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PRICING MECHANISMS FOR COOPERATIVE STATE ESTIMATION

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ABSTRACT

The conflict between cooperation in distributed state estimation and the resulting leakage of private state information (competitive privacy) is studied for an interconnected two regional transmission organizations (RTOs) model of the grid. Using an information theoretic rate-distortion-leakage (RDL) tradeoff model, each RTO communicates at a rate chosen to optimize an objective function that is dependent on two opposing quantities: a rate-distortion based pricing function that encourages cooperation, and a leakage function that impedes it. It is shown that strictly non-zero pricing incentives are required to achieve non-trivial target distortions.

***Index Terms*—** Competitive privacy, distributed state estimation, rate-distortion-leakage tradeoff, pricing mechanisms

1. INTRODUCTION

The electric power industry is undergoing profound changes as greater emphasis is placed on the importance of a smarter grid that supports sustainable energy utilization. Technically, power system estimation and control are likely to involve many more fast information gathering and processing devices (e.g. Phasor Measurement Units) [1]. Economically, the deregulation of the electricity industry has led to the creation of many regional transmission organizations (RTOs) within a large interconnected power system [2]. Both technical and economic drivers suggest the need for more distributed estimation and control in power system operations.

Traditionally, state estimation requires a central coordinator that estimates the state using measurements from all the RTOs. More recently, a distributed approach to this problem has been considered; however, distributed state estimation research has dominantly focused on two-tier hierarchical models [3] in which each local control center (e.g., an RTO) estimates independently, and at a higher level, a central coordinator receives the estimation results from the individual areas and coordinates them to obtain a system-wide solution. However, this model does not scale with increasing measurement rates due to communication and reliability challenges inherent in systems with one coordination center. Furthermore, the physical interconnectedness of the RTOs makes the problem of wide area monitoring and control important and immediate. This requirement of estimating state precisely and often is driving the need for a *fully distributed approach* (without a central coordinator) to state estimation wherein the local control centers *interactively* estimate the system state as a whole.

In [4], the authors introduce a *competitive privacy* notion at the level of the RTOs to capture the conflict between sharing data for distributed estimation (utility/benefit to all RTOs) and withholding data for economic and end-user privacy reasons. Assuming that the RTOs are willing to cooperate, they present an information-theoretic rate-distortion-leakage framework for a two-RTO network and show that each RTO has to tolerate a level of information leakage proportional to the fidelity (distortion) required at the other RTO. A natural question that arises is whether such a cooperation can be achieved via incentives. In this paper, we address this question by introducing a pricing-based objective function at each RTO that depends on two opposing quantities: a rate-distortion based pricing function that encourages cooperation, and a leakage function that impedes it. We show that strictly non-zero pricing incentives are required to achieve non-trivial target distortions.

This paper is organized as follows: in Sec. 2, we describe our model and in Sec. 3 we develop our main results. We illustrate our results in Sec 4 and conclude in Sec. 5.

2. SYSTEM MODEL AND PROBLEM FORMULATION

In [4], the authors present a linearized noisy Gaussian measurement model at the RTOs; thus, a vector of measurements at each RTO is a noisy linear function of the system states, i.e. $Y_1 = X_1 + \alpha X_2 + Z_1,\ Y_2 = \beta X_1 + X_2 + Z_2$ where $\alpha > 0,\ \beta > 0$. The states, X_1, X_2, are assumed to be independently Gaussian distributed and the additive Gaussian noises, $Z_1,\ Z_2$, are assumed to be independent of the RTO states and of fixed variances σ_1^2, σ_2^2 . RTO $j,\ j \in \{1,2\}$,

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encodes (quantizes) its measurement vector at a rate (in bits per measurement) R_j. The encoding is subject to satisfying two constraints: a distortion requirement of the state estimate at the other RTO (cooperation) and a leakage requirement (privacy) on the information leaked about its state to the other RTO. Let D_j and L_j denote the fidelity (distortion) and leakage requirements of the estimate at RTO j, $j \in \{1,2\}$ where D_j is the mean square error between the original and reconstructed state vectors and L_j is the average mutual information between the state vector at each RTO and the revealed data and measurement vector at the other RTO. We refer the reader to [4] for additional details. For notational convenience, throughout the sequel, we develop the results for RTO j and denote the other RTO by $i \in \{1,2\} \setminus \{j\}$.

For this model, the authors show the optimality of an encoding scheme in which each RTO encodes its measurements to satisfy the distortion constraint at the other RTO and tolerates a resulting level of privacy leakage. We summarize the resulting the rate-distortion-leakage tradeoff in the following theorem.

Theorem 2.1 *[4] The rate-distortion-leakage tradeoff* $(R_1, R_2, D_1, D_2, L_1, L_2)$ *is given as follows. For RTO* j, $j \in \{1,2\}$, *and* $i \in \{1,2\} \setminus \{j\}$, *we have*

- $D_i < D_{\max,i}$:

$$R_j = \frac{1}{2}\log\left(\frac{c_j m_j^2}{D_i - D_{\min,i}}\right), \text{ and} \tag{1}$$

$$L_j = \frac{1}{2}\log\left(\frac{m_j^2}{m_j^2 D_{\min,j} + n_j^2(D_i - D_{\min,i})}\right); \tag{2}$$

- $D_i \geq D_{\max,i}$: $R_j = 0$ *and* $L_j = \log\left(V_i/(V_i - q_j)\right)/2$,

where $q_1 = \beta$, $q_2 = \alpha$, $V_1 = 1 + \alpha^2 + \sigma_1^2$, $V_2 = 1 + \beta^2 + \sigma_2^2$, $E = \alpha + \beta$, $c_j = \frac{V_1 V_2 - E^2}{V_i}$, $n_1 = \frac{V_2 - \beta E}{V_1 V_2 - E^2}$, $n_2 = \frac{V_1 - \alpha E}{V_1 V_2 - E^2}$, $m_1 = \frac{\alpha V_2 - E}{V_1 V_2 - E^2}$, $m_2 = \frac{\beta V_1 - E}{V_1 V_2 - E^2}$ *and*

$$D_{\min,1} = 1 - \frac{(\beta^2 V_1 + V_2 - 2\beta E)}{(V_1 V_2 - E^2)}, \tag{3a}$$

$$D_{\min,2} = 1 - \frac{(V_1 + \alpha^2 V_2 - 2\alpha E)}{(V_1 V_2 - E^2)}, \tag{3b}$$

$$D_{\max,j} = 1 - \frac{1}{V_j}. \tag{3c}$$

Assuming perfect knowledge of the system parameters at both RTOs, we model the state estimation problem as two independent optimization problems.

We write $\mathcal{R}_j(\overline{D}_i)$ to denote the *feasible set* of rates R_j at RTO j for which a target distortion $\overline{D}_i$ can be achieved at RTO i. From the achievable rate-distortion-leakage tradeoff in [4] we have that, if $\overline{D}_i < D_{\max,i}$, then $\mathcal{R}_j(\overline{D}_i) = \left[\log\left(c_j m_j^2/(\overline{D}_i - D_{\min,i})\right)/2, \infty\right)$, else $\mathcal{R}_j(\overline{D}_i) = [0,\infty)$. Let $R_{\min,j}\left(\overline{D}_i\right) \triangleq \log\left(c_j m_j^2/(\overline{D}_i - D_{\min,i})\right)/2$ denote the minimal rate at which RTO j encodes its observation to guarantee a distortion $\overline{D}_i$ at RTO i.

We define an *objective function* $u_j : \mathcal{R}_j \to [0,\infty)$ as:

$$u_j(R_j) = -L_j(D_i(R_j)) + h_j(R_j) \tag{4}$$

where $D_i(R_j)$ represents the distortion-rate function at RTO i, $L_j(D_i(R_j))$ represents the leakage of information at RTO j when it transmits at rate R_j to satisfy a distortion requirement D_i at RTO i, and $h_j(R_j)$ is the economical reward that RTO j receives for cooperating with RTO i at rate R_j. We write $(R_j^*, L_j^*(R_j^*))$ to denote the rate-leakage pair that maximizes the objective function.

3. PRICING MECHANISMS

Before presenting the proposed pricing mechanisms, we briefly analyse the particular case where $h_j(\cdot) = 0$. The objective of each RTO is to choose its own communication rate that will minimize its leakage. From Theorem 2.1, we have that:

$$L_j(R_j) = -\frac{1}{2}\log\left(D_{\min,j} + c_j n_j^2 2^{-2R_j}\right).$$

Note that $L_j(R_j)$ strictly increases with R_j. This implies that the optimal strategy is $R_j^* = 0$ if $\overline{D}_i \geq D_{\max,i}$, and $R_j^* = R_{\min,j}(\overline{D}_i)$, otherwise. Thus, the achievable distortion pair is the minimum required distortions $(\overline{D}_1, \overline{D}_2)$ and the achievable leakages $L_j^*\left(R_j^*\right)$, $j \in \{1,2\}$, are given in (2) and replacing D_i with $\overline{D}_i$. Thus, in the absence of any incentives to cooperate, minimizing the leakage will either lead to a trivial solution ($R_j^* = 0$, $j \in \{1,2\}$) or an enforced cooperation to achieve a desired target distortion pair $(\overline{D}_1, \overline{D}_2)$.

3.1. Linear pricing

We start by analyzing a simple pricing scheme where the incentive RTO j receives for cooperating increases linearly with the rate R_j. In this case, the objective function of RTO j is given by $u_j(R_j) = -L_j(D_i(R_j)) + p_j R_j$. We observe that $\lim_{R_j \to \infty} u_j(R_j) = \infty$, i.e., the rewarding mechanism completely dominates the leakage term and, thus, the RTOs are incentivized to exchange data at infinitely large rates.

Since each RTO is cooperating to achieve a target distortion at the other RTO, we now consider a pricing mechanism that explicitly takes this into account. We consider the following objective function:

$$\begin{aligned} u_j(R_j) &= -L_j(D_i(R_j)) + p_j(D_{\max,i} - D_i(R_j)) \quad (5) \\ &= \frac{1}{2}\log\left(D_{\min,j} + c_j n_j^2 2^{-2R_j}\right) - \\ &\quad p_j\left(D_{\min,i} + c_j m_j^2 2^{-2R_j}\right) + p_j D_{i,max}, \quad (6) \end{aligned}$$

where in (5) RTO j is rewarded proportionally to the reduction in the distortion at the other RTO ($D_{\max,i} - D_i(R_j)$), and

(6) results from substituting (1) and (2) in (5). We thus have $\lim_{R_j\to\infty} u_j(R_j) = 1/2\log(D_{\min,j}) + p_j(D_{\max,i} - D_{\min,i}) < \infty$ and therefore it is not necessarily optimal for RTO j to perfectly share its observation.

From the objective function and its first order derivative, we further classify the optimal solution in different categories as described below. To this end, we define

$$\begin{aligned} T_{1,j} &\triangleq \frac{n_j^2}{(2\ln 2)m_j^2 D_{\min,j}}, \\ T_{2,j} &\triangleq \frac{n_j^2}{(2\ln 2)[n_j^2(\overline{D}_i - D_{\min,i}) + m_j^2 D_{\min,j}]}, \\ T_{3,j} &\triangleq \log\left(1 + \frac{n_j^2}{m_j^2}\frac{\overline{D}_i - D_{\min,i}}{D_{\min,j}}\right) \Big/ \left[2(\overline{D}_i - D_{\min,i})\right]. \end{aligned}$$

I. $p_j \geq T_{1,j}$: for this case $u_j(R_j)$ increases with R_j, and thus, $R_j^* \to \infty$. This means that, if the price is high enough, then the leakage term is dominated by the economic incentives.

II. $p_j < T_{1,j}$: here $u_j(R_j)$ decreases for $R_j \leq \tilde{R}_j(p_j) \triangleq \frac{1}{2}\log\left(\frac{(2\ln 2)p_j c_j n_j^2 m_j^2}{n_j^2 - (2\ln 2)p_j m_j^2 D_{\min,j}}\right)$ and increases for $R_j > \tilde{R}_j(p_j)$. We now have two sub-cases depending on the relative order of $\tilde{R}_j(p_j)$ and $R_{\min,j}(\overline{D}_i)$ (recall that $R_{\min,j}(\overline{D}_i)$ is the inferior bound of the feasible rate set).

1. $p_j < T_{2,j} < T_{1,j}$: here $\tilde{R}_j(p_j) < R_{\min,j}(\overline{D}_i)$ which implies that, in the feasible domain, $u_j(R_j)$ increases with R_j and the optimal rate $R_j^* \to \infty$.

2. $T_{2,j} \leq p_j < T_{1,j}$: here $\tilde{R}_j(p_j) \geq R_{\min,j}(\overline{D}_i)$, i.e., $\tilde{R}_j(p_j)$ lies inside of the feasible domain. Since the function is convex, the optimal rate is on the boundary. If $T_{2,j} \leq p_j \leq T_{3,j}$, then $u_j(R_{\min,j}(\overline{D}_i)) \geq \lim_{R_j\to\infty} u_j(R_j)$ and the optimal rate is $R_j^* = R_{\min,j}(\overline{D}_i)$, else (if $T_{3,j} < p_j < T_{1,j}$) the optimal rate is $R_j^* \to \infty$.

Thus, depending on the system parameters and p_1, p_2, the optimal rates are on the borders of the feasible rate sets: either $R_j^* = R_{\min,j}(\overline{D}_i)$ or $R_j^* \to \infty,\ j \in \{1,2\}$ which can viewed as a solution involving a *hard decision*. Next we consider a non-linear pricing model that will allow us a smoother manipulation of the optimal rates w.r.t. the prices.

3.2. Non-linear pricing

Since the communication (source coding) rates are proportional to the logarithm of distortions, we now consider a logarithmic pricing function. The non-linear objective function of RTO j is:

$$\begin{aligned} u_j(R_j) &= -L_j(R_j) - p_j\log(D_i(R_j)) + p_j\log(D_{\max,i}) \quad (7)\\ &= \frac{1}{2}\log\left(D_{\min,j} + c_j n_j^2 2^{-2R_j}\right) - p_j\log\left(D_{\min,i} + c_j m_j^2 2^{-2R_j}\right) + p_j\log(D_{\max,i}). \quad (8) \end{aligned}$$

Let $T_{0,j} = 0.5$, and $T_{1,j} \triangleq \frac{1}{2}\frac{D_{\min,i}n_j^2}{D_{\min,j}m_j^2}$,

$$\begin{aligned} T_{2,j} &\triangleq \frac{1}{2}\frac{\overline{D}_i n_j^2}{m_j^2 D_{\min,j} + n_j^2(\overline{D}_i - D_{\min,i})}, \\ T_{3,j} &\triangleq \log\left(1 + \frac{n_j^2}{m_j^2}\frac{\overline{D}_i - D_{\min,i}}{D_{\min,j}}\right) \Big/ \left[2\log\left(1 + \frac{\overline{D}_i - D_{\min,i}}{D_{\min,i}}\right)\right]. \end{aligned}$$

Following a similar analysis as the previous subsection, we obtain:

I. $p_j \geq \max\{T_{0,j}, T_{1,j}\}$: for this case $u_j(R_j)$ increases with R_j, and thus $R_j^* \to \infty$.

II. $p_j \leq \min\{T_{0,j}, T_{1,j}\}$: here $u_j(R_j)$ decreases with R_j, and thus $R_j^* = R_{\min,j}(\overline{D}_i)$. If the price is lower than a certain threshold, then the leakage term dominates the economic incentives.

III. $p_j \in (T_{1,j}, T_{0,j})$: here $u_j(R_j)$ is decreasing for $R_j \leq \tilde{R}_j(p_j)$ and increasing for $R_j \geq \tilde{R}_j(p_j)$ where

$$\tilde{R}_j(p_j) \triangleq \frac{1}{2}\log\left(\frac{c_j n_j^2 m_j^2(1 - 2p_j)}{2p_j m_j D_{\min,j} - n_j^2 D_{\min,i}}\right).$$

We now have two sub-cases depending on the relative order of $\tilde{R}_j(p_j)$ and $R_{\min,j}(\overline{D}_i)$.

1. $T_{2,j} < p_j < T_{0,j}$: here $R_{\min,j}(\overline{D}_i) \leq \tilde{R}_j(p_j)$ and $\tilde{R}_j(p_j)$ lies inside of the feasible domain. Since the function is convex, the optimal rate is on the boundary. If $T_{3,j} < p_j < T_{0,j}$, then $u_j(R_{\min,j(\overline{D}_i)}) < \lim_{R_j\to\infty} u_j(R_j)$ and the optimal rate is $R_j^* \to \infty$, else (if $T_{2,j} < p_j \leq T_{3,j}$) the optimal rate is $R_j^* = R_{\min,j}(\overline{D}_i)$.

2. $T_{1,j} < p_j \leq T_{2,j}$: for this case $R_{\min,j} \geq \tilde{R}_j(p_j)$ which implies that, in the feasible domain, $u_j(R_j)$ is increasing with R_j and $R_j^* \to \infty$.

IV. $p_j \in (T_{0,j}, T_{1,j})$: here $u_j(R_j)$ increases for $R_j \leq \tilde{R}_j(p_j)$ and decreases for $R_j \geq \tilde{R}_j(p_j)$. The optimal rate will depend again on the relative order of $\tilde{R}_j(p_j)$ and $R_{\min,j}(\overline{D}_i)$.

1. $T_{2,j} < p_j < T_{1,j}$: here $R_{\min,j}(\overline{D}_i) \leq \tilde{R}_j(p_j)$, i.e. $\tilde{R}_j(p_j)$ lies inside of the feasible domain. Since the function is concave, the optimal rate is $R_j^* = \tilde{R}_j(p_j)$ which explicitly depends on p_j.

2. $T_{0,j} < p_j \leq T_{2,j}$: here $R_{\min,j}(\overline{D}_i) \geq \tilde{R}_j(p_j)$ which implies that, in the feasible domain, $u_j(R_j)$ is decreasing with R_j and the optimal rate is $R_j^* = R_{\min,j}(\overline{D}_i)$.

Notice that cases III and IV are mutually exclusive depending on the relative order of $T_{0,j}$ and $T_{1,j}$. Also, we obtain that, depending on the system parameters and for a certain price range the optimal rate R_j^* depends explicitly on p_j. This dependence will be illustrated via numerical simulations.

4. NUMERICAL SIMULATIONS

We illustrate our results for the non-linear pricing model for the following parameters: $\alpha = 0.5$, $\beta = 0.9$, $\sigma_1^2 = 0.1$, $\sigma_2^2 = 0.1$. We choose the target distortions as $\overline{D}_j = D_{\min,j} + (D_{\max,j} - D_{\min,j})/2$, $j \in \{1,2\}$. For these parameters we have: $D_{\min,2} = 0.3088$, $D_{\max,2} = 0.4764$, $D_{\min,1} = 0.2183$, $D_{\max,1} = 0.2593$, $T_{1,1} = 1.5093$, $T_{2,1} = 1.0548$. $T_{1,1}$ and $T_{2,1}$ alongside with $T_{0,1} = 0.5$ are the vertical asymptotes.

We plot the optimal rate for RTO 1 as function of the p_1 in Fig. 1. If the price is low ($p_1 < T_{2,1}$) the optimal rate is $R_1^* = R_{\min,1}(\overline{D}_2)$. If the price is high ($p_1 \geq T_{1,1}$) the optimal rate is $R_1^* \to \infty$. If $T_{2,1} \leq p_1 < T_{1,1}$, the solution is non-trivial and depends explicitly on p_1, i.e., $R_1^* = \tilde{R}_1(p_1)$. In Fig. 2, we observe that if p_1 is below a certain threshold, the leakage term dominates the economic incentive and the resulting utility is negative. The optimal utility is increasing with p_1. Therefore, even though the leakage is increasing with p_1, this is compensated by the pricing function. Finally, in Fig. 3, we observe that the distortion of RTO 2 decreases with p_1. Most interestingly, in contrast to the linear pricing model, an RTO can be incentivized to reveal more than needed to achieve the target distortion thereby further reducing the distortion of the other RTO. Furthermore, any distortion $D_2 \in (D_{\min,2}, \overline{D}_2]$ can be achieved by a unique value of the price p_1.

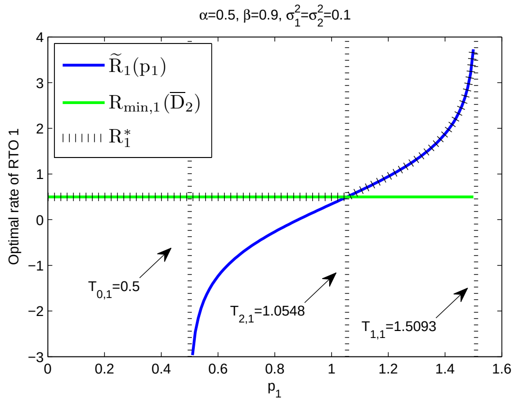

Fig. 1. Optimal rate as functions of the price: $R_1^*(p_1)$

5. CONCLUSIONS

Based on the concept of *competitive privacy* [4], we have presented a pricing based optimization framework to study the distributed state estimation problem at the RTO level. We have shown that, in the absence of pricing incentives, privacy takes precedence over cooperation. Using linear and non-linear pricing functions, we have shown that it is possible to incentivize the RTOs to cooperate and achieve their desired distortion levels. Future work includes coupling the

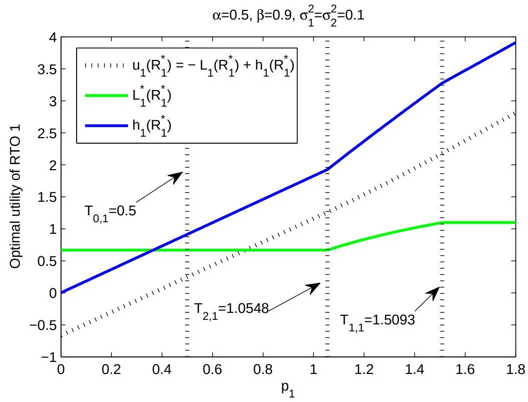

Fig. 2. Optimal utility, leakage, pricing of RTO 1 as functions of the price: $u_1(R_1^*(p_1))$, $L_1^*(R_1^*(p_1))$, $h_1(R_1^*)$

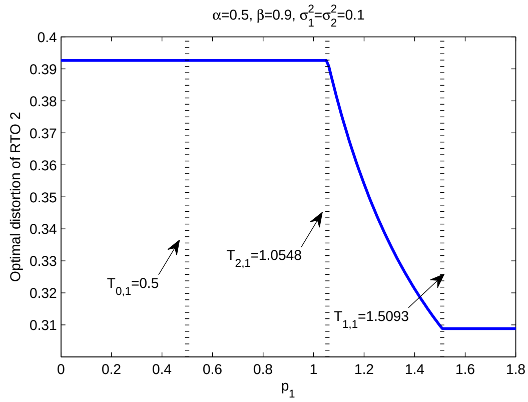

Fig. 3. Optimal distortion of RTO 2 as function of the price: $D_2(R_1^*(p_1))$

optimization problems which can lead to game theoretic solutions.

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