A semi-active controller tuning and application to base seismically-isolated structures

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***Abstract*— This paper proposes a modified version of Leitmann and co-authors' classical result on the stabilization of uncertain nonlinear systems. In particular, for usual models of structure dynamics in earthquake engineering it is shown that applying a specific control law drives the state variables into a ball around the origin (arbitrarily chosen) in finite time as long as the radius of the ball is not lower than a limiting value. In addition estimates of this limiting ball radius and the time limit for arbitrary ball radius are provided. The semi-active control thus provides the control designer with interesting design parameters. It is also an attempt to explicitly use pseudoacceleration floor response spectrum as a performance criterion. Though not limited to two-degree-of-freedom structures, this semi-active control is applied to these plant models for simplicity and illustrated through simulations.**

I. Introduction

Earthquake protection of structures (e.g., buildings in civil engineering, power generation facilities like nuclear installations or special equipments) has been studied for decades and is still an active area of research in earthquake engineering. A practical solution for base isolated structures uses controlled devices situated at the base level and consisting of actuators that should not require large amount of energy, for instance, for security reasons. Such requirements lead to semi-active control design. Most of previous works used a two-step procedure: a reference is first calculated using an *active* control law then this reference is approximated at the best utilizing available devices. The reader may refer to [15] where LQR techniques are used to calculate the above mentioned reference.

In the present paper, the two degree of freedom structure considered by Politopoulos and Pham in [15] is revisited using an extension of a theorem by Leitmann and co-authors [8]. Contrary to [8] where a nonlinear *active* control is designed, in the present paper a nonlinear *semi-active* control (SAC) strategy is proposed. In addition, robustness aspects are addressed by allowing uncertainties in main parts of the model of the structure.

More specifically, the main result in [8] is first revisited by focusing in semi-active instead of active control. Some bounds on trajectories as well as time estimates of time to reach such bounds are provided in the present paper. Under specified conditions it is possible to state that all system trajectories are driven to a neighborhood of the origin within finite time, and estimates of theses limiting quantities are calculated. Moreover, the main objective of control is to keep the so-called *floor response spectrum* as low as possible. Floor spectrum is known from earthquake protection literature. Most previous works use performance criterion in terms of relative or absolute coordinates. In this paper, a tentative use of the floor spectrum is proposed. This is done by having recourse to modal coordinates [16, §2.3], [2, §12], [17, §3.4]. In most literature dealing with vibration, modal description is mainly used for analysis, while we use it for control synthesis purpose. For two-degree-of-freedom structure model, penalties are introduced on modal coordinates that are responsible for generating higher values on the floor spectrum.

Recall that, in seismic protection literature, control systems are commonly classified in one of the following three types [16], [17], [7]: passive, semi-active and active. From a physical point of view, semi-active devices are basically passive devices, but contrary to the latter, they have time-dependent characteristics that can be adjusted in real time and therefore provide more flexibility in designing control vibration isolation solutions. Compared to active actuators, they require small amounts of external energy source to be driven, meet safety requirements and are cheaper for maintenance. In addition, reduced-size installations are necessary to drive and operate them.

The paper is organized as follows: The model of the structure is briefly recalled in the next section, followed by some details on the floor spectrum concept. Then in Section IV a theoretical result is provided that addresses the stabilization of uncertain nonlinear systems. It is this result which is applied in Section V to obtain the proposed semi-active control. Finally simulations are shown illustrating the performance of the semi-active controller. They present improvements over passive control, for some seismic signals, notably around the second eigenfrequency.

II. Plant model

The class of base isolated structures that are considered in this work is described by the following type of equa-

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tions

$$M\,\ddot{z}_{\mathrm{r}} + (C + C_{\mathrm{c}}(c_{\mathrm{A}}))\;\dot{z}_{\mathrm{r}} + K\,z_{\mathrm{r}} = -M\,1_{n\times 1}\,\ddot{x}_{\mathrm{g}}(t) \qquad (1)$$

where the structure is modeled as an n-lumped-mass with horizontal displacements relative to the ground denoted by z_{r}, and where $\ddot{x}_{\mathrm{g}}(t)$ represents total ground level acceleration due to the horizontal seismic motion.

The three terms in (1) denote inertia, the damping component of the structure dynamics which includes a semi-active viscous damper located at the base level and acting solely in horizontal direction, and the third term stands for the structure stiffness dynamics. The semi-active device is assumed to be an ideal damper whose time-varying viscous damping coefficient c_{A} is the control input. According to its physical meaning, it should keep bounded $c_{\mathrm{A}}(t) \in [0, c_{\mathrm{A}}^{\max}]$, with $c_{\mathrm{A}}^{\max} > 0$. More complex models of semi-active actuators may be considered as, for instance, in [15] where, in addition, hysteresis and spring components are present.

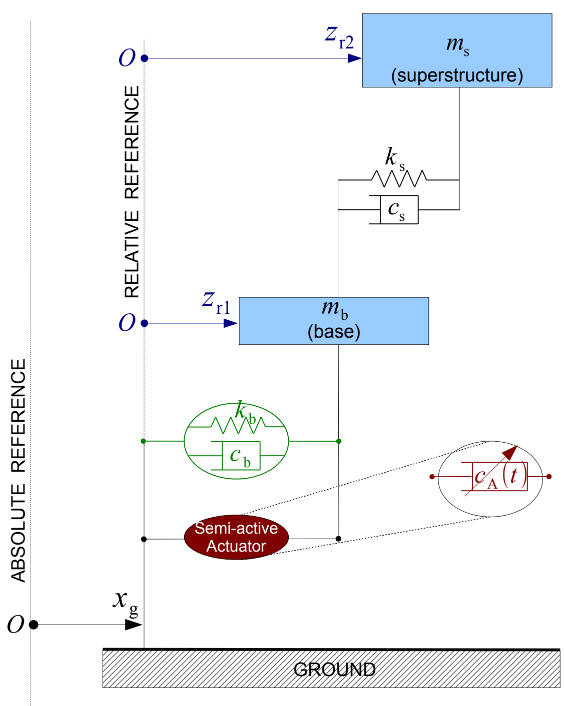

Fig. 1: A schematic diagram of a 2-degree-of-freedom structure in horizontal motion, with its relative coordinates

For a 2-degree-of-freedom lumped-mass depicted in Fig. 1, the matrices M, C, $C_{\mathrm{c}}(c_{\mathrm{A}})$ and K are given by

$$M = \begin{pmatrix} m_{\mathrm{b}} & 0 \\ 0 & m_{\mathrm{s}} \end{pmatrix}, \quad K = \begin{pmatrix} k_{\mathrm{b}} + k_{\mathrm{s}} & -k_{\mathrm{s}} \\ -k_{\mathrm{s}} & k_{\mathrm{s}} \end{pmatrix},$$

$$C = \begin{pmatrix} c_{\mathrm{b}} + c_{\mathrm{s}} & -c_{\mathrm{s}} \\ -c_{\mathrm{s}} & c_{\mathrm{s}} \end{pmatrix}, \quad C_{\mathrm{c}}(c_{\mathrm{A}}) = \begin{pmatrix} c_{\mathrm{A}} & 0 \\ 0 & 0 \end{pmatrix},$$

with all coefficients, real positive numbers, that can easily be identified in Fig. 1.

In practice, the variation of $c_{\mathrm{A}}(t)$ corresponds to the opening-closing of an orifice within the architecture of the viscous damper device. For more details, one may see [18].

Notation $1_{n\times 1}$ stands for column vector with 1 as coefficients, and 1_n will be used hereafter to designate the identity matrix of size n.

III. Floor response spectrum

The objective of control design is to achieve sufficiently low values of the so-called *floor response spectrum*, for all seismic disturbance $\ddot{x}_{\mathrm{g}}(t)$. In control literature, this is a perturbation attenuation problem with a specific performance criterion.

For the reader who is not familiar with earthquake engineering literature the following is reported: floor response spectrum, by definition (see § 25.1 of [2], Chapter 6 of [1], or § 7.3 of [11] for more details), is the function

$$(\omega, \zeta, v) \;\longmapsto\; \mathrm{PSA}(\omega, \zeta, v) = \omega^2 \max_{t \geq t_0} |y(t)| \qquad (2)$$

where

$$\ddot{y} + 2\zeta\omega\dot{y} + \omega^2 y = v(t),\;\; y(t_0) = 0,\;\; \dot{y}(t_0) = 0, \qquad (3)$$

and $v(t)$ is typically the absolute acceleration of one of the n floors of structure model. In this paper we are only concerned with base level, therefore use the quantity $\ddot{z}_{\mathrm{r}1}(t) + \ddot{x}_{\mathrm{g}}(t)$ instead of $v(t)$. When the signal $v(t)$ is clear from context, $\mathrm{PSA}(\omega, \zeta, v)$ is simply denoted by $\mathrm{PSA}_{\zeta}(\omega)$. This notation stands for pseudo-acceleration (PSA) and emphasizes the fact that the unit of measurement is $^{\mathrm{m}}/_{\mathrm{s}^2}$

The explicit solution of (3) is given by:

$$y(t) = \frac{1}{\omega_{\mathrm{d}}} \int_{t_0}^{t} e^{-\zeta\omega(t-\tau)} \sin\left(\omega_{\mathrm{d}}(t-\tau)\right)\, v(\tau)\;\mathrm{d}\tau \qquad (4)$$

with $\omega_{\mathrm{d}} = \omega\sqrt{1-\zeta^2}$.

As is clear from its definition (2), floor spectrum is *not* an explicit performance criterion that can be easily handled to obtain control laws. Multiple reasons can be cited, some of them are: (i) the nonlinear nature of the result, with respect to additivity of multiple input signals; (ii) even for simple input signals, like monochromatic and Heaviside step functions, the resulting $\mathrm{PSA}_{\zeta}(\omega)$ is a non-convex function.

This is why in most papers dealing with this topic, closed-loop performance is expressed in terms of cost functions based on maximum displacement, velocity and acceleration of the structure in response to a number of given seismic signals [12]. In present work a better insight to explicit use of floor spectrum criterion is proposed.

To face the problem of $\ddot{x}_{\mathrm{g}}(t)$ being unknown, and since it is needed for calculating $v(t)$ and further on (4), one can think of (at least) two strategies:

- even if earthquakes are known to be unpredictable, try to use explicit mathematical models of $\ddot{x}_{\mathrm{g}}(t)$, the so-called artificial signals [19], based on some *a priori* knowledge, e.g., semi-empirical laws related to some local geographical sites,

- use available past records of signals $\ddot{x}_{\mathrm{g}}(t)$ and compute numerically the solutions of equation (3) and use it in (2) to get approximated values.

Both situations emphasize the difficulty to construct a general controller based on floor spectrum. Therefore, the following control design approach is proposed: use *modal coordinates* in lieu of relative or absolute ones, and introduce penalties on various modes that are known to give high values on floor spectrum. This is detailed in Section V.

IV. A robust control design theorem

The following result is largely inspired by the work of G. Leitmann and co-authors, see for instance [8], [3], [10], [5], [6].

Theorem 1: Consider systems described by the following type of equations

$$\begin{aligned}\dot{x} &= (A + \Delta A(t))\, x + f(x,\nu) + \\ &\quad (B(x) + \Delta B(t,x,\nu))\, c_{\mathrm{A}}(t) + D\,\nu(t), \\ &\qquad x(0) = x_0\end{aligned} \tag{5}$$

with state variable $x(t) \in \mathbb{R}^n$, control input $c_{\mathrm{A}}(t) \in \mathbb{R}^m$, and nonvanishing continuous-time perturbations $\nu(t) \in \mathbb{R}^l$. Matrices A and D are known constant ones, B is a *known* continuous function of x, structural uncertainties on plant model and actuator, respectively, ΔA and ΔB are *unknown* continuous functions of their arguments. Assume f and ρ to be known continuous functions of their arguments and that

(i) matrix A is Hurwitz, P is the unique symmetric positive-definite solution of $P\,A + A'\,P = -Q$, given arbitrary symmetric positive definite Q ,

(ii) functions ν, f, ΔA, ΔB are bounded with respective bounds $\nu^{\max}$, $f^{\max}$, $\Delta A^{\max}$, $\Delta B^{\max}$,

(iii) $\Delta A^{\max} < \lambda_{\min}(Q)/\lambda_{\max}(P)/2$,

(iv) $\rho(x) \in [0, \rho^{\max}]$,

(v) $\varepsilon > 0$,

(vi) and let

$$b_0 = \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)}\, \frac{f^{\max} + \Delta B^{\max}\,\rho^{\max} + \|D\|\nu^{\max}}{\dfrac{1}{2}\dfrac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} - \Delta A^{\max}}\,. \tag{6}$$

If the control input is set as follows

$$c_{\mathrm{A}}(x) = \max\left(0_{m\times 1}, p(x)\right) \tag{7}$$

$$p(x) = \begin{cases} -\dfrac{B(x)'P\,x}{\|B(x)'P\,x\|}\rho(x), & \|B(x)'P\,x\| \geq \varepsilon, \\ -\dfrac{B(x)'P\,x}{\varepsilon}\rho(x), & \|B(x)'P\,x\| < \varepsilon, \end{cases} \tag{8}$$

then all closed-loop trajectories $x(t)$ are bounded, and, for any initial conditions x_0 satisfying $\sqrt{x_0{}'Px_0} > \sqrt{\lambda_{\min}(P)}\, b_0$, and for all $b > b_0$ the trajectories $x(t)$ are driven into balls of radius b within time

$$T = \frac{\ln\left(\dfrac{\sqrt{x_0{}'Px_0} - \sqrt{\lambda_{\min}(P)}\, b_0}{\sqrt{\lambda_{\min}(P)}\,(b - b_0)}\right)}{\dfrac{1}{2}\dfrac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} - \Delta A^{\max}}\,. \tag{9}$$

The proof of this theorem was constructed using uniform boundedness and uniform ultimate boundedness tools as in Theorem 4.18 of [9] and page 1141 of [3]. It is not included here for lack of space. However, we will briefly provide some details which will be used in the following section, to justify the different choices of controller parameters. To calculate (9) we have used the Lyapunov-like function

$$V(x) = x'Px$$

and its derivative along system (5) trajectories:

$$\begin{aligned}\dot{V}(x) &= \dot{x}'Px + x'P\dot{x} \\ &= -x'Qx + 2x'PB(x)c_{\mathrm{A}}(x) + \\ &\quad 2x'P\left[\Delta A(t)\,x + f(x,\nu) + \Delta B(t,x,\nu)c_{\mathrm{A}}(x) + D\nu\right]\end{aligned} \tag{10}$$

The following are comments on the differences between Thm 1 and its inspiring results in [8]. In works [8], [3], [10] matching conditions provide the means to dominate the cumulative effect of perturbations and structured uncertainties on plant model and actuator, by using a sufficiently strong active control law (8). In [8], [3], [10] when $\varepsilon = 0$ in (8) and the lower branch of control law is removed it can be shown that global asymptotic stability (GAS) of origin can be ensured. This means that the energy dissipation mechanism is ensured for all bounded perturbation signals, and for all $t \geq 0$.

The proposed SAC in Thm. 1 is not designed to ensure GAS of origin in any particular situation. Actually, hypotheses of Thm. 1 are not sufficient to prove GAS of origin. The controller design function $\rho(x)$ can be chosen arbitrarily small in Thm. 1 which is not the case in [8], [3], [10] where a necessary lower bound, $\rho(x) \geq \rho^{\min} > 0$ is calculated based on maximum amplitude value of perturbation. This allowed us to freely tune the function $\rho(x)$ in order to attain performance criterion like floor response pseudoacceleration spectrum.

It is worth noting that the controller (7) is a *continuous* function of variable x.

Summarizing, the theorem provides this very interesting design tool: application of the controller allows to bring system trajectories into bounds b within time T as long as $b > b_0$ where the limiting bound b_0 is of course a pretty complex function of model uncertainties and controller design parameters. In addition the theorem says that the higher the ratio $\lambda_{\min}(Q)/\lambda_{\max}(P)$ is the higher amount of uncertainty ΔA can be tolerated. However, clearly, the desirable minimization of b_0 and T

with respect to controller design parameters is not simple given the complexity of the dependance of b_0 and T on model uncertainties and controller design.

V. Semi-active control

Following the result of Thm. 1, energy dissipation mechanism is depicted by $\dot{V}(x)$ in (10). It consists of three terms: the first one can arbitrarily be fixed by the user, by properly choosing matrix Q, in accordance with control objective of floor spectrum reduction. It is a matter of simple calculation to show that the second term is nonpositive for all x. Actually, $p(x)$ from (7) was chosen to provide maximal effort, minimum time response, based on this second term of $\dot{V}$. The third term is sign uncertain and is responsible for the coupling between uncertainties and perturbation, on the one hand, and controller parameters on the other hand.

Our efforts in shaping energy loss mechanism, such that floor sectrum reduction objective should be achieved, are concentrated on the right choice of Q matrix.

To compensate the lack of an explicit expression relating the control law and floor spectrum, information related to physical phenomena is used instead: modal shapes and their influence to floor spectrum.

A. SAC tuning

As already mentioned in Section II it is a standard practice in earthquake engineering to use modal coordinates as follows.

Let (ω_i^2, φ_i) $i = 1, \ldots, n$ be the couples of eigenvalues and right eigenvectors associated with the symmetric matrices M and K, and M is positive-definite: for each i, ω_i and φ_i verify

$$K\,\varphi_i = \omega_i^2\,M\,\varphi_i\,, \quad {\varphi_i}'\,M\,\varphi_j = \delta_{ij}\,, \quad {\varphi_i}'\,K\,\varphi_j = \omega_i^2\,\delta_{ij}\,,$$

where δ_{ij} is the Kronecker symbol. ω_i and coefficients of φ_i are all real numbers. Let (ω_i^2, φ_i) be numbered such that $\omega_1^2 \le \omega_2^2 \le \cdots \le \omega_n^2$. The reader may refer to [4, §11] for more details on the algebra of symmetric positive definite generalized eigenvalue problem.

Let

$$\phi = (\varphi_1\;\varphi_2\;\cdots\;\varphi_n)$$

be the matrix of *modal shapes*, and let q be the *modal coordinates* vector defined such that

$$z_{\mathrm{r}}(t) = \phi\,q(t) = \varphi_1\,q_1(t) + \varphi_2\,q_2(t) + \cdots + \varphi_n\,q_n(t).$$

Though not limited to two-degree-of-freedom structures, this semi-active control is applied to the case $n = 2$ for simplicity in the sequel.

Modal coordinates transformations are used in the following way to influence the floor spectrum. Let W be a negative definite function, used to construct the first term in (10),

$$W = -\gamma_1 q_1^2 - \gamma_2 q_2^2 - \gamma_3 \dot{q}_1^2 - \gamma_4 \dot{q}_2^2\,,$$

with positive real parameters $\gamma_i > 0$. Parameters γ_1 and γ_3 are used to penalize the effect of first vibrational mode, in terms of generalized modal displacement and velocity, respectively. Similarly γ_2 and γ_4 are used to penalize the effect of second vibrational mode. By setting $-x'Qx = W$ with

$$x = \begin{pmatrix} z_{\mathrm{r}} \\ \dot{z}_{\mathrm{r}} \end{pmatrix}\,,$$

direct calculation lead to the following bloc diagonal matrix

$$Q = \mathrm{diag}\Big(M'\left(\gamma_1\varphi_1{\varphi_1}' + \gamma_2\varphi_2{\varphi_2}'\right)M\,,\quad M'\left(\gamma_3\varphi_1{\varphi_1}' + \gamma_4\varphi_2{\varphi_2}'\right)M\Big)\,,$$

and the structure model (1) can be rewritten as

$$\dot{x} = A\,x + B(x)\,c_{\mathrm{A}} + D\,\nu\,, \quad x(0) = 0_{2n\times 1}\,, \tag{11}$$

where

$$A = \begin{pmatrix} 0_2 & 1_2 \\ -M^{-1}K & -M^{-1}C \end{pmatrix}\,,$$

$$B(x) = \begin{pmatrix} 0_{2\times 1} \\ -e'x/m_{\mathrm{b}} \\ 0 \end{pmatrix}\,,$$

$$D = \begin{pmatrix} 0_{2\times 1} \\ -1_{2\times 1} \end{pmatrix}\,,$$

$$\nu = \ddot{x}_{\mathrm{g}}\,,$$

and

$$e' = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}\,.$$

Notations $0_{n\times 1}$, 0_n stand, respectively, for column vector of dimension n and matrices of size $n \times n$ with 0 as coefficients; notation $1_{n\times 1}$, stands for column vector with 1 as coefficients, and 1_n is the identity matrix of size n.

System (11) is of the form (5) allowing to apply Theorem 1.

B. Features and remarks

Control law calculated in subsection V-A, guarantees the required constraint specification $c_{\mathrm{A}}(t) \in [0, c_{\mathrm{A}}^{\max}]$ if one fixed $\rho(\cdot) \equiv c_{\mathrm{A}}^{\max}$. In this situation the semi-active controller is generating at all time instants the maximum calculated damping control force $-c_{\mathrm{A}}(t)\dot{z}_{\mathrm{r}1}(t)$.

On the other hand, a different approach to choosing $\rho(\cdot)$ is

$$\rho(x) = c_{\mathrm{A}}^{\max}\left(1 + \tanh\left(-|q_2(x)|\right)\right) \tag{12}$$

motivated by the objective of floor response spectrum reduction around the second eigenfrequency.

The idea in (12) is to avoid amplification of second mode's response, as it becomes more and more prominent by reducing gradually the control force at the base level. Actually this is a compromise solution, meaning that reduction of peaks around second eigenfrequency on floor spectrum curve is done in the detriment of first peak.

This method concerning controller synthesis and tuning can easily be applied on structure model (1) rewritten in absolute coordinates, instead of relative ones as is done in this paper. In this situation, the new plant model system is still of the form (5), allowing for Thm. 1 to be applied. The feedback control law c_{A} will be a function depending on absolute coordinates. Numerical simulation results not shown in this paper, illustrate similar performance in terms of floor spectrum with those presented in section VI.

VI. Simulation results

In this section we illustrate the performance of the proposed semi-active control through the 2-degree of freedom structure already detailed throughout the previous section. The parameters (in consistent units) which are used in the simulations are as follows: $m_{\mathrm{b}} = 0.25$; $m_{\mathrm{s}} = 1$; $c_{\mathrm{b}} = 0.3927$; $k_{\mathrm{b}} = 12.3370$; $c_{\mathrm{s}} = 1.8850$; $k_{\mathrm{s}} = 355.3058$; $c_{\mathrm{A}}^{\max} = 2\,\Delta\xi_{\mathrm{b}}\,\sqrt{k_{\mathrm{b}}(m_{\mathrm{b}} + m_{\mathrm{s}})}$; $\Delta\xi_{\mathrm{b}} = 0.20$; $\xi_{\mathrm{b}} = 0.05$.

Numerical simulation parameters are fixed to: time-duration of 15 seconds; sampling and computation time is 10^{-4} seconds; fourth-order Runge-Kutta fixed-step numerical solver. Floor spectrum numerical algorithm is taken from [13] and its damping ratio is set to $\zeta = 2\%$.

Multiple control scenarios are proposed for comparison purposes:

- NC (no control): this is the uncontrolled structure dynamic response to the seismic excitation. Specifically, the structure may be thought as with base isolation consisting of low damping rubber bearing (LDRB) device, with a damping coefficient ξ_{b}.
- PC (passive control): an additional damping is put at the base level consisting of a high damping rubber bearing (HDRB), so that the total equivalent damping coefficient is $\xi_{\mathrm{b}} + \Delta\xi_{\mathrm{b}}$.
- SAC from subsection V-A: corresponds to implementing control law (7) on plant model (11), with parameters $\varepsilon = 10^{-2}$, $\gamma_1 = \gamma_3 = 3$, $\gamma_2 = \gamma_4 = 10^4$, $\rho(\cdot) \equiv c_{\mathrm{A}}^{\max}$.

Instead of real records of seismic signal it is found more illustrative to use artificial ones for $\ddot{x}_{\mathrm{g}}(t)$. The reader may find them in the appendix.These signals suit the ground response pseudoacceleration spectrum specification of Cadarache rock site, in southern France. They were scaled in amplitude to reach a maximum absolute value of $0.6\,g \approx 5.88\,\mathrm{m/s^2}$, with $g = 9.81\,\mathrm{m/s^2}$. The reason in doing so is to ensure controller to be within the full working range of 0 to $c_{\mathrm{A}}^{\max}$. Otherwise, if signals are too weak compared to a reference, namely the signal used to calibrate controller parameters, the effect of adding a SAC force is negligible with respect to the natural behavior of structure. On the contrary, if seismic signals are too strong the SAC may saturate and it will act as PC with high damping, which again is not wanted.

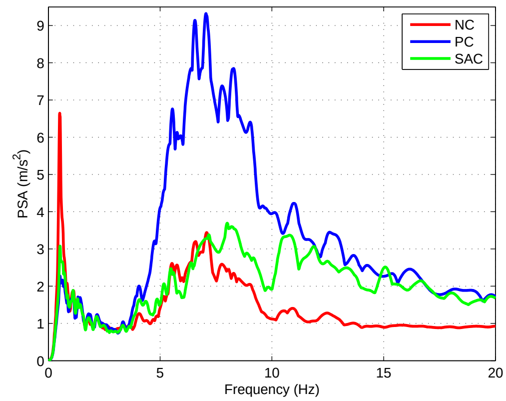

Fig. 2: Floor response spectrum at the base in response to seismic signal record no. 1

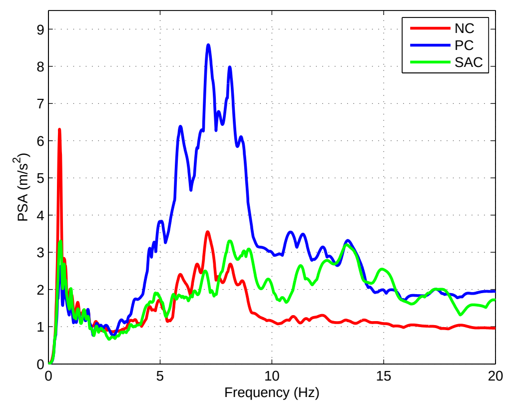

Fig. 3: Floor response spectrum at the base in response to seismic signal record no. 2

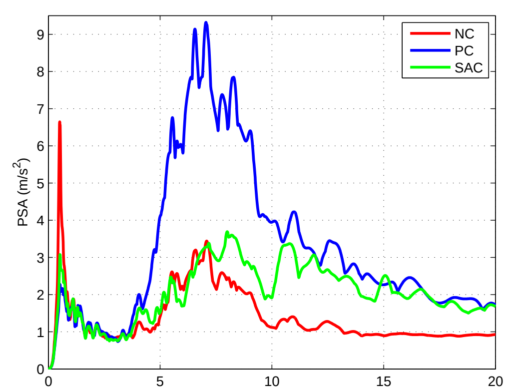

Fig. 4: Floor response spectrum at the base in response to seismic signal record no. 3

In other words, prior to implementing in practice this SAC, it is necessary to have some *a priori* knowledge on the maximum amplitude of seismic signal to hit the structure. In earthquake engineering, this information is often used in seismic characterization and is called peak ground acceleration. On the other hand, it is also required by Thm. 1 as in its second assumption.

Moreover, the selection of seismic signals is made such that they are wide-band sufficiently rich in

- low spectral content around first eigenfrequency so that we can notice a pretty large first peak on floor spectrum curve is observed when using NC,
- higher spectral content around second eigenfrequency, otherwise, phenomena related to amplification of second mode of vibration, e.g., when using PC, might not be visible in terms of floor spectrum. Often, this is the case for earthquake signals recorded on stiff soil and rock sites.

The controller parameters are calibrated once for all three simulations. They are calculated with the seismic signal in Fig. 2. The other two figures then serve for validation and to support further discussions on controller capabilities. Since the choice of controller parameters has been made with respect to only one seismic signal, one may wonder whether or not the result concerning adjustment controller parameters is globally available. By looking at Figs. 3–4, one can notice a visible advantage and improvement when using SAC over PC in terms of floor spectrum evaluation especially around second eigenfrequency. However, in practice locally available data (characteristics of known recorded signals or artificially generated ones, e.g., in terms of maximum amplitude, energy, etc., and geographical site) should be used to fine tune controller parameters.

Although not shown here, in terms of floor spectrum gauge we do not have up-to-date any concluding results that the SAC proposed in this paper is better than other semi-active control strategies already presented in the literature, like the one in [15].

Extended simulations with randomly chosen natural, historical, seismic events showed that, as a worst case behavior of this SAC device with fixed adjustment parameters, the performance in terms of floor response spectra will be at least as good as PC.

Further work: In this paper we have used two-degree-of-freedom structure models that are simple enough and therefore control-oriented, so that they might allow implementation in real-time scenarios. However, real world three-dimensional (3D) applications are much more complex than what is discussed and presented here and consequently will be more demanding in terms of control law design. For instance, the following should be taken into account for control design in a future stage of our work: (i) rotation [14] of the structure model; (ii) nonlinearities of the isolation systems, like friction; (iii) time delays and saturation in force of the SAC actuator.

VII. Conclusions

In this paper Leitmann and co-authors's results on the stabilization of uncertain nonlinear systems and on earthquake structure protection both have been revisited. Bounds are calculated guaranteeing some interesting control design parameters. This result is more user friendly. The major achievement lies in the adaptation of control law and system dynamics towards solving SAC problems. A second contribution of this paper consisted in presenting a method for choosing SAC parameters, based on vibrational modes analysis. One reason for proceeding in this direction is that performance in closed-loop of structure response to unknown seismic signals is evaluated qualitatively in terms of floor response pseudoacceleration spectrum. It is showed that working with information related to balls radius size in state space coordinates might provide efficient and potential means to reduce floor response spectra.

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