Petri-Net Simulation Model of a Nuclear Component Degradation Process

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Abstract: Multi physical state modeling (MPSM) is a novel approach being investigated for estimating the reliability of components and systems in the context of probabilistic risk assessment (PRA). The approach integrates multi-state modeling, which describes the degradation process by transitions among discrete states (e.g. initial, micro-crack, rupture, etc) and physical modeling by (physical) equations that govern the degradation process. In practice, the degradation process is non-Markovian and its transition rates are time-dependent and influenced by external factors such as temperature and stress. Under these conditions, it is in general difficult to derive the state probabilities analytically.

On the contrary, Petri nets provide a flexible modeling framework for describing degradation processes with arbitrary transition rates. In this paper, we build a Petri net in support of Monte Carlo simulation of the stochastic aging behavior of a nuclear component undergoing stress corrosion cracking. The results are compared with analytical results derived in a previous work of literature.

Keywords: Degradation process, Multi-state model, Petri net, Monte Carlo simulation.

1. INTRODUCTION

Degradation processes have been intensively studied in the reliability engineering community [1-4]. In general, the degradation models can be classified into analytical models [2-5] and simulation models [1, 6, 7]. The analytical degradation models can be further classified into the following three groups:

- Statistical models of time to failure (e.g. lifetime distribution [2]).
- Models describing the evolution of a measurable quantity indicating time-dependent degradation, and failure upon reaching a threshold value (e.g. Brownian motion [4] and gamma process [3]).
- Multi-state models of degradation [5].

Multi-state models (MSM) [8] are frequently applied for component degradation process modeling, since they fit practically to component aging processes in real life situations when there is a range of levels from perfect functioning to complete failure. To model the dynamics of the degradation process, Markov [9] and semi-Markov models [10] have been used. In the Markov models, the transition rates between states are constant, which means that the degradation process is memoryless.

In some recent works [11, 12], the non-homogenous continuous time Markov model (NHCTMM) has been introduced to account for the aging effects of un-repairable components or systems. The component degradation process is also possibly dependent upon other external factors (e.g. temperature, stress) [13].

Multi-state modeling requires estimating the transition rates from field data. In practice, it can be difficult or even impossible to collect relevant data especially for the highly reliable devices (e.g. nuclear components, aerospace devices, etc).

To overcome some of the problems mentioned above, a novel approach based on multi-state physics modeling has been proposed [14], in which the transition rates are described by physics functions rather than estimated from service data. The resulting model is non-Markovian since the transition rates are time-dependent and uncertain. To solve the problem, in the original work by [14] a state-space enrichment approach has been used upon discretization of the component lifetime into equally sized time intervals, during each of which the transition rate remains constant. Then, the component degradation process is converted into a discrete time Markov chain (DTMC) residing in a largely enriched state space described by a tuple $\boldsymbol{S}_e = (\boldsymbol{S}, \boldsymbol{t}_c)$, where $\boldsymbol{S}$ is the vector of original component degradation states and $\boldsymbol{t}_c$ is the vector of discretized holding times at each state.

In this work, we propose an integrated simulation framework for modeling the stochastic aging behavior of components. Such framework is supported by a stochastic Petri net (SPN), which provides a flexible model representation scheme for describing the state transition process. Uncertain external influencing factors (e.g. temperature and stress) are included. A Monte Carlo (MC) simulation algorithm is proposed to realize the degradation transition process and compute the state probability distributions.

The rest of the paper is organized as follows. Section 2 presents the formal definition of the multi-state physics model, with consideration of time-dependent transition rates and uncertain external influencing factors. Section 3 introduces the stochastic Petri nets. Section 4 presents the integrated framework and the detailed MC simulation procedures of the integrated model. In Section 5, the real-world case study from Unwin et al. (2011) is used as an application, and a comparison is made with the state-space enrichment technique. Section 6 concludes the work and points out possible future extensions.

2. MULTI-STATE PHYSICS MODELLING OF COMPONENT DEGRADATION PROCESSES

Under the framework of multi-state modeling, the dynamics of component degradation is described by transitions among a finite number of discrete degradation states $\boldsymbol{S} = \{s_0, s_1, \dots, s_M\}s$. The solution of the multi-state model is the state probability vector at any time instant t, $\boldsymbol{P}(t) = \{p_0(t), p_1(t), \dots, p_M(t)\}$, given the transition rates $\lambda_{i,j}$ from state i to state j. In MSPM, the transition rate $\lambda_{i,j}(t, \boldsymbol{\theta})$ is a function of time t, for given values of the physical factors $\boldsymbol{\theta}$. This function can be formulated based upon material science knowledge about the degradation physics of the component (e.g. the crack development process [13]). With the consideration of degradation physics, the following assumptions are made for MSPM:

- The component consists of (M+1) states where states '0' and 'M' represent the complete failure state and perfect functioning state, respectively. The generic intermediate state i ($0<i<M$) is a degradation state where the component is partially functioning.
- The initial state (at time t = 0) of the component is M.
- Repair can be performed on the intermediate states. Once the component is in complete failure (e.g. rupture), it is no longer repairable.
- The transition rates $\lambda_{i,j}(t, \boldsymbol{\theta})$ from state i to state j is a function of time and of the external influencing factors $\boldsymbol{\theta}$, whose values may not be precisely known.

The transition rate is defined as:

$$\lambda_{i,j}(t, \boldsymbol{\theta}) = \lim_{\Delta t \to 0} \frac{\Pr\left(\emptyset(t+\Delta t)=j \mid \emptyset(t)=i, \boldsymbol{\theta}\right)}{\Delta t} \tag{1}$$

where $\boldsymbol{\theta} = \{\theta_0, \theta_1, \dots, \theta_N\}$ is the vector of the external influencing factors, which can be random variables (the probability density function (PDF) of $\boldsymbol{\theta}$ is denoted as $p(\boldsymbol{\theta})$), and $\emptyset(t)$ is a discrete function representing the stochastic degradation process and taking values from the state space $\boldsymbol{S}$.
The target of multi-state modeling is to solve the state probability vector $\boldsymbol{P}(t) = \{p_0(t), p_1(t), \dots, p_M(t)\}$ with:

$$p_i(t) = \int \Pr(\emptyset(t) = i \mid \boldsymbol{\theta})\, p(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{2}$$

The integral at the right-hand side of (2) is over all possible values of $\boldsymbol{\theta}$. If the transition rates are constant, the state probabilities can be obtained by solving the ordinary differential equations corresponding to the state space diagram. Solving analytically the Markov model with time-dependent transition rates and possibly random $\boldsymbol{\theta}$ is a difficult task [15]. Then, approximation methods are introduced. One used by several researchers [14, 15] amounts to discretizing the component lifetime into intervals and assuming a constant value of transition rate in each interval. By doing so, the description of the stochastic process is converted into a discrete time Markov chain (DTMC) with a significantly enriched state space which includes discrete time steps, and characterized by a sparse transition matrix. For example, given a component with 6 original degradation states and a lifetime of 100 years discretized in time steps of 0.5 years, the approximation method would generate a

transition matrix of $(6 \times 200)^2$ entries. Alternatively, the simulation modeling framework offers a promising way of solution.

3. STOCHASTIC PETRI NETS

Petri nets (PNs), coined by Carl Petri [16], are an adaptive, versatile and yet simple graphical modeling tool for representing dynamic systems. PNs have successful applications in the reliability modeling of various systems. A PN is a bipartite directed graph with two types of nodes in which abstract objects (tokens), drawn as bold-faced dots, are moved, created or diminished. The two types of nodes are: places (states) $p_i^{PN}(i = 0, \dots, M)$, which are circular and usually denote the states of the system being modeled, and transitions $t_j^{PN}(j = 1, \dots, N)$, which are bars and denote the transitions corresponding to actions or events that result to a state change. Places are linked only to transitions using directed arcs $a_{p_i^{PN} t_j^{PN}}$, and vice versa. It is possible for a place to have multiple arcs to or from the transition, which can be condensed down to a single arc with a weight or multiplicity denoted by a slash through the arc with a number next to it. If there is no slash, the weight is usually assumed to be 1 (it is also the default weight value).

The tokens, which represent objects in the model, are stored in places. The movements of the tokens passing between places represent the transitions in the system. The transition t_j^{PN} is enabled only if the weight of each incoming arc is at most equal to the number of tokens at the corresponding input place. In original PNs, the transitions are assumed to be immediate. The stochastic Petri nets (SPNs) introduce delays of a transition which can be either immediate (in which case the transition bar is solid), deterministically time-delayed, or randomly time-delayed based on a pre-defined probability distribution. Once the time period has passed and the transition remains enabled, the switching will take place. The switching will remove the number of tokens in each input place corresponding to the weight of the relevant incoming arcs and create the number of tokens in each output place corresponding to the weights of the relevant outgoing arcs.

The SPN is often used as a model representation tool: it can be internally converted to a continuous time Markov chain (CTMC) for solution, when the time delay at each transition follows an exponential distribution [17]; the Monte Carlo (MC) simulation is used to solve the SPN directly, when time delays are random [18]. For the MSPM formulation embraced in this study of a stochastic degradation process, MC simulation is used to solve the SPN.

4. THE INTEGRATED SIMULATION MODEL

A graphical sketch of the integration of SPN and uncertain external influencing factors is given in Fig 1. The transition rates are dependent on the values of $\boldsymbol{\theta}$ which is a vector of *n* random parameters with joint probability distribution $p(\boldsymbol{\theta})$. Given the general form of the transition rate (1), the total rate of departure from state *i* is:

$$\lambda_i(t, \boldsymbol{\theta}) = \sum_{\substack{j=0 \\ j \neq i}}^{M} \lambda_{i,j}(t, \boldsymbol{\theta}) \tag{3}$$

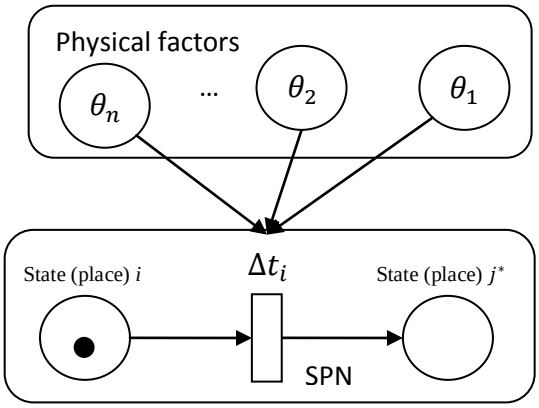

Fig 1. Sketch of the integrated model

4.1 Basics of the Monte Carlo simulation model

To obtain the state probability in (2), the following M differential equations need to be solved given the realizations of $\boldsymbol{\theta}$ (the detailed theoretical analysis for Monte Carlo simulation for ICTMC can be found in [19]):

$$\frac{d}{dt}p_i(t|\boldsymbol{\theta}) = \sum_{\substack{k=0 \\ k\neq i}}^{M} p_k(t|\boldsymbol{\theta})q_{k,i}(t|\boldsymbol{\theta})\lambda_k(t,\boldsymbol{\theta}) - p_i(t|\boldsymbol{\theta})\lambda_i(t,\boldsymbol{\theta}) \tag{4}$$

where $i = 1, \ldots, M$ and

$$q_{k,i}(t|\boldsymbol{\theta}) = \lambda_{k,i}(t,\boldsymbol{\theta})/\lambda_k(t,\boldsymbol{\theta}) \tag{5}$$

The quantity $q_{j,i}(t|\boldsymbol{\theta})$ is regarded as the conditional probability that given the transition out of state j at time t and the values of $\boldsymbol{\theta}$, the transition arrival state will be i. To rewrite equation (4) into integral form, an integrating factor $M_i(t,\boldsymbol{\theta}) = \exp\left[\int_0^t \lambda_i(t',\boldsymbol{\theta})dt'\right]$ is used. Multiplying both sides of (4) by the integrating factor, we obtain:

$$\frac{d}{dt}[p_i(t|\boldsymbol{\theta})M_i(t,\boldsymbol{\theta})] = M_i(t,\boldsymbol{\theta})\sum_{\substack{k=0 \\ k\neq i}}^{M} p_k(t|\boldsymbol{\theta})q_{ki}(t|\boldsymbol{\theta})\lambda_k(t,\boldsymbol{\theta}) \tag{6}$$

Taking the integral of both sides, we obtain:

$$p_i(t)M_i(t,\boldsymbol{\theta}) = p_i(0) + \int_0^t \left[M_i(t',\boldsymbol{\theta})\sum_{\substack{k=0 \\ k\neq i}}^{M} p_k(t'|\boldsymbol{\theta})q_{ki}(t'|\boldsymbol{\theta})\lambda_k(t',\boldsymbol{\theta})\right]dt' \tag{7}$$

Substituting $M_i(t,\boldsymbol{\theta})$ with $\exp\left[\int_0^t \lambda_i(t',\boldsymbol{\theta})dt'\right]$, we obtain the following expression of $p_i(t|\boldsymbol{\theta})$:

$$p_i(t|\boldsymbol{\theta}) = p_i(0)\exp\left[-\int_0^t \lambda_i(t',\boldsymbol{\theta})dt'\right] + \int_0^t \exp\left[-\int_{t'}^t \lambda_i(t'',\boldsymbol{\theta})dt''\right]\sum_{\substack{k=0 \\ k\neq i}}^{M} p_k(t'|\boldsymbol{\theta})q_{ki}(t'|\boldsymbol{\theta})\lambda_k(t',\boldsymbol{\theta})\,dt' \tag{8}$$

In the MC simulation, the probability distribution function $p_i(t|\boldsymbol{\theta})$ is not sampled directly. Instead, the process holding time at state i is sampled and then the transition from state i to another state j is determined. This procedure is repeated until the accumulated holding time reaches the predefined time horizon. The resultant time sequence consists of the holding times at different states.

To sample the holding time, the probability density (or total frequency) of departing state i, $\psi_i(t|\boldsymbol{\theta})$, needs first to be obtained by multiplying $\lambda_i(t,\boldsymbol{\theta})$ both sides of (8):

$$\begin{aligned}\psi_i(t|\boldsymbol{\theta}) &= \lambda_i(t,\boldsymbol{\theta})p_i(t|\boldsymbol{\theta}) \\ &= p_i(0)\cdot\lambda_i(t,\boldsymbol{\theta})\cdot\exp\left[-\int_0^t \lambda_i(t',\boldsymbol{\theta})dt'\right] \\ &\quad + \int_0^t \lambda_i(t,\boldsymbol{\theta})\cdot\exp\left[-\int_{t'}^t \lambda_i(t'',\boldsymbol{\theta})dt''\right]\cdot\sum_{\substack{k=0 \\ k\neq i}}^{M}\psi_k(t'|\boldsymbol{\theta})q_{ki}(t'|\boldsymbol{\theta})\,dt' \\ &= p_i(0)f_i(t|0,\boldsymbol{\theta}) + \sum_{\substack{k=0 \\ k\neq i}}^{M}\int_0^t \psi_k(t'|\boldsymbol{\theta})q_{ki}(t'|\boldsymbol{\theta})f_i(t|t',\boldsymbol{\theta})dt'\end{aligned} \tag{9}$$

where

$$f_i(t|t',\boldsymbol{\theta}) = \lambda_i(t,\boldsymbol{\theta})\exp\left[-\int_{t'}^t \lambda_i(t'',\boldsymbol{\theta})dt''\right] \qquad t \geq t' \tag{10}$$

is defined as the conditional PDF that the process will depart state *i* at time *t*, given that the process is at state *i* at time t' and the values of the external influencing factors $\boldsymbol{\theta}$. Equation (9) indicates that the PDF $\psi_i(t|\boldsymbol{\theta})$ consists of the sum of contributions from the random walks with transitions passing through all the states (including state *i*) from time 0 to *t*, given the values of $\boldsymbol{\theta}$. To obtain the marginal distribution $\psi_i(t)$, the conditional distribution is multiplied with the PDF of $\boldsymbol{\theta}$ and the results is integrated over all possible values of $\boldsymbol{\theta}$:

$$\psi_i(t) = \int \psi_i(t|\boldsymbol{\theta})\, p(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{11}$$

Based on equation (11), the MC simulation procedure mentioned above can be derived. The CDF of departure time t given that it is at state *i* at time t' is denoted as:

$$F_i(t|t') = \int F_i(t|t', \boldsymbol{\theta}) p(\boldsymbol{\theta}) d\boldsymbol{\theta} = 1 - \int \exp\left[-\int_{t'}^{t} \lambda_i(t'', \boldsymbol{\theta}) dt''\right] p(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{12}$$

Given t' and (12), the departure time *t* can be sampled through direct inversion sampling, acceptance-rejection sampling, and other sampling techniques [20].

Following the departure, the marginal transition probabilities to any other state $j = \{0, \ldots M | j \neq i\}$ are calculated as:

$$q_{i,j}(t) = \int \frac{\lambda_{i,j}(t,\boldsymbol{\theta})}{\lambda_i(t,\boldsymbol{\theta})} p(\boldsymbol{\theta}) d\boldsymbol{\theta} \tag{13}$$

and a uniformly distributed random number *U* is sampled in the interval [0,1]: if $\sum_{k=0}^{j^*-1} P_{i,k} < U < \sum_{k=0}^{j^*} P_{i,k}$, then the transition to state j^* is activated and occurs at time t_i units of time. After time $\Delta t_i = t - t'$ a new token will appear at place j^* and the token at place *i* is removed.

4.2 The simulation procedures

The simulation procedures are presented in this Section. Prior to the simulation, incorporation of the external influencing factors should be carried out through the following steps: 1) formulate the functions describing the physics of the transition rates; 2) identify the external influencing factors θ_i (e.g. temperature, stress); 3) define the distribution functions, $p(\boldsymbol{\theta})$ representing the uncertainties in the values of these factors.

The algorithm for the simulation of the process of component degradation on the time horizon $[0, t_{max}]$ is sketched in the following pseudo-code:

Initialize the system by allocating a token onto place $i = M$ (initial state of perfect performance), setting the time $t = 0$ (initial time) and the total number of replications N_{max}
Set $t' = 0$
Set $n = 1$
While $n < N_{max}$
    While $t < t_{max}$
        Sample a realization of the external influencing factors $\boldsymbol{\theta}$ from the joint probability function $p(\boldsymbol{\theta})$
        Sample a departure time t from the distribution function $F_i(t|t', \boldsymbol{\theta})$
        Sample a random number *U* from the uniform distribution in [0, 1]
        For each outgoing transition ($j = 0,1,\ldots,M, j \neq i$)
            Calculate the transition probability $q_{i,j}(t|\boldsymbol{\theta})$
            If $\sum_{k=0}^{j^*-1} q_{i,k} < U < \sum_{k=0}^{j^*} q_{i,k}$
            Then activate the transition to state j^*
            End If
        End For
        Set $t' = t$

Remove the token from place i and add a new token onto place j^*

End While

Set $n = n + 1$

End While

Subsequent to the execution of the simulation algorithm, an estimate $\widehat{\boldsymbol{P}}(t) = \{\widehat{p_0}(t), \widehat{p_1}(t), \dots, \widehat{p_M}(t)\}$ of the state probability vector is computed by dividing the total number of visits to each state by the total number of simulations N_R: $\widehat{\boldsymbol{P}}(t) = \frac{1}{N_{max}}\{n_0(t), n_1(t), \dots, n_M(t)\}$, where $\{n_i(t)|i = 0, \dots, M, t \leq t_{max}\}$ is the total number of visits to state i at time t. It is noted that the derived distributions $p(\boldsymbol{\theta})$ and $F_i(t|t')$, may have complicated mathematical expressions; under these circumstances, the Markov Chain Monte Carlo technique can be used to sample random values [1].

5. EXPERIMENTS AND RESULTS

5.1 Case study

The case study refers to the cracking process in an Alloy 82/182 dissimilar metal weld in a primary coolant system of a nuclear power plant [14]. Cracks can grow from the inner to the outer diameter of the dissimilar metal welds in one of the three major morphologies: axial, radial, and circumferential. The latter two types can lead to the rupture of the component. The crack growth has two steps (1) crack initiation, (2) crack propagation. The radial crack mainly grows outward from the initiation site towards the outer diameter; the process can lead to a leak and potentially to rupture. The circumferential crack grows relatively evenly around the circumference, potentially leading to a rupture. The Petri net representing the multi-state physics model of the crack growth process, is given in Fig. 3. The detailed information about the definitions of the transition rates can be found in [14]. It is noted that in their formulation, the transition rates $\varphi_1, \varphi_2, \varphi_3$ and φ_4 are time-dependent.

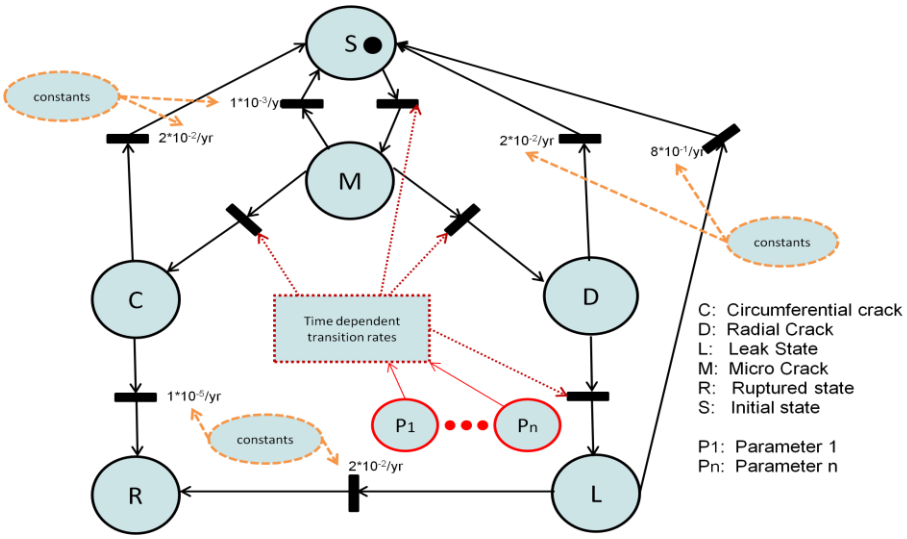

Fig 3. Petri net of the multi-state physics model of the crack growth process in Alloy 82/182 dissimilar metal weld

5.2 Results

The simulation model supported by a Petri net description of the degradation process has been applied to the case study presented above, with the same parameter settings as in [14]. It is noted that in the original study, the uncertainties of the external influencing factors (e.g. temperature and pressure) have not been modeled in the crack initiation process and have been inexplicitly modeled in the crack propagation process by means of a uniform distribution of the transition rate $\dot{a}_M$.

The simulation model has been executed $N_{max} = 10^6$ times over a component lifetime $t_{max} = 80$ years, in line with the original study. To investigate the convergence of the simulation model, the 10^6 realizations have

been subdivided into N = 20 subsamples of 50000 each. The sample mean and variance of the estimated state probabilities are calculated. At t = 80 years, the variances are 0.6749×10^{-8}, 0.776×10^{-8}, 0.0352×10^{-8}, 0.0106×10^{-8}, 0.0037×10^{-8}, and 0.0337×10^{-8} for 'initial', 'micro-crack', 'circumferential', 'radial', 'leak', and 'rupture' states, respectively. Similar results are found at different time moments. The good stabilization of state probabilities at $t = 80$ is manifested. Similar convergence curves are obtained at different time moments but are not presented here, due to space limitation.

For comparison with the state-space enrichment technique proposed in the original study of [14], the state probabilities resultant from 10^6 simulation runs are shown in Figure 4 as functions of time. Comparing to the results from the original study, it is observed that the shapes and trends of the probability curves are similar to those of state-space enrichment (with different step sizes), in that in all plots there is: 1) an early, rapid transition from the initial state to the micro crack state; 2) a monotonic increase in the probability of the rupture state.

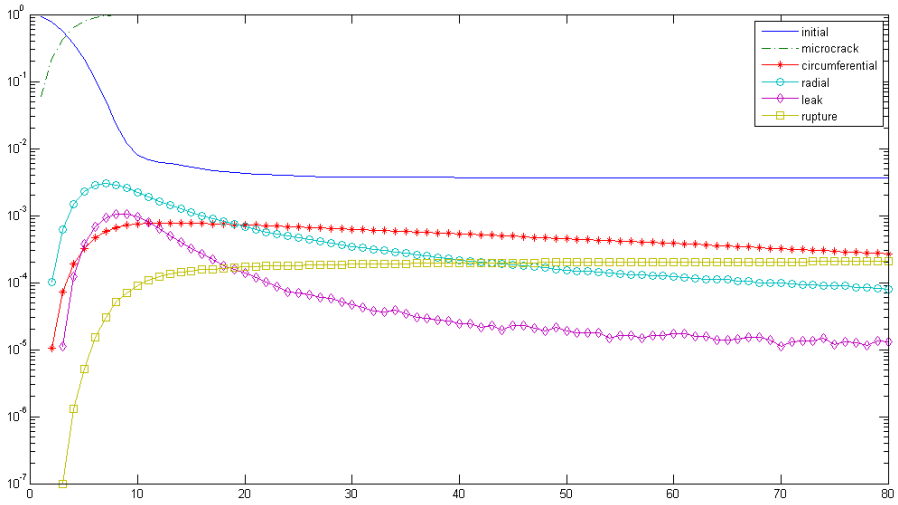

Fig 4. State probabilities obtained by simulation

The numerical comparisons on the state probability values at year 80 are reported in Table 1. The differences between the simulation and state-space enrichment methods increase as the step size in the latter is reduced. This confirms that the state-space enrichment method is sensitive to the step size, as expected.

Table 1. Comparison of simulation results with state-space enrichment results (state probability values at year 80)

	Simulation	State-space enrichment Step size = 1 year	State-space enrichment Step size = 0.5 year	State-space enrichment Step size = 0.1 year
Initial state probability	***0.0036***	0.0033	0.0035	0.0042
Micro crack probability	***0.9958***	0.9956	0.9938	0.9897
Circumferential crack probability	***2.72e-4***	1.95e-4	4.72e-4	0.0030
Radial crack probability	***7.78e-5***	6.38e-05	1.41e-4	8.47e-4
Leak probability	***1.18e-5***	8.93e-06	2.14e-5	0.00014
Rupture state probability	***2.07e-4***	1.38e-4	3.47e-4	0.002199

5.3 Uncertain external influencing factors

As explained in Section 4, the SPN-supported simulation framework is able to explicitly accommodate the uncertainties in the external influencing factors. To show this, as example we assign a truncated normal distribution to the temperature T values of the Weibull scale parameter τ which is used in the rate function of the first transition. According to [21], τ has the following relationship with temperature: $\tau = 9.2 \times 10^7 \times \sigma^{-7} \times \exp\left(\frac{129}{8.314\times10^{-3}\times T}\right)$, where σ is the stress constant. The truncated normal distribution is defined as: $p(T) = \frac{\phi(T-598.15)}{\phi(623.15-598.15)-\phi(573.15-598.15)}$, where $\phi(\cdot)$ denotes the PDF of a normal distribution. Without loss of generality, the variance is assumed to be equal to one. The simulation results are displayed in Figure 6, in terms of mean values of the state probabilities (solid lines) together with their 95% confidence intervals (dashed lines). The results show that the confidence intervals are larger at lower probability values, which implies a larger variance on rare events.

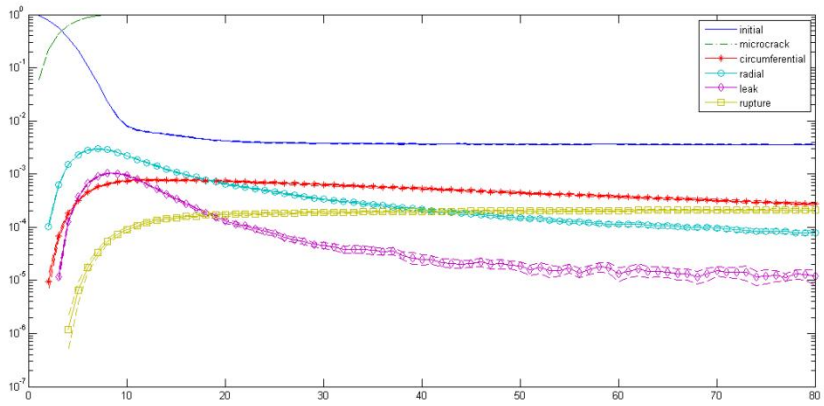

Fig 5. SPN simulation accommodating uncertainty in temperature

Numerical comparisons are reported in Table 2 in terms of the state probability mean values at year 80 and the relative differences of the results obtained by simulation with and without uncertain factor. It is observed that the uncertainty in temperature has significant impacts on the mean value of the initial state probability, as expected.

Table 2. Comparison of simulation results w/o uncertain factors (state probability mean values at year 80)

	Without uncertain factors	Uncertain temperature	Relative differences
Initial state probability	***0.0036***	0.012	-236.17%
Micro crack probability	***0.9958***	0.9873	0.86%
Circumferential crack probability	***2.72e-4***	2.96e-4	-8.82%
Radial crack probability	***7.78e-5***	9.15e-5	-17.61%
Leak probability	***1.18e-5***	1.55e-5	-31.36%
Rupture state probability	***2.07e-4***	2.14e-4	-3.38%

6. CONCLUSIONS AND FUTURE WORKS

An SPN-supported simulation framework has been proposed to solve the multi-state physics model (MSPM) describing a component degradation process with time-dependent transition rates and uncertain external influencing factors. SPN provides a flexible tool for representing the dynamics of degradation processes, and the

simulation solution allows handling time-dependent transition rates and uncertain influencing factors, without analytical complications.

The framework has been applied with success on a nuclear component undergoing stress corrosion cracking. The comparison with analytical approximated results has been satisfactory and the framework has been shown capable of indeed explicitly accommodating the uncertainties in the external influencing factors. Future research work is envisaged on the following aspects: 1) how to represent and propagate the uncertainties in the external influencing factors when there is insufficient data to estimate probability distributions; 2) other application cases may include multiple influencing factors with dependencies; 3) extension of the model for application to multiple degradation processes of components and systems.

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