Predictive Control for Trajectory Tracking and Decentralized Navigation of Multi-Agent Formations

Ionela Prodan, Sorin Olaru, Cristina Stoica, Silviu-Iulian Niculescu

Int. J. Appl. Math. Comput. Sci., , Vol. , No. , –
DOI:

PREDICTIVE CONTROL FOR TRAJECTORY TRACKING AND DECENTRALIZED NAVIGATION OF MULTI-AGENT FORMATIONS

IONELA PRODAN [*,**], SORIN OLARU [*], CRISTINA STOICA [*], SILVIU-IULIAN NICULESCU [**]

[*] SUPELEC Systems Sciences (E3S)
Automatic Control Department, 3 Joliot Curie, 91192, Gif sur Yvette, France
e-mail: ionela.prodan, sorin.olaru, cristina.stoica@supelec.fr

[**]Laboratory of Signals and Systems, SUPELEC - CNRS,
3 Joliot Curie, 91192, Gif sur Yvette, France
e-mail: ionela.prodan, silviu.niculescu@lss.supelec.fr

This paper addresses a predictive control strategy for a particular class of multi-agent formations with a time-varying topology. The goal is to guarantee tracking capabilities with respect to a reference trajectory which is pre-specified for an agent designed as the *leader*. Then, the rest of the agents, designed as *followers*, track the position and orientation of the leader. In real-time, a predictive control strategy enhanced with the Potential Field methodology is used in order to derive a feedback control action based only on local information within the group of agents. The main concern is that the interconnections between the agents are time-varying, affecting the neighborhood around each agent. The proposed method exhibits effective performance validated through some illustrative examples.

Keywords: Multi-Agent Systems, Linear Systems, Model Predictive Control (MPC), potential function, polyhedral function.

1. Introduction

Control and coordination of multi-agent systems, such as pedestrians in the crowd, vehicles, spacecraft and unmanned vehicles, are emerging as a challenging field of research. The advances in network design, information, control synthesis and sensors technology allow nowadays large groups of agents to be coordinated and controlled in an effective manner for various tasks in evaluating the safety of the social infrastructures (Helbing *et al*., 2000), (Fang *et al*., 2010), efficient flow of traffic (Van den Berg *et al*., 2004), (Baskar *et al*., 2006), water control for irrigation canals, water supply and sewer networks (Overloop *et al*., 2010), (Negenborn *et al*., 2009) and deep space observation (Mesbahi and Hadaegh, 2001), (Massion *et al*., 2008). In addition, there exist several classes of multi-agent systems where the interconnections between the agents could be time-varying (e.g. traffic control, pedestrian behavior etc.). Guaranteeing stability with the existing cooperative control techniques is still an open problem for multi-agent systems with time-varying (constrained) topologies. This paper addresses a new methodology based on predictive control in order to answer to some of these difficulties; illustrative examples prove the interest of the proposed methodology.

Collision avoidance is often the most difficult problem in the context of managing multiple agents, since certain (static or dynamic) constraints are non-convex. A common point of most publications dealing with the collision avoidance problem is the hypothesis of *punctiform* agents, which is far from the conditions in real world applications. In many of them the relative positioning between agents becomes important, such as large interferometer construction from multiple telescopes (Schneider, 2009) or the air traffic management, two aircraft are not allowed to approach each other closer than a specific alert distance.

A key idea for the treatment of collision avoidance problems is represented by *Mixed-Integer-Programming* (MIP) (Osiadacz *et al*., 1990), (Richards and How, 2002), (Bemporad and Morari, 1999), which has the ability to include non-convex constraints and discrete decisions in the optimization problem. However, despite its modeling capabilities and the availability of good solvers, MIP has serious numerical drawbacks. As stated in (Garey and

Johnson, 1979), mixed-integer techniques are NP-hard, i.e. the computational complexity increases exponentially with the number of binary variables used in the problem formulation. A method for reducing the number of binary variables is detailed in (Stoican *et al.*, 2011) with an application to the obstacles avoidance problem. Yet, the fundamental limitation of MIP complexity remains redoubtable.

A different class of methods for collision avoidance problems uses artificial potential fields (Khatib, 1986) to directly obtain feedback control actions steering the agents over the entire workspace. One shortcoming of this approach is the possible generation of traps (local minima). Relevant research on generating so-called navigation functions that are free from local minima is available in the literature (Rimon and Koditschek, 1992). However, generating a navigation function is computationally involved and thus not suitable for many navigation problems.

There is a large literature dedicated to the formation control for collections of vehicles using the potential field approach. The authors of (Jadbabaie *et al.*, 2003) and (Tanner *et al.*, 2007) investigate the motions of vehicles modeled as double integrators. Their objective is for the vehicles to achieve a common velocity while avoiding collisions with obstacles and/or agents assumed to be points. The derived control laws involve graph Laplacians for an associated undirected graph and also nonlinear terms resulting from artificial potential functions. In (Roussos and Kyriakopoulos, 2010) a decentralized navigation of multiple agents operating in a spherical workspace is considered. Navigation functions are used to derive control laws for point-like agents with an associated disc of predefined radius around them.

In the present paper, we revise the preliminary results (Prodan *et al.*, 2010), (Prodan *et al.*, 2011) and introduce enhancements in the control design method which enables the decentralized decision making for a leader/followers group of agents. The aim of this work is twofold:

- First, to provide a generic framework for non point-like shapes which may define obstacles and/or safety regions around an agent.

- Second, to offer a novel control strategy derived from a combination of constrained receding horizon and potential field techniques for the trajectory tracking problem, applied to multi-agent systems with time-varying topologies.

To the best of the authors' knowledge, there does not exist a similar method in the open literature. The methods that we propose can be applied to various practical applications (e.g. motion control of wheeled mobile robots (Michalek *et al.*, 2009), path following control of nonholonomic mobile manipulators (Mazur and Szakiel, 2009), the control of a mobile offshore base viewed as a string of modules that have to be kept aligned (Girard *et al.*, 2001)).

First, we introduce two different constructions which take into consideration the shape of the convex region associated to a safety region of an agent or an obstacle. The proposed constructions can be further used with the various potential or navigation functions existing in the literature in order to have a complete multi-agent system. Second, through the rest of the paper a leader/followers strategy for the trajectory tracking problem is proposed. The agents are required to follow a pre-specified trajectory while keeping a desired inter-agent formation in time. We consider polyhedral safety regions for the agents and obstacles. A specified trajectory is generated for the leader using the differential flatness formalism (Fliess *et al.*, 1995). Differentially flat systems are well suited to problems requiring trajectory planning as it circumvents the complexity of differential equations formalism by transforming the model description in a algebraic form, more suitable for open-loop control design. The most important aspect of flatness in our context (predictive control) is that it reduces the problem of trajectory generation to finding a trajectory of the so called flat output of the system through the resolution of a system of algebraic equations. Furthermore, for the followers, we propose a Potential Field method which aims to follow the group leader and respect the formation specifications. These are realized through the use of a receding horizon approach (Camacho and Bordons, 2004), (Rossiter, 2003), (Mayne *et al.*, 2000), both for the leaders and followers.

This paper is organized as follows. Section 2 presents two constructions that take into account the shape of a convex region defining an obstacle and/or a safety region around an agent. Considering the dynamics of the agents, Section 3 presents the trajectory tracking problem for a leader/followers formation. A reference trajectory is generated for the leader and using predictive control the tracking error is minimized. For the followers, a potential function is embedded within Model Predictive Control (MPC) in order to achieve the group formation with a collision free behavior. Further on, Section 4 presents illustrative simulation results. And finally, several concluding remarks are drawn in Section 5.

The following notations will be used throughout the paper. Given a vector $v \in \mathbb{R}^n$, $\|v\|_\infty := \max_{i=1,\cdots,n} |v_i|$ denotes the infinity norm of v. Minkowski's addition of two sets $\mathcal{X}$ and $\mathcal{Y}$ is defined as $\mathcal{X} \oplus \mathcal{Y} = \{x + y : x \in \mathcal{X},\ y \in \mathcal{Y}\}$. The interior of a set S, $Int(S)$ is the set of all interior points of S. The collection of all possible n_c combinations of binary variables will be noted $\{0,1\}^{n_c} = \{(b_1, \ldots, b_{n_c}) :\ b_i \in \{0,1\}, \forall i = 1, \ldots, n_c\}$. Denote as $\mathbb{B}_p^n = \{x \in \mathbb{R}^n : \|x\|_p \leq 1\}$ the unit ball of norm p, where $\|x\|_p$ is the p-norm of vector x. Let $x_{k+1|k}$ denote the value of x at time instant $k+1$, predicted upon the information available at time $k \in \mathbb{N}$.

2. PREREQUISITES

For safety and obstacle avoidance problems the feasible region in the space of solutions is a non-convex set. Usually this region is considered as the complement of a (union of) convex region(s) which describes an obstacle and/or a safety region. Due to their versatility and relative low computational complexity the polyhedra are the instrument of choice in characterizing these regions.

Let us define a bounded convex set in its polyhedral approximation, a polytope $S \subset \mathbb{R}^n$ through the implicit half-space description:

$$S = \left\{ x \in \mathbb{R}^n : h_a x \leq k_a,\ \ a = 1, \cdots, n_h \right\}, \quad (1)$$

with $h_a \in \mathbb{R}^{1\times n}$, $k_a \in \mathbb{R}$ and n_h the number of half-spaces. We focus on the case where $k_a > 0$, meaning that the origin is contained in the strict interior of the polytopic region, i.e. $0 \in Int(S)$.

By definition, every supporting hyperplane for the set S in (1):

$$\mathcal{H}_a = \{x \in \mathbb{R}^n : \ h_a x = k_a,\ a = 1, \cdots, n_h\} \quad (2)$$

will partition the space into two disjoint[1] regions:

$$\begin{aligned} \mathcal{R}^+(\mathcal{H}_a) &= \{x \in \mathbb{R}^n : \quad h_a x \leq \quad k_a\} \quad (3)\\ \mathcal{R}^-(\mathcal{H}_a) &= \{x \in \mathbb{R}^n : \ -h_a x \leq -k_a\} \quad (4) \end{aligned}$$

$\mathcal{R}_a^+$ and $\mathcal{R}_a^-$ denote in a simplified formulation the complementary regions associated to the a^{th} inequality of (1).

In the following, we are interested in measuring the relative position of an agent to such a region. In other words, we require a function which measures if and when a given state is inside or outside the polyhedral set (1). The forthcoming constructions will be introduced in a *repulsive potential function* to take into account the shape of the convex region in terms of (1). The repulsive potential will be further used in a predictive control context in order to derive a control action such that the collision avoidance inside the formation is satisfied.

2.1. Polyhedral function. Consider the class of (symmetrical) piecewise linear functionals defined using the specific shape of a polyhedral set. The following definitions will be instrumental for the rest of the paper.

Definition 1 (Minkowski function – (Blanchini, 1995)) *Any bounded convex set S induces a Minkowski function defined as*

$$\mu(x) = inf \left\{ \alpha \in \mathbb{R},\ \alpha \geq 0 : \ x \in \alpha S \right\} \quad (5)$$

[1] The relative interiors of these regions do not intersect but their closures have as common boundary the affine subspace $\mathcal{H}_a$.

Definition 2 (Polyhedral function – (Blanchini, 1995)) *A polyhedral function is the Minkowski function of the polyhedral bounded convex set S defined in* (1)*. This function has the following expression:*

$$\mu(x) = \|Fx\|_\infty, \quad (6)$$

where $F \in \mathbb{R}^{n_h \times n}$ is a full column matrix with $F_a = \frac{h_a}{k_a}$, $a = 1, \cdots, n_h$.

In fact, any polytope can be defined in terms of the Minkowski function (5). Indeed there always exists a full column matrix $F \in \mathbb{R}^{n_h\times n}$ such that the polytope S in (1) is equivalently defined as

$$S = \left\{ x \in \mathbb{R}^n : \mu(x) \leq 1 \right\}, \quad (7)$$

with $\mu(x)$ defined by (6). From the avoidance point of view, the Minkowski function (5) denotes the inclusion of a value x to the given polytope (7) if $\mu(x) \in [0, 1]$. Conversely, if $\mu(x) > 1$ then x is outside the polytope (7).

Remark 1 *Note that if $k_a < 0$ in* (1)*, the origin is not contained in the strict interior of the polytopic region, i.e. $0 \notin Int(S)$, then the polyhedral function can be brought to the form* (6) *by imposing*

$$F_a = \frac{h_a(x - x_s)}{k_a - h_a x_s},\ \ a = 1, \cdots, n_h, \quad (8)$$

with $x_s \in \mathbb{R}^n$ the analytic center of the polytope (1)*.*

Note that, the polyhedral function (6) is piecewise affine and continuous. This means that each of the inequalities which compose its definition can provide the maximum, an explicit description of these regions being

$$X_a = \left\{ x \in \mathbb{R}^n : \tfrac{h_a}{k_a} x > \tfrac{h_b}{k_b} x, \forall\, a \neq b,\ a, b = 1, \cdots, n_h \right\}. \quad (9)$$

The entire space can thus be partitioned in a union of disjoint regions X_a which are representing in fact cones with a common point in the origin (respectively in x_s for the general case evoked in Remark 1).

Practically, the polyhedral function (5) can be represented in the form

$$\mu(x) = F_a x,\ \ \forall\, x \in X_a,\ \ a = 1, \cdots, n_h \quad (10)$$

and the piecewise affine gradient is defined as:

$$\bigtriangledown \mu(x) = F_a,\ \ \forall\, x \in X_a,\ \ a = 1, \cdots, n_h. \quad (11)$$

Remark 2 *Strictly speaking the generalized gradient* (11) *is multivalued (the Minkowski function induced by a polytope is not differentiable in the classical sense, rather* *it is differentiable almost everywhere). However, an univocal candidate can be selected for the computations and such an approach is used in the rest of the paper. We mention that alternatively the explicit use of multivalued expression of the gradient would not bring computational difficulties as long as the range of variation is bounded and can be represented by the extreme values in practice.*

2.2. Sum function. The polyhedral functions presented in the previous section represent the basis for the construction of "exclusion" functions (or penalty functions).

Consider again the polytope defined in (1), and a piecewise linear function (introduced in (Camacho and Bordons, 2004)):

$$\psi(x) = \sum_{a=1}^{n_h} (h_a x - k_a + |h_a x - k_a|). \qquad (12)$$

The function (12) is zero inside the convex region (1) and increases linearly in the exterior, as the distance to the frontier is augmented.

The definition (12) describes in fact a continuous piecewise affine function over a partition of the state-space. Over each of the polyhedral cells composing this partition, the absolute values of $|h_a x - k_a|$ are constant resulting in a fixed affine form for $\psi(x)$. In order to explicitly describe the regions composing the partition, several additional theoretical notions will be introduced.

Definition 3 (Hyperplane arrangements – (Ziegler, 1995)) *A collection of hyperplanes* $\mathbb{H} = \{\mathcal{H}_a\}$ *with* $a = 1, \cdots, n_h$ *partition the space in an union of disjoint cells defined as follows:*

$$\mathcal{A}(\mathbb{H}) = \bigcup_{l=1,\ldots,\gamma(n_h)} \underbrace{\left(\bigcap_{a=1,\cdots,n_h} \mathcal{R}^{\sigma_l(a)}(\mathcal{H}_a) \right)}_{A_l}, \qquad (13)$$

where $\sigma_l \in \{-,+\}^{n_h}$ *denotes all feasible combinations of regions* (3) *and* (4) *obtained for the hyperplanes in* $\mathbb{H}$ *and* $\gamma(n_h)$ *denotes the number of feasible cells.*

Note that the number of regions in the hyperplane arrangement is usually much greater than the number of regions (9) associated to the polyhedral function (10).

Therefore, the piecewise affine function (12) can be alternatively described as:

$$\psi(x) = 2 \sum_{\substack{a=1 \\ \sigma_l = \{+\}}}^{n_h} (h_a x - k_a), \; \forall \, x \in \mathcal{A}_l, \; l = 1, \cdots, \gamma(n_h). \qquad (14)$$

The piecewise affine gradient of (14) is defined as:

$$\bigtriangledown \psi(x) = 2 \sum_{\substack{a=1 \\ \sigma_l = \{+\}}}^{n_h} h_a^T, \; \forall \, x \in \mathcal{A}_l, \;\; l = 1, \cdots, \gamma(n_h). \qquad (15)$$

2.3. Exemplification for the construction of repulsive potential functions. In this subsection the previous theoretical tools will be integrated in order to describe two types of piecewise affine functions which measure the position of a state with respect to the frontier of a polyhedral set defined in (1).

In order to exemplify their influence in a collision avoidance problem, we propose several repulsive potential functions constructed through the use of the formulations (10) and (14). The potential functions take into account the shape of a convex region as in Figure 1, which can define a safety region for an agent and/or an obstacle. For the given convex region, Figure 2 and Figure 3 illustrate the polyhedral function and the sum function defined according to (10) and (14), respectively.

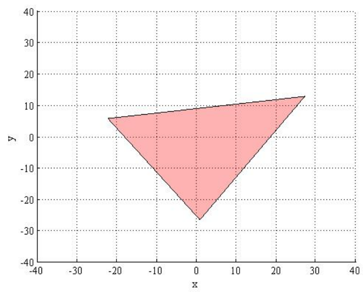

Fig. 1. A convex region.

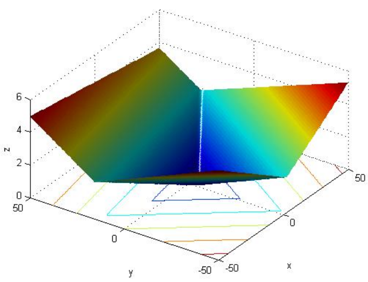

Fig. 2. Polyhedral function (10) of the convex region in Figure 1.

Furthermore, for the control design purpose, the construction based on the polyhedral function defined in (10) is proposed for the generation of a repulsive potential:

$$V_\mu(\mu(x)) = c_1 e^{-(\mu(x) - c_2)^2}, \qquad (16)$$

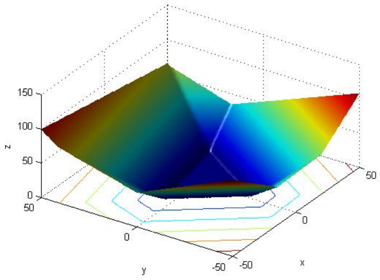

Fig. 3. Sum function (14) of the convex region in Figure 1.

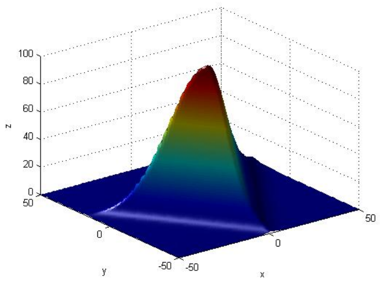

Fig. 4. Repulsive potential using the polyhedral function in Figure 2.

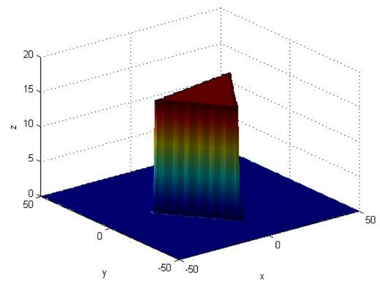

Fig. 5. Repulsive potential using the sum function in Figure 3.

where the parameters c_1 and c_2 are positive constants representing the strength and effect ranges of the repulsive potential.

An alternative repulsive potential using the sum function described in (14) will be given by:

$$V_\psi(\psi(x)) = \frac{c_3}{(c_4 + \psi(x))^2}, \tag{17}$$

with c_3 and c_4 positive constants representing the strength and effect ranges of the repulsive potential (17).

Both, Figure 4 and Figure 5 illustrate the proposed functions (16), (17), respectively for the given convex region in Figure 1. As it can be seen, both functions have a high value inside the polytopes and a low value outside them. The repulsive potential will be further used in order to derive a control action such that the collision avoidance inside the formation is satisfied.

3. TRAJECTORY TRACKING FOR A LEADER/FOLLOWERS FORMATION

This section presents the formation trajectory tracking problem. The agents are required to follow a pre-specified trajectory while preserving a tight inter-agent formation in time. Each agent has an associated polyhedral safety region as defined in (1). Using a leader/followers approach, we generate a reference trajectory for the leader and formulate a receding horizon optimization problem in order to minimize the tracking error. For the followers, we propose a gradient method combined with a receding horizon approach which aims to follow the group leader and respect the collision avoidance formation specifications.

A set of N_a linear systems (vehicles, pedestrians or agents in a general form) will be used to model the behavior of individual heterogeneous agents. The i^{th} system is described by the following continuous time dynamics:

$$\dot{x}^i(t) = A_{c,i}x^i(t) + B_{c,i}u^i(t) \quad i = 1, \cdots, N_a, \tag{18}$$

where $x^i(t) \in \mathbb{R}^n$ are the state variables and $u^i(t) \in \mathbb{R}^m$ is the control input vector for the i^{th} agent. The components of the state are: the position $p^i(t)$ and the velocity $v^i(t)$ of the i^{th} agent such that $x^i(t) = [p^i(t)\ v^i(t)]^T$.

The problem of generating a reference trajectory for the leader (i.e. $i = l$ in (18)) is next summarized, along the line in (Van Nieuwstadt and Murray, 1998).

3.1. Trajectory generation. The idea is to find a trajectory $(x^l(t), u^l(t))$ that steers the model of the leader (18) with $i = l$ from an initial state x_0 to a final state x_f, over a fixed time interval $[t_0, t_f]$. Using the flatness theory (Fliess *et al.*, 1995), (Van Nieuwstadt and Murray, 1998), (Suryawan *et al.*, 2010), the system is parameterized in terms of a finite set of variables $z^l(t)$ and a finite number of their derivatives:

$$\begin{aligned} x^l(t) &= \xi(z^l(t), \dot{z}^l(t), \cdots, z^{l,(q)}(t)), \\ u^l(t) &= \eta(z^l(t), \dot{z}^l(t), \cdots, z^{l,(q)}(t)), \end{aligned} \tag{19}$$

where $z^l(t) = \Upsilon(x^l(t), u^l(t), \dot{u}^l(t), \cdots, u^{l,(q)}(t))$ is called the flat output[2]. The generation of a reference trajectory will be based on the class of polynomial functions. Using the parametrization (19) and imposing boundary constraints for the evolution of the differentially flat systems (De Doná *et al.*, 2009) one can generate a reference trajectory $z^l_{ref}(t)$ by the resolution of a linear system of

[2]Hereafter we assume that the characteristics necessary for flat trajectory (controllability and the existence of a flat output) are respected for the leader agent.

equalities. Therefore, the corresponding reference state and input for the system (18), with $i = l$ are obtained by replacing the reference flat output $z^l_{ref}(t)$, with $t \in [t_0, t_f]$ in (19):

$$\begin{aligned} x^l_{ref}(t) &= \xi(z^l_{ref}(t), \dot{z}^l_{ref}(t), \cdots, z^{l,(q)}_{ref}(t)), \quad (20) \\ u^l_{ref}(t) &= \eta(\dot{z}^l_{ref}(t), \ddot{z}^l_{ref}(t), \cdots, z^{l,(q)}_{ref}(t)), \end{aligned}$$

where $t \in [t_0, t_f]$.

In the rest of the paper, we use the discrete analog to the reference signals in (20). That is, a corresponding discrete-time model for the equations (18) is constructed upon a chosen sampling period T_s by considering the time instants $t_k = kT_s$:

$$x^i(k+1) = A_i x^i(k) + B_i u^i(k), \quad k \in \mathbb{N}, \quad i = 1 : N_a, \tag{21}$$

where $x^i(0)$ corresponds to the boundary condition in (20) and $u^i(k) = u^i(t_k)$. The pairs (A_i, B_i) are given by:

$$A_i = e^{A_{c,i}T_s}, \quad B_i = \int_0^{T_s} e^{A_{c,i}(T_s-\theta)} B_{c,i} d\theta.$$

Considering the discrete-time model of the leader (21) with $i = l$, we compare the measured state and input variables with the reference trajectory $(x^l_{ref}(k), u^l_{ref}(k))$ which satisfies the nominal dynamics:

$$x^l_{ref}(k+1) = A_l x^l_{ref}(k) + B_l u^l_{ref}(k). \tag{22}$$

Further on, the tracking error between the leader's state (21) and the state reference (22) becomes:

$$\tilde{x}^l(k+1) = A_l \tilde{x}^l(k) + B_l \tilde{u}^l(k), \tag{23}$$

with $\tilde{u}^l(k) = u^l(k) - u^l_{ref}(k)$, $\tilde{x}^l(k) = x^l(k) - x^l_{ref}(k)$.

Since the reference trajectory is available beforehand, an optimization problem which minimizes the tracking error for the leader can be formulated in a predictive control framework (Goodwin *et al.*, 2006), (Maciejowski, 2002). Consequently, the leader must follow the reference trajectory from the initial position to the desired position, using the available information over a finite time horizon in the presence of constraints [3].

3.2. Predictive control for the leader. In what follows we present the predictive control problem, where an optimization is performed to compute the control law for the leader. The discrete model of the leader (i.e. $i = l$ in (21)) is used in a predictive control context which permits the minimization of the tracking error.

A finite receding horizon implementation of the optimal control law is typically based on the real-time construction of a control sequence $\tilde{\mathbf{u}}^l = \{\tilde{u}^l(k|k), \tilde{u}^l(k+1|k), \cdots, \tilde{u}^l(k+N_l-1|k)\}$ that minimizes the finite horizon quadratic objective function:

$$\begin{aligned} \tilde{\mathbf{u}}^* = \arg\min_{\tilde{\mathbf{u}}^l}(\|\tilde{x}^l(k+N_l|k)\|_P + \quad (24) \\ + \sum_{s=1}^{N_l-1} \|\tilde{x}^l(k+s|k)\|_Q + \sum_{s=0}^{N_l-1} \|\tilde{u}^l(k+s|k)\|_R), \end{aligned}$$

subject to:

$$\begin{cases} \tilde{x}^l(k+s+1|k) = A_l \tilde{x}^l(k+s|k) + B_l \tilde{u}^l(k+s|k), \\ \tilde{x}^l(k+s|k) \in X_l, \quad s = 1, \cdots, N_l, \\ \tilde{u}^l(k+s|k) \in U_l, \quad s = 1, \cdots, N_l, \end{cases} \tag{25}$$

Here $Q = Q^T \succeq 0$, $R \succ 0$ are positive definite weighting matrices, $P = P^T \succeq 0$ defines the terminal cost and N_l denotes the prediction horizon for the leader. The optimization problem (24) has to be solved subject to the dynamical constraints (25). In the same time, other security or performance specifications can be added to the system trajectory. These physical limitations (velocity, energy or forces) are stated in terms of hard constraints on the internal state variables and input control action as in (25). Note that the sets X_l, U_l have to take into account the reference tracking type of problem delineated in (24). Thus, the absolute limitations have to be adjusted according to the reference signals. In the original state space coordinates, these constraints will describe a tube around the reference trajectory. A finite horizon trajectory optimization is performed at each sample instant, the first component of the resulting control sequence being effectively applied. Then, the optimization procedure is reiterated using the available measurements based on the receding horizon principle (Camacho and Bordons, 2004).

3.3. Decentralized predictive control for the followers. In this subsection, we present a control strategy which is a combination of MPC and Potential Field control approach. The goal is to control the agents to achieve a formation while following the specified trajectory. The repulsive potential functions introduced in (16) and (17) produce a potential field. The negative gradient of this potential is the key element towards a collision free behavior for the agents. Globally, an attractive component of the potential function aims at maintaining a given formation. In this context, we provide a practical control design method which enables the decentralized decision making for a leader-followers group of agents. The proposed method exhibits effective trajectory tracking perfor-

[3] The flat trajectory can also be generated to enforce obstacle avoidance at the trajectory planning stage. In this framework the obstacles can be modeled in terms of a convex safety region around each agent, as in (1). Even if the reference trajectory is generated over the entire interval $[t_0, t_f]$, intermediary points can be added along the trajectory in order to avoid obstacles at a specific time subinterval by redesigning of the flat trajectory.

mances while avoiding the centralized design which can be computationally demanding[4].

Consider that each agent has an associated convex safety region as defined in (1). Before actually defining the potential function, let us recall a general result relative to convex sets which will be further used in the optimization problem:

Proposition 1 *For any two convex sets $S_1, S_2 \in \mathbb{R}^n$ the next relations are equivalent:*

1. $S_1 \cap S_2 = \emptyset$
2. $\{0\} \notin S_1 \oplus \{-S_2\}$

Sketch of proof: It suffices to note that if the origin is inside the set $S_1 \oplus \{-S_2\}$ then, necessarily, there exists $x_1 \in S_1$ and $x_2 \in S_2$ such that $x_1 - x_2 = 0$ q.e.d.

Corollary 1 *Consider the agents i and j with the associated safety regions S_i, S_j as defined in (1). The agent i with the associated position p^i does not intersect agent j with the position p^j if an only if $p^i \notin S_{ij}(p^j)$, where*

$$S_{ij}(p^j) \triangleq \{p^j\} \oplus S_j \oplus \{-S_i\}, \tag{26}$$

with $i = 1, \cdots, N_a$, $i \neq j$.

Remark 3 *For the ease of the computation, the agents are considered as unidimensional points in the position space. Even if they are characterized by a nonempty region $R_i \subset \mathbb{R}^n$, one can define the set in (1) as $\tilde{\mathcal{S}}_i \triangleq \mathcal{S}_i \oplus R_i$, where R_i denotes the region describing the i^{th} agent.*

Let us now assume the steering policy for each follower agent (i.e. $i \neq l$ in (21)) based only on local state information from its nearest neighbors.

Definition 4 (Neighboring graph (Tanner *et al.*, 2007)) *An undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ represents the nearest neighboring relations and consists of:*

- *a set of vertices (nodes) $\mathcal{V} = \{n_1, n_2, \cdots, n_{N_a}\}$ indexed by the agents in the group;*
- *a set of edges $\mathcal{E} = \{(n_i, n_j) \in \mathcal{V} \times \mathcal{V} : n_i \leftrightarrow n_j\}$, containing unordered pairs of nodes that represent neighboring relations.*

The set of neighbors of agent i with $i = 1, \cdots, N_a$ and $i \neq l$ can be defined as follows:

$$\mathcal{N}_i(k) \triangleq \{j = 1, \cdots, N_a : \|p^i(k) - p^j(k)\| \leq r,\ i \neq j\}, \tag{27}$$

[4]Besides computational expenses, the logistic difficulties can be mentioned: a centralized approach means that there exists a center which needs information from all the agents and has to send the control action to all agents. Such a construction may be difficult to implement and be prone to errors (e.g. the case of a radio linked system of agents and a non-neglected physical obstacle which cuts the communication inside a subgroup).

where r is the radius of the ball centered in p^i. Since the agents are in motion, their relative distances can change with time, affecting their neighboring sets (27). For each agent i, we define an inter-agent potential function which aims to accomplish the following objectives:

1) collision avoidance between agents;
2) convergence to a group formation and following the leader.

To be specific, in our problem, the following inter-agent potential function is used:

$$V_i(p^i, v^i) = \beta_r V_i^r(p^i) + \beta_a V_i^a(p^i, v^i),\ \forall i \in \mathcal{N}_i. \tag{28}$$

The two components of the potential function account for the objectives presented above and β_r, β_a are weighting coefficients for each objective. For the i^{th} agent the total potential is formed by summing the potentials terms corresponding to each of its neighbors. Consequently, in our approach, the potential functions are designed as follows:

1) $V_i^r(p^i)$ denotes the repulsive potentials that agent i sense from its neighbors:

$$V_i^r(p^i) = \sum_{j \in \mathcal{N}_i} V_{ij}^r(p^i) \tag{29}$$

To implement this, the concepts introduced in Subsection 2.3, specifically the potential functions (16) or (17) are taken into account:

$$V_{ij}^r(p^i) = \frac{c_3}{(c_4 + \psi_{ij}(p^i))^2},\ i \neq j,\ i \neq l, \tag{30}$$

where $\psi_{ij}(p^i)$ is the sum function (14) induced by the polyhedral set defined in (26). Note that the repulsive component (30) takes into account the safety regions (26) associated to both the followers and the leader.

2) $V_i^a(p^i, v^i)$ denotes the attractive component between agents in order to achieve a formation and to follow the leader:

$$V_i^a(p^i, v^i) = \sum_{j \in \mathcal{N}_i} V_{ij}^a(p^i, v^i) + \|p^l - p^i\|, \tag{31}$$

for all $i \in \mathcal{N}_i$ and $i \neq l$.
The second component denotes the relative distance between the leader and the followers. The first component $V_{ij}^a(x_i)$ has the following form:

$$V_{ij}^a(p^i, v^i) = \log(\psi_{ij}^2(p^i)) + \beta_v(v^i - v^j), \tag{32}$$

where β_v denotes a weighting coefficient for which the agents velocities are synchronizing.

As it can be observed in similar works based on the potential function methods, the parameters of the potential field have to be tuned experimentally. It will be seen in the simulations that the collision avoidance is realized for the chosen parameters. Note also that for a potential function, a piecewise affine gradient can be computed using the results in Section 2. As in (Rimon and Koditschek, 1992), (Tanner *et al.*, 2007) the negative value of the gradient can be applied in order to derive a control action for agent i. The direct approach has several shortcomings mentioned in the Section 1.

In the following, we reformulate the optimization problem (24) for the followers, by using the potential-based cost function described in (28). A control sequence $\mathbf{u}^i = \{u^i(k|k), u^i(k+1|k), \cdots, u^i(k+N_f-1|k)\}$ which minimizes the finite horizon nonlinear objective function:

$$\mathbf{u}^* = \arg\min_{\mathbf{u}^i} \left(\sum_{s=0}^{N_f} V_i(p^i(k+s|k), v^i(k+s|k) \right). \tag{33}$$

Here N_f denotes the prediction horizon for the followers. In the optimization problem (33) we need to know the future values of the neighboring graph and the values of the state for the corresponding neighbors. All these elements are time-varying and difficult to estimate. For the ease of computation we assume the following:

- The neighboring graph is considered to be constant along the prediction horizon, that is,

$$\mathcal{N}_i(k+s|k) \triangleq \mathcal{N}_i(k) \tag{34}$$

- The future values of the followers state are considered constant

$$x^j(k+s|k) \triangleq x^j(k) \tag{35}$$

- An estimation of the leader's state is provided by the equation (23)

$$x^l(k+s|k) \triangleq \tilde{x}^l_{ref}(k+s) \tag{36}$$

The equations (34)–(36) represent only rough approximation of the future state of the agents. Obviously, the MPC formulation can be improved by using prediction of the future state of the neighboring agents. Where feasible, this prediction may be provided by the agents themselves (Dunbar and Murray, 2006). Here a simplified approach was implemented for the followers (by assuming constant predictions) and using the reference trajectory for the leader.

Remark 4 *The time-varying nature of the neighboring graph and the fact that the future values of the neighboring states and the leader state are unknown represent some of the computational limitations of the presented scheme. Moreover, the resulting cost function is nonlinear and, more than that, non-convex. This means that the numerical solution may suffer from the hardware limitations and may not correspond to the global optimum.*

Remark 5 *The receding horizon technique (33) uses a discrete-time optimal control sequence parameterized by the discrete counterpart of the reference trajectory (20). As such, the usual performance, stability and robustness properties of the predictive control can be invoqued only for the discrete time closed-loop behavior. The approach presented in this paper does not consider the intersample phenomena which can be handled using polytopic inclusions or alternative over-approximations (see the work in (Gielen et al., 2010; Heemels et al., 2010)).*

4. SIMULATIONS

This section proposes two simulation examples in order to better illustrate the proposed techniques.

Consider a set of N_a heterogeneous agents in two spatial dimensions with the dynamics described by:

$$A_i = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -\frac{\nu_i}{m_i} & 0 \\ 0 & 0 & 0 & -\frac{\nu_i}{m_i} \end{bmatrix}, \; B_i = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{1}{m_i} & 0 \\ 0 & \frac{1}{m_i} \end{bmatrix} \tag{37}$$

where $[x^i \; y^i \; v^i_x \; v^i_y]^T$, $[u^i_x \; u^i_y]^T$ are the state and the input of each system. The components of the state are: the position (x^i, y^i) and the velocity (v^i_x, v^i_y) of the i^{th} agent, $i = 1, \cdots, N_a$. The parameters m_i, ν_i are the mass of the agent i and the damping factor, respectively[5].

In the first example, we consider a punctiform agent operating in an environment with obstacles designed as convex regions. Then, for controlling the agent to maneuver successfully in the hostile environment we use first a gradient approach. Second, we introduce the potential function in a predictive control optimization problem such that the collision avoidance is satisfied. In the second example, we illustrate the trajectory tracking of a leader/followers formation. For avoiding collisions inside the formation, we consider agents with associated safety regions designed also as convex sets.

Example 1: Consider one agent (i.e. $N_a = 1$ in (37)) in two spatial dimensions described by the dynamics (37), with $m_1 = 45$kg, $\nu_1 = 15$Ns/m. Let the position component of the agent be constrained by three obstacles defined by (1). We consider a potential function as in (28)

[5]Note that the matrices A_i, B_i correspond to the continuous system (18). After the initial stage of computing a flat trajectory, we will use discretized matrices, obtained as in Subsection 3.1 with a discretization time $T_s = 0.1$ s.

($\beta_r = 1$, $\beta_a = 1$), with two components: a repulsive potential (17) with $c_3 = 1$, $c_4 = 0.25$ and an attractive potential (32). The potential function generates a potential field depicted in Figure 6. First, we calculate the gradient of the potential function which is piecewise affine as in (15). The negative value of the gradient is applied in order to derive a control action for the agent. We obtain that the obstacles are usually avoided, but there are situations when the constraints are not satisfied, or the control action obtained through the negative gradient has unrealistic values. For these reasons, we introduce the potential function in a predictive control framework as in (33), with a prediction horizon $N_f = 2$. We obtain that the obstacles are always avoided. Figure 7 illustrates several trajectories of the agent with a random initial position.

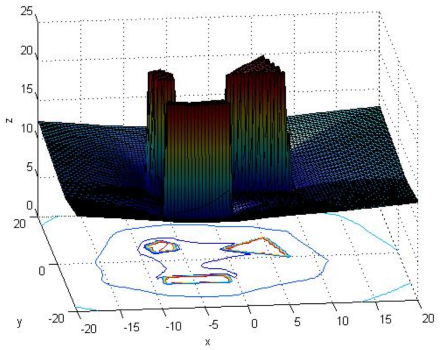

Fig. 6. Potential filed in a workspace with 3 obstacles.

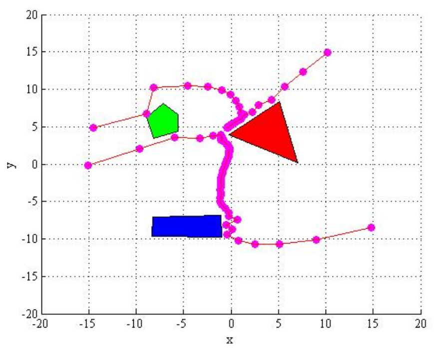

Fig. 7. Agent trajectories using predictive control.

Example 2: Consider five agents (i.e. $N_a = 5$ in (37)) described by the dynamics (37), with $m_1 = 45$kg, $m_2 = 60$kg, $m_3 = 30$kg, $m_4 = 50$kg, $m_5 = 75$kg, $\nu_1 = 15$Ns/m, $\nu_2 = 20$Ns/m, $\nu_3 = 18$Ns/m, $\nu_4 = 35$Ns/m, $\nu_5 = 23$Ns/m. The initial positions and velocities of the agents are chosen randomly. We associate to each agent a polyhedral safety region as in (1). For the sake of illustration we will choose identical safety zones for each agent. We take arbitrarily $l = 1$ to be the leader which has to be followed by the rest of the agents $i = 2, \ldots, 5$ ($i \neq l$). Figure 8 illustrates the potential filed generated for the considered group of agents.

For the leader we generate through flatness methods, state and input references (20) and for both types of agents we use MPC in order to construct the control action. A quadratic cost function as defined in (24) is used for the leader. Figure 9 illustrates the reference trajectory (in blue) and the time evolution of the leader (in red) along the trajectory. Satisfactory tracking performances for the given reference trajectory are obtained with a prediction horizon $N_l = 10$.

For the followers we consider a potential function as the cost function in the optimization problem (33), with a prediction horizon $N_f = 2$. The potential will be constructed such that both the following of the leader and the maintaining of a formation are respected. The neighborhood radius is set to $r = 8$m, the weighting coefficients are $\beta_r = 1$, $\beta_a = 10$, $c_3 = 1$, $c_4 = 0.25$, $\beta_v = 15$. The effectiveness of the present algorithm is confirmed by the simulation depicted in Figure 9, where the evolution of the agents is represented at three different time instances. The agents successfully reach a formation and follow the leader without trespassing each other safety regions.

We note that we prefer a smaller prediction horizon for the followers than the one used for the leader. This is justified by the fact that the trajectory of the leader is more important and that any additional prediction step for the potential function (which is not quadratic) incurs significant computational complexity.

Example 3: We build upon the previous example and we consider additionally obstacle avoidance. Furthermore, we redesign the reference trajectory such that it avoids stationary and a priori known obstacles. More precisely we add control points which steer the reference trajectory from the interdicted region (for further details see footnote 3 and (Prodan *et al.*, 2012)).

In Figure 10 the original reference (in blue) illustrates the flat trajectory which does not take into account the obstacle. On the other hand, by adding an additional control point we were able to construct a trajectory which avoids the obstacle (in red). Satisfactory tracking performances for the given reference trajectory are obtained with a prediction horizon $N_l = 10$, as well as in the previous example.

5. CONCLUSIONS

This paper presents the trajectory tracking problem of multiple agents. Convex safety regions are associated to each agent in order to solve the collision avoidance problem. First, the notion of polyhedral function is recalled and further introduced in a potential function which accounts for the associated safety region. Sec-

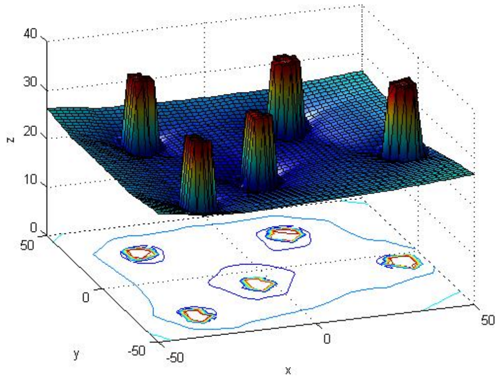

Fig. 8. Potential field in a workspace with 5 agents.

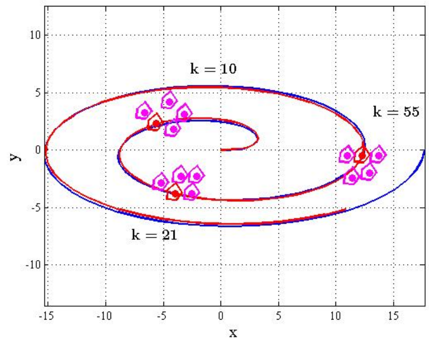

Fig. 9. Trajectory tracking of the leader/followers formation at different time instances with their safety regions (leader in red, followers in magenta).

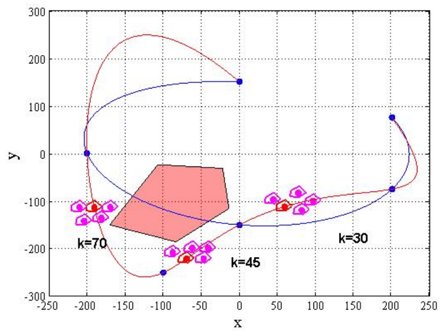

Fig. 10. Obstacle avoidance and trajectory tracking of the leader/followers formation at different time instances with their safety regions (leader in red, followers in magenta).

ond, in real-time, a receding horizon control design and a leader/followers strategy are adopted for driving the agents into a formation with collision free behavior. For the leader, a flat trajectory is generated and a receding horizon optimization problem is solved in order to minimize the tracking error. For the followers, decentralized control method is introduced by combining of Model Predictive Control and Potential Field concepts. Two kinds of potential terms are distinguished in the cost function of the followers. The repulsive potential term accounts for the collision avoidance between the agents and the attractive potential term which guarantees the convergence to a formation and the following of the leader.

Future work will focus on the investigation of the robust stability properties of the multi-agent system in presence of disturbances and uncertainties, this problem being known to be particularly intricate without strict assumptions on the time-varying properties of the interconnection graph.

ACKNOWLEDGEMENTS

The research of Ionela Prodan is financially supported by the EADS Corporate Foundation (091-AO09-1006).

References

Baskar, L., De Schutter, B. and Hellendoorn, H. (2006). Decentralized traffic control and management with intelligent vehicles, *Proceedings of the 9th TRAIL Congress 2006TRAIL in MotionCD-ROM*.

Bemporad, A. and Morari, M. (1999). Control of systems integrating logic, dynamics, and constraints, *Automatica* **35**: 407–428.

Blanchini, F. (1995). Nonquadratic Lyapunov functions for robust control, *Automatica* **31**(3): 451–461.

Camacho, E. and Bordons, C. (2004). *Model predictive control*, Springer Verlag.

De Doná, J., Suryawan, F., Seron, M. and Lévine, J. (2009). A flatness-based iterative method for reference trajectory generation in constrained NMPC, *Nonlinear Model Predictive Control* pp. 325–333.

Dunbar, W. and Murray, R. (2006). Distributed receding horizon control for multi-vehicle formation stabilization, *Automatica* **42**(4): 549–558.

Fang, Z., Song, W., Zhang, J. and Wu, H. (2010). Experiment and modeling of exit-selecting behaviors during a building evacuation, *Statistical Mechanics and its Applications* **389**(4): 815–824.

Fliess, M., Lévine, J., Martin, P. and Rouchon, P. (1995). Flatness and defect of non-linear systems: introductory theory and examples, *International Journal of Control* **61**(6): 1327–1361.

Garey, M. and Johnson, D. (1979). *Computers and intractability. A guide to the theory of NP-completeness. A Series of Books in the Mathematical Sciences*, WH Freeman and Company, San Francisco, California.

Gielen, R., Olaru, S., Lazar, M., Heemels, W., Van de Wouw, N. and Niculescu, S. (2010). On polytopic inclusions as a modeling framework for systems with time-varying delays, *Automatica* **46**(3): 615–619.

Girard, A., de Sousa, J. and Hedrick, J. (2001). Coordinated control of agent formations in uncertain, dynamic environments, *Proceedings of the European Control Conference, Porto, Portugal*.

Goodwin, G., Seron, M., Middleton, R., Zhang, M., Hennessy, B., Stone, P. and Menabde, M. (2006). Receding horizon control applied to optimal mine planning, *Automatica* **42**(8): 1337–1342.

Heemels, W., Van De Wouw, N., Gielen, R., Donkers, M., Hetel, L., Olaru, S., Lazar, M., Daafouz, J. and Niculescu, S. (2010). Comparison of overapproximation methods for stability analysis of networked control systems, *Proceedings of the 13th ACM international conference on Hybrid systems: computation and control*, ACM, pp. 181–190.

Helbing, D., Farkas, I. and Vicsek, T. (2000). Simulating dynamical features of escape panic, *Nature* **407**(6803): 487–490.

Jadbabaie, A., Lin, J. and Morse, A. (2003). Coordination of groups of mobile autonomous agents using nearest neighbor rules, *Automatic Control, IEEE Transactions on* **48**(6): 988–1001.

Khatib, O. (1986). Real-time obstacle avoidance for manipulators and mobile robots, *The International Journal of Robotics Research* **5**(1): 90.

Maciejowski, J. (2002). *Predictive control: with constraints*, Pearson Education.

Massion, I., Keviczky, T. and Verhaegen, M. (2008). New approaches to distributed control of satellite formation flying, *Proceedings 3rd International Symposium on Formation Flying, Missions and Technologies*.

Mayne, D., Rawlings, J., Rao, C. and Scokaert, P. O. (2000). Constrained model predictive control: Stability and optimality, *Automatica* **36**: 789–814.

Mazur, A. and Szakiel, D. (2009). On path following control of nonholonomic mobile manipulators, *International Journal of Applied Mathematics and Computer Science* **19**(4): 561–574.

Mesbahi, M. and Hadaegh, F. (2001). Formation flying control of multiple spacecraft via graphs, matrix inequalities, and switching, *Journal of Guidance, Control, and Dynamics* **24**(2): 369–377.

Michalek, M., Dutkiewicz, P., KiełCzewski, M. and Pazderski, D. (2009). Trajectory tracking for a mobile robot with skid-slip compensation in the vector-field-orientation control system, *International Journal of Applied Mathematics and Computer Science* **19**(4): 547–559.

Negenborn, R., van Overloop, P., Keviczky, T. and De Schutter, B. (2009). Distributed model predictive control of irrigation canals, *Networks and Heterogeneous Media (NHM)* **4**(2): 359–380.

Osiadacz, A., Nemhauser, G. L. and Wolsey, L. A. (1990). Integer and combinatorial optimization, *International Journal of Adaptive Control and Signal Processing* **4**(4): 333–334.

Overloop, P., Negenborn, R., Schutter, B. and Giesen, N. (2010). Predictive Control for National Water Flow Optimization in The Netherlands, *Intelligent Infrastructures* pp. 439–461.

Prodan, I., Bencatel, R., Olaru, S., Sousa, J., Stoica, C. and Niculescu, S. (2012). Predictive control for autonomous aerial vehicles trajectory tracking, *In Proceedings of the Nonlinear Model Predictive Control Conference*, available upon request.

Prodan, I., Olaru, S., Stoica, C. and Niculescu, S. (2010). Collision avoidance and path following for multi-agent dynamical systems, *Control Automation and Systems (ICCAS), 2010 International Conference on*, IEEE, pp. 1930–1935.

Prodan, I., Olaru, S., Stoica, C. and Niculescu, S.-I. (2011). Predictive control for tight group formation of multi-agent systems, *Proceedings of the 18th IFAC World Congress*, Milano, Italy, pp. 138–143.

Richards, A. and How, J. (2002). Aircraft trajectory planning with collision avoidance using mixed integer linear programming, *Proceedings of the 21th American Control Conference*, Anchorage, Alaska, USA, pp. 1936–1941.

Rimon, E. and Koditschek, D. (1992). Exact robot navigation using artificial potential functions, *IEEE Transactions on Robotics and Automation* **8**(5): 501–518.

Rossiter, J. (2003). *Model-based predictive control: a practical approach*, CRC.

Roussos, G. and Kyriakopoulos, K. (2010). Completely Decentralised Navigation of Multiple Unicycle Agents with Prioritization and Fault Tolerance, pp. 2996–3000.

Schneider, J. (2009). Pathway Toward a Mid-Infrared Interferometer for the Direct Characterization of Exoplanets, *Arxiv preprint arXiv:0906.0068* .

Stoican, F., Prodan, I. and Olaru, S. (2011). On the hyperplanes arrangements in mixed-integer techniques, *Proceedings of the 30th American Control Conference*, San Francisco, CA, USA, pp. 1898–1903.

Suryawan, F., De Dona, J. and Seron, M. (2010). Methods for trajectory generation in a magnetic-levitation system under constraints, *18th Mediterranean Conference on Control and Automation*, pp. 945–950.

Tanner, H., Jadbabaie, A. and Pappas, G. (2007). Flocking in fixed and switching networks, *Automatic Control, IEEE Transactions on* **52**(5): 863–868.

Van den Berg, M., Hegyi, A., De Schutter, B. and Hellendoorn, J. (2004). A macroscopic traffic flow model for integrated control of freeway and urban traffic networks, *Proceedings of the 43th IEEE Conference on Decision and Control*, Vol. 3, Atlantis, Paradise Island, Bahamas, pp. 2774–2779.

Van Nieuwstadt, M. and Murray, R. (1998). Real-time trajectory generation for differentially flat systems, *International Journal of Robust and Nonlinear Control* **8**(11): 995–1020.

Ziegler, G. M. (1995). *Lectures on polytopes*, Springer-Verlag.