagat.* (08)
- **Henriksson, Lambert, Lesselier** MUSIC-type algorithm for eddy-current non-destructive evaluation of small defects in metal plates *ENDE (XIV) IOS* (11)

Eddy-Current (Diffusive) Imaging (1)

Asymptotics (*valid for voids small vs. skin depth*) of secondary H field, for P spherical voids centered at \mathbf{x}_j , bobbin centered at \mathbf{r}_n :

$$\mathbf{H}_s^{(n)}(\mathbf{r}) = \mathbf{H}^{(n)}(\mathbf{r}) - \mathbf{H}_0^{(n)}(\mathbf{r}) = - \sum_{j=1}^P \left[i\omega\epsilon_2 \mathbf{G}_r^{me}(\mathbf{r}, \mathbf{x}_j) \cdot \mathbf{M}_j \left(\frac{\epsilon_j}{\epsilon_2}, V_j \right) \mathbf{E}_0^{(n)}(\mathbf{x}_j) \right]$$

Generalized polarization tensors:

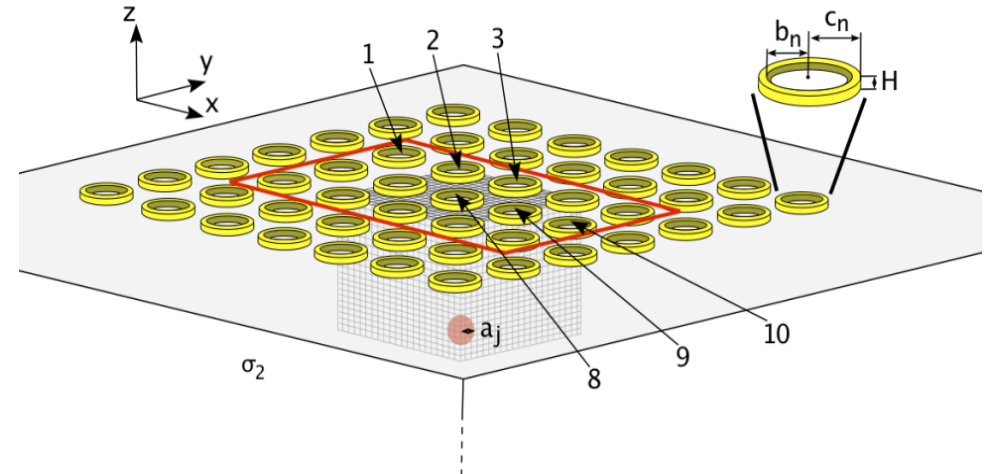
$$\epsilon_2 = \epsilon_0 + i \frac{\sigma_2}{\omega}$$

$$\epsilon_j = \epsilon_0$$

$$\mathbf{M}_j \left(\frac{\epsilon_j}{\epsilon_2}, V_j \right) = \frac{3(\epsilon_j - \epsilon_2)}{2\epsilon_2 + \epsilon_j} |V_j| \mathbf{I}, \text{ where } |V_j| = \frac{4\pi a_j^3}{3}$$

δ_s skin-depth

**Bobbin radius small enough:
dyadic Green's function
 $\mathbf{G}_t^{em}(\mathbf{r}_n, \mathbf{x}_j) \approx \text{primary } \mathbf{E}_0^{(n)}(\mathbf{x}_j)$**



Eddy-Current (Diffusive) Imaging (2)

MUSIC

Singular Value Decomposition \Leftarrow **Multistatic Response matrix \mathbf{A}** \Leftarrow **Secondary magnetic field (M receiver & N transmit array elements)**

$$G_r^{me}(\mathbf{x}_j) = k^2 [\mathbf{G}_r^{me}(\mathbf{r}_1, \mathbf{x}_j) \cdot \hat{\mathbf{z}}, \dots, \mathbf{G}_r^{me}(\mathbf{r}_M, \mathbf{x}_j) \cdot \hat{\mathbf{z}}]^t$$

$$G_t^{em}(\mathbf{x}_j) = k^2 [\mathbf{G}_t^{em}(\mathbf{x}_j, \mathbf{r}_1) \cdot \hat{\mathbf{z}}, \dots, \mathbf{G}_t^{em}(\mathbf{x}_j, \mathbf{r}_N) \cdot \hat{\mathbf{z}}]^t$$

$$\mathbf{A} = \sum_{j=1}^P G_r^{me}(\mathbf{x}_j) \mathbf{M}_j G_t^{em}(\mathbf{x}_j) = \mathbf{U} \Sigma \mathbf{V}^*$$

Number of non-zero singular values for j^{th} defect \Leftarrow **rank of matrices $\mathbf{G}_r^{me}(\mathbf{x}_j)$ and $\mathbf{G}_t^{em}(\mathbf{x}_j)$**

\Rightarrow **Image function $\mathbf{W}(\mathbf{x})$** \Leftarrow **orthogonal projectors onto noise subspaces associated to left & right signal subspaces \mathbf{U}_s and \mathbf{V}_s**

$$\mathbf{W}(\mathbf{x}) = \frac{1}{\|\mathbf{P}_r(G_r^{me}(\mathbf{x}) \cdot \mathbf{a})\|^2} + \frac{1}{\|\mathbf{P}_t(G_t^{em}(\mathbf{x}) \cdot \mathbf{a})\|^2}$$

$$\mathbf{P}_r = (\mathbf{I} - \mathbf{U}_s \mathbf{U}_s^*)$$

$$\mathbf{P}_t = (\mathbf{I} - \mathbf{V}_s \mathbf{V}_s^*)$$

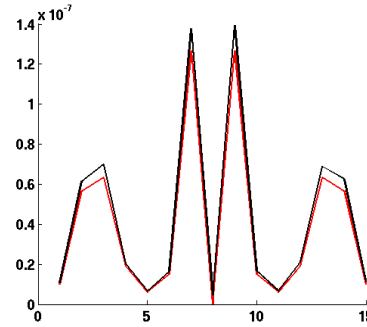
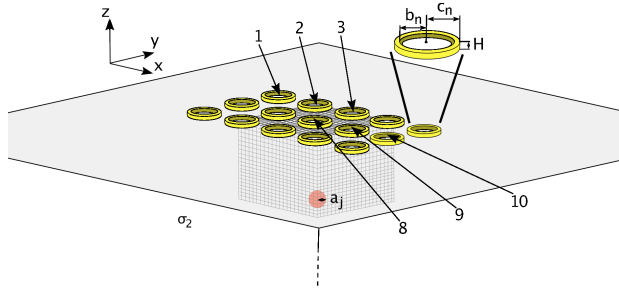
$$\mathbf{a} = (1, 1, 1)$$

Numerical Validation of Asymptotic Signals

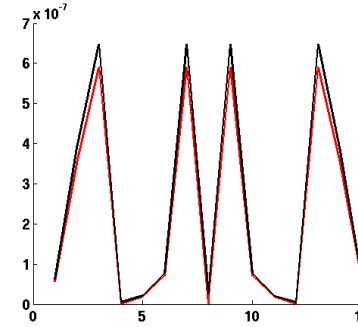
$$\mathbf{H}_s^{(n)}(\mathbf{r}_m) \cdot \hat{\mathbf{z}} \approx \Delta Z_{nm}$$

$$\Delta Z_{nm} = -\frac{1}{I_n I_m} \int_{V_j} \mathbf{E}_0^{(m)}(\mathbf{r}) \cdot \mathbf{J}_j^{(n)}(\mathbf{r}) d\mathbf{r}$$

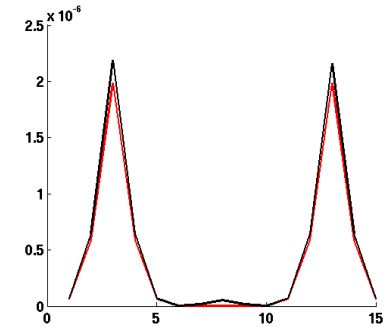
**Spherical void
(radius $\delta_s/10$)
center of search
zone, $\delta_s/2$ deep**



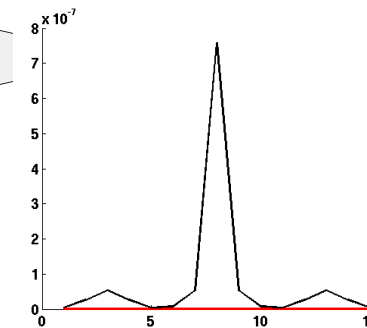
(a)



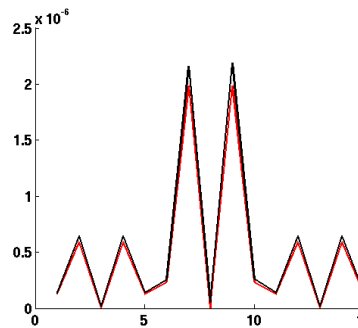
(b)



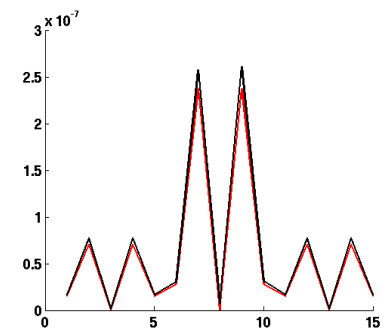
(c)



(d)



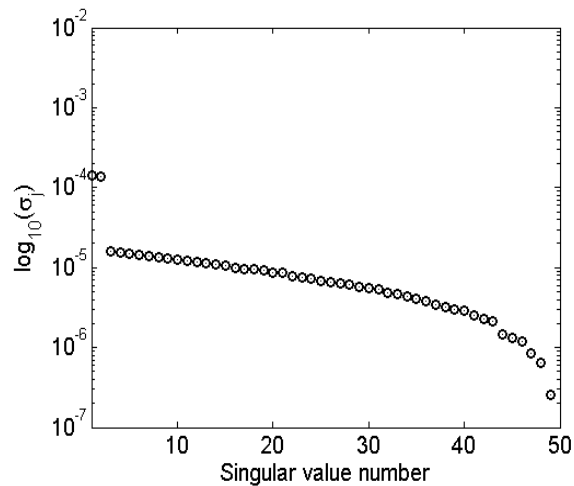
(e)



(f)

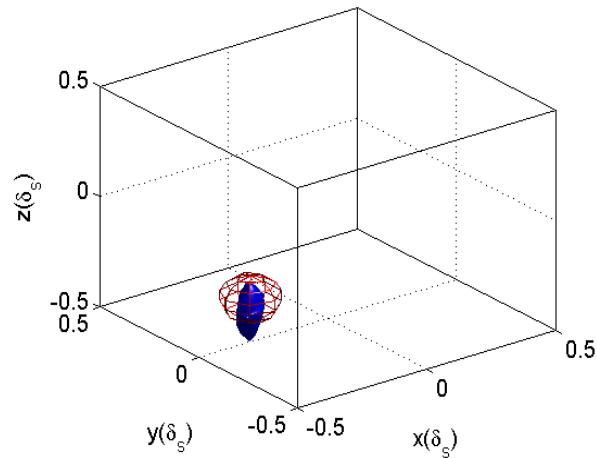
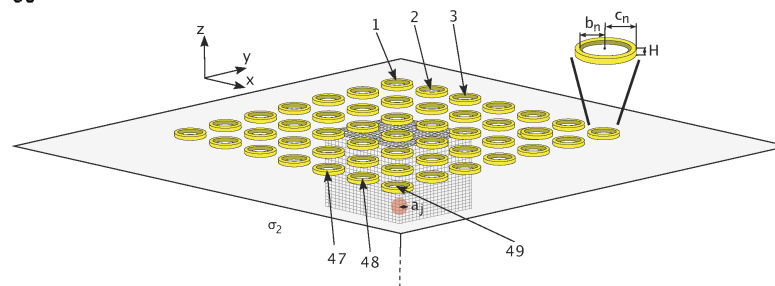
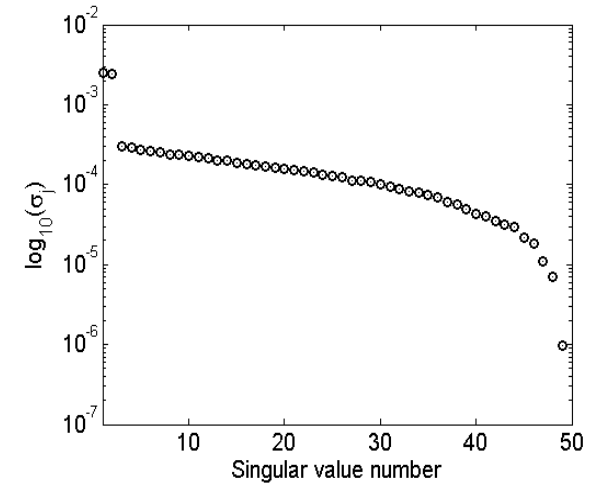
Impedance variation CIVA (black) & z-polar. asymptotic H-field with primary \mathbf{E}_0 using \mathbf{G}_t^{em} (red), along 15 (3×5) receiving bobbins when exciting from bobin (a) 1, (b) 2, (c) 3, (d) 8, (e) 9, (f) 10

MUSIC Imaging from Noisy Asymptotic Data (1)

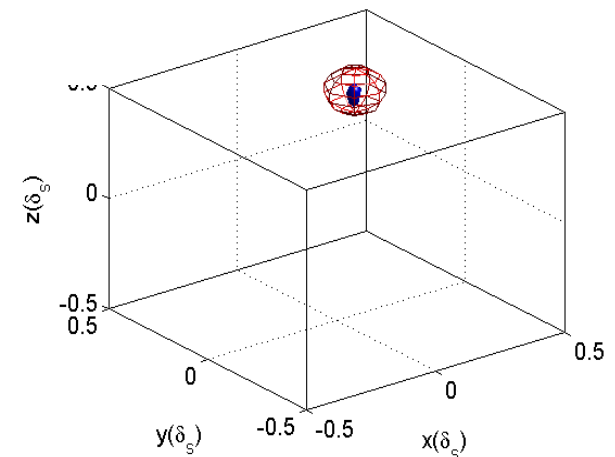


Spherical void (radius $\delta_s/10$) at arbitrary location in search space.

Asymptotic data, 10 dB Gaussian noisy

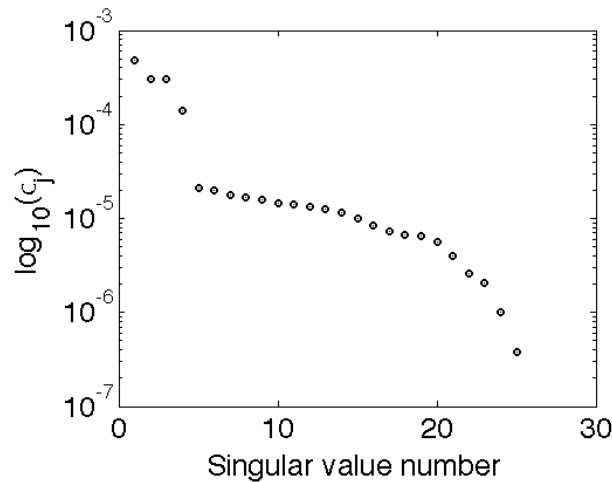


Depth:
 $3\delta_s/4$ (left)
 $\delta_s/4$ (right)



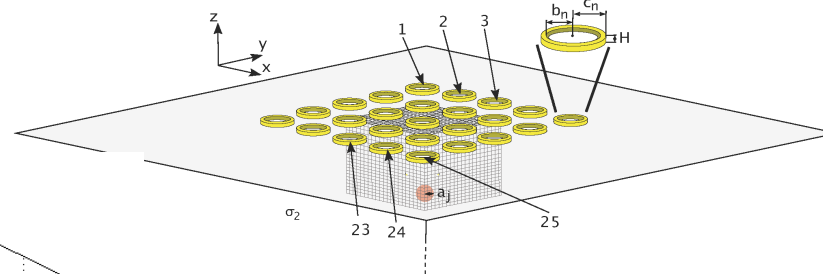
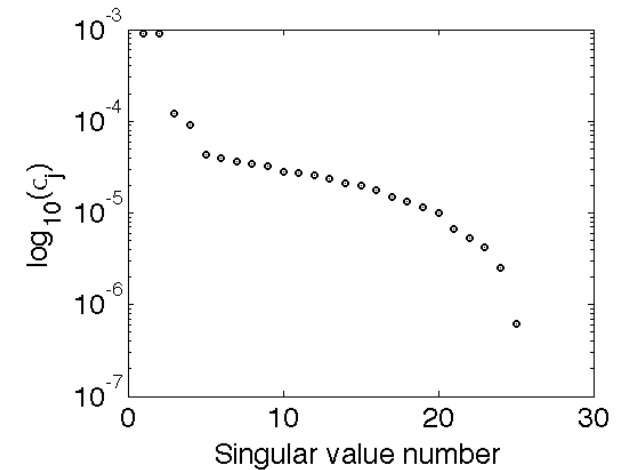
Singular value distribution & MUSIC images with full 7 x 7 array set-up
 $[\delta_s/2$ separation between each center, radii $\delta_s/6.5$]

MUSIC Imaging from Noisy Asymptotic Data (2)

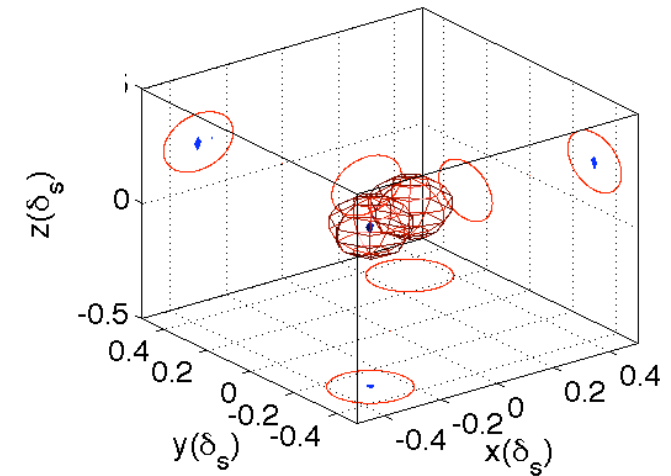
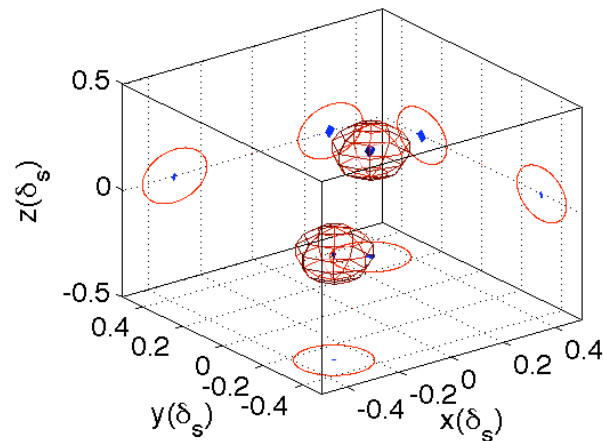


Two spherical voids
(radii $\delta_s/10$) at same
or different depths in
search space.

Asymptotic data,
20 dB Gaussian noisy

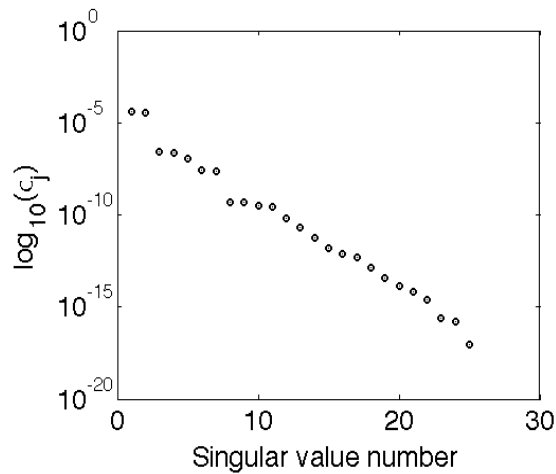


Depths:
0.5 & 0.5 δ_s (left)
0.4 & 0.6 δ_s (right)



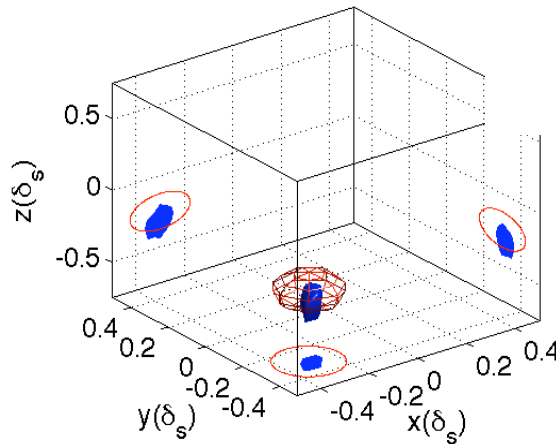
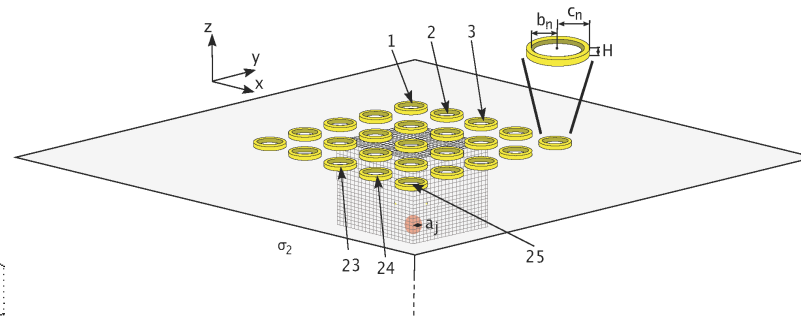
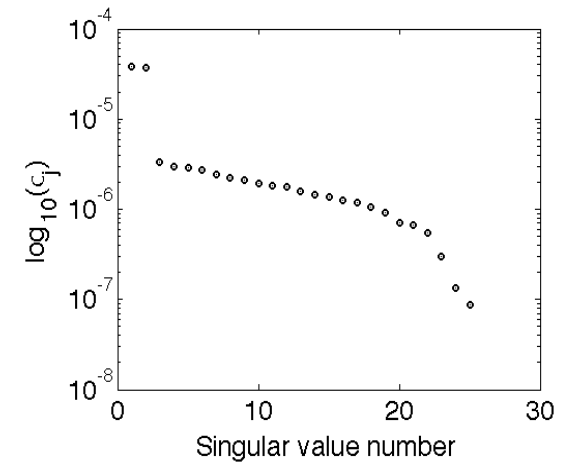
Singular value distribution & MUSIC images with full 5 x 5 array set-up
[$3\delta_s/2$ separation between each center, radii $\delta_s/6.5$]

MUSIC Imaging from Noiseless & Noisy CIVA Data

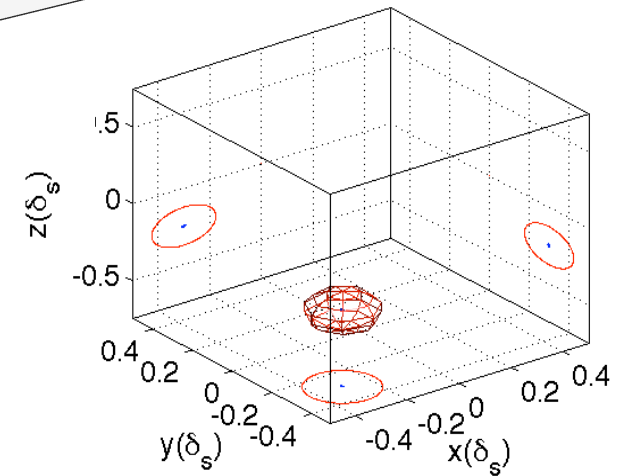


One spherical void
(radius $\delta_s/10$) at depth
 $3\delta_s/4$

CIVA noiseless data
(left) & **15 dB Gaussian**
noisy data (right)

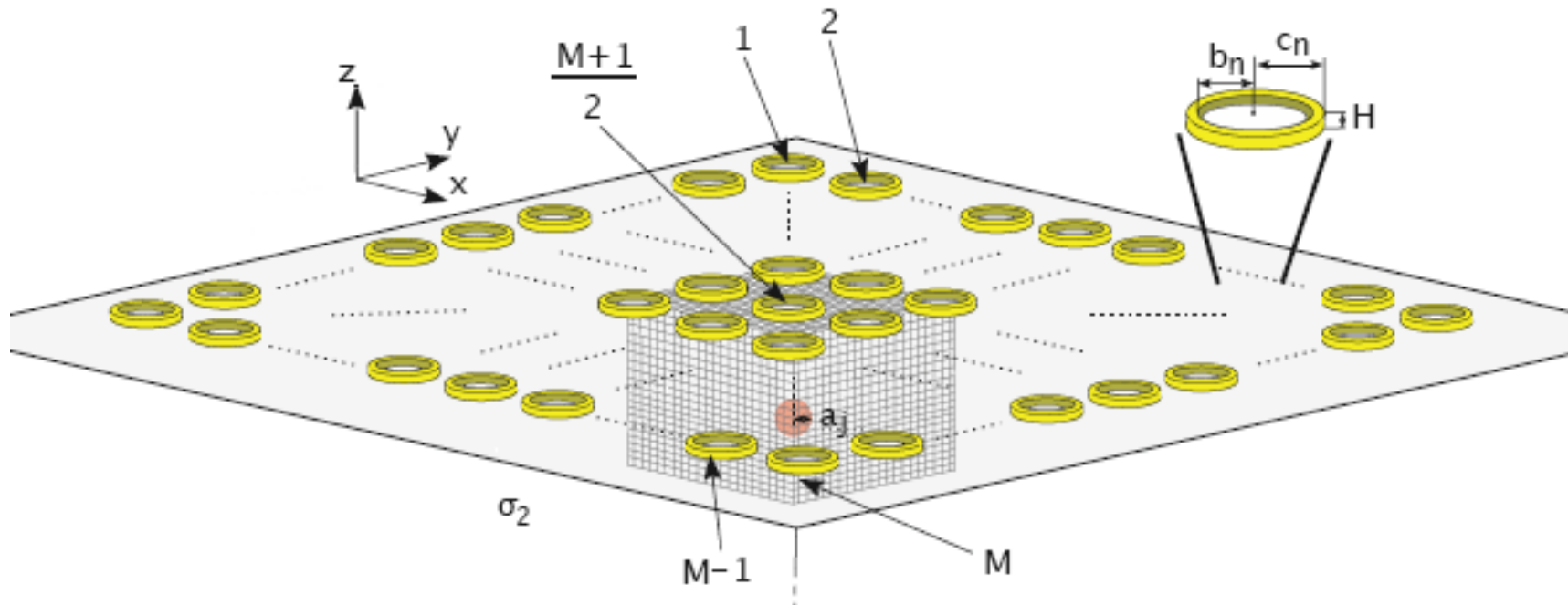


Extra singular values
correlated to geometry
of void, seem to drown
into noise



Singular value distribution & MUSIC images with full 5 x 5 array set-up
 $[3\delta_s/2$ separation between each center, radii $\delta_s/6.5]$

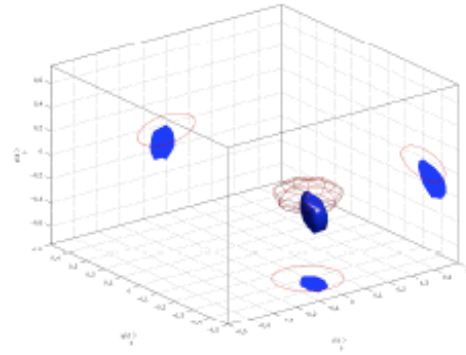
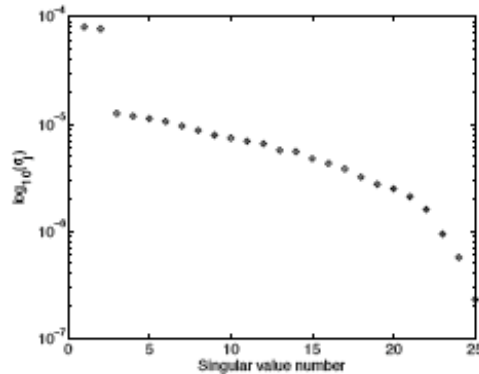
Changing Size & Density of Array (1)



$\delta_s = 1.58 \text{ mm}$

- Each bobbin: inner & outer radii $b_n = \delta_s/6.5$, $c_n = \delta_s/6$, thickness $H = \delta_s/15.8$, lift-off $\delta_s/15.8$
- Total number of bobbins $M = 16, 25, 36 \text{ \& } 49 \rightarrow$ array 4×4 to 7×7
- $(\Delta x, \Delta y)$ separation between bobbin centers varied from $2\delta_s/7$ to $2\delta_s$

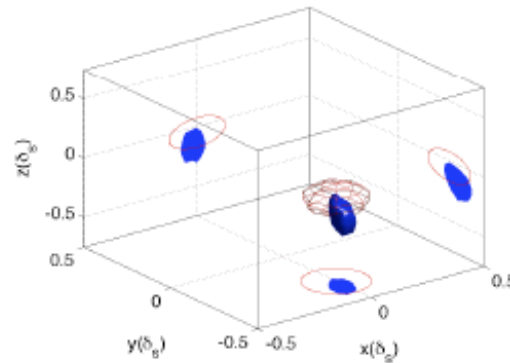
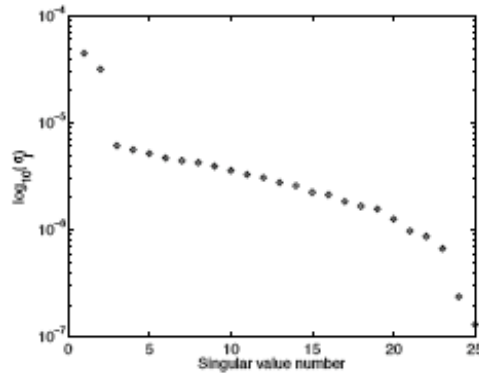
Changing Size & Density of Array (2)



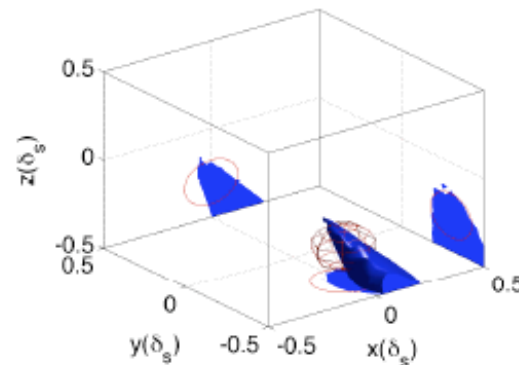
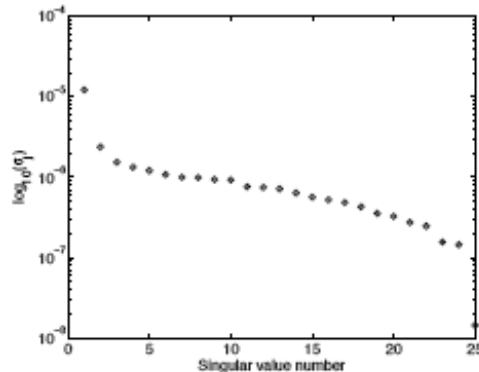
array 5 x 5, 10 dB noise

$(\Delta x, \Delta y)$ separation

$= 2\delta_s/3$ (side $\approx 8\delta_s/3$)

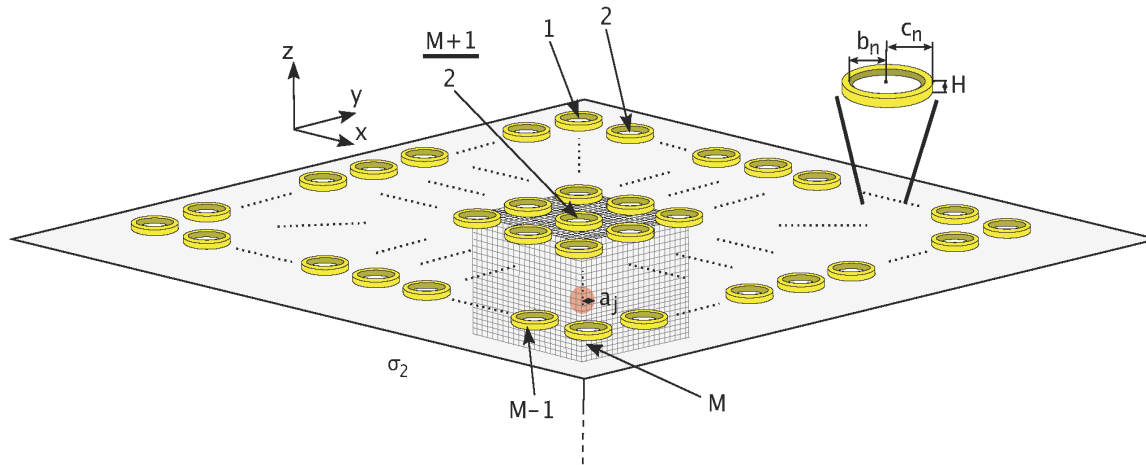


$= \delta_s$ (side $\approx 4\delta_s$)



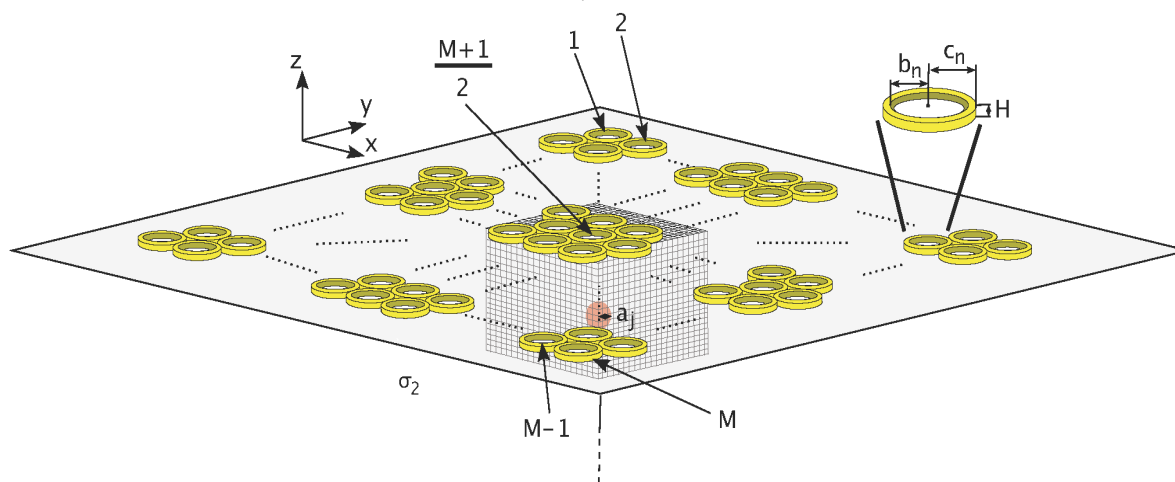
$= 2\delta_s$ (side $\approx 8\delta_s$)

Changing Array Topology (1)



Uniform array

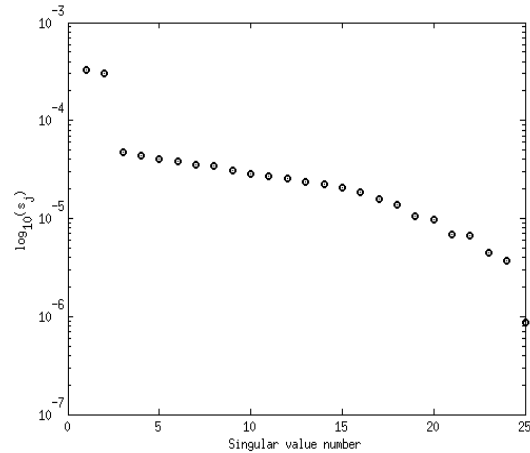
Uniform spreading of bobbins, with fixed separation between



Tightly packed array

Bobbins next to each other, separation $\Delta x = 2c_n$ (outer radius) and Δy as small as possible

Changing Array Topology (2)

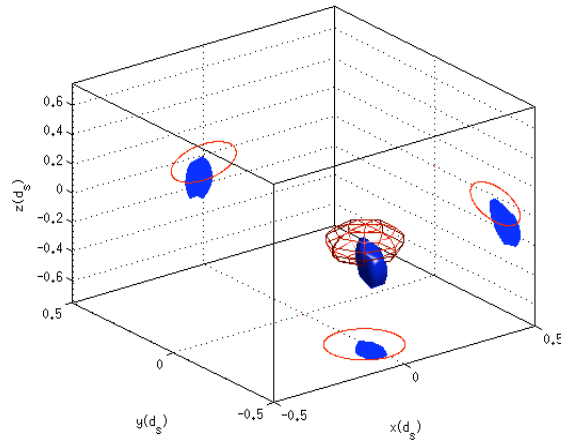


One spherical void
(radius $\delta_s/10$) at depth
 $4\delta_s/5$

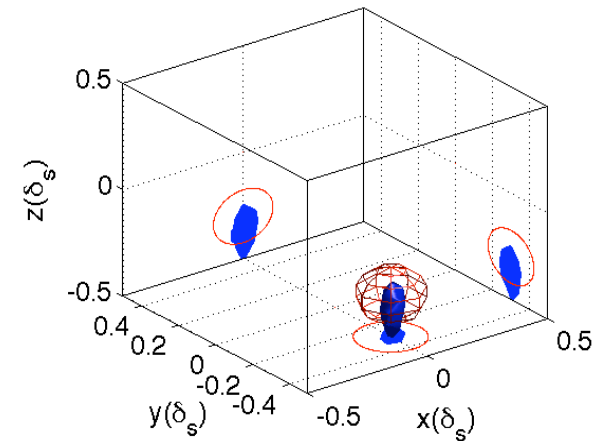
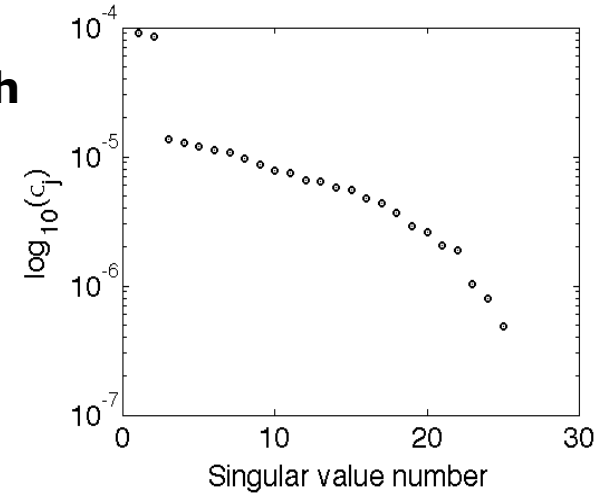
10 dB noisy data

5 x 5 array, which is

either
uniform, with $2\delta_s/3$
separation
(left)



or
packed, with
separation $\Delta x = \delta_s/3$
(right)



Recap

- Asymptotic secondary magnetic field **OK as long as bobbin size and radius of defect small vs. skin-depth** of conductive plate.
- Successful **MUSIC images achieved** for one/two spherical voids with both **asymptotic** and **CIVA** data.
- A **5 x 5 uniform bobbin array** with separation $\approx 2\delta_s/3$ suffices in most cases for noise up to 10 dB. Density and size to be matched.
- Lost information due to **smaller number** of bobbins also **compensated** by **increased array area** (up to certain point).
- A **tightly packed array** might improve **robustness** against noise a bit.
- Image isolevels (here 70% of max) adjustable as well (*not shown here*) yet not so essential.
- Singular values **sensitive** to noise level of data.

Future Works

- Managing multi-frequency datasets (e.g., 3 sets in EC-NDT)

Use of another, more appropriate imaging functional

Improve robustness vs. noise

- Characterization of defects via retrieval of Polarization Tensors

Polarization Tensors (PT) retrieved for ellipsoidal equivalents

Unless higher order of PT, no separation shape/electrical parameters

- Extension to crack-like (planar) defects

Need of simple direct model, e.g., Pavo & Lesselier, *T-Mag* 06.

Need of sound imaging theory, e.g., like 3-D counterpart of

Park, Lesselier MUSIC-type imaging of a thin penetrable inclusion from its multi-static response matrix *Inverse Problems* (09)

Park, Lesselier Electromagnetic MUSIC-type imaging of perfectly conducting, arc-like cracks at single frequency *J. Comput. Phys.* (09)