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ABSTRACT

Recently, it was shown that transceiver hardware impairments have a detrimental impact on the performance of communication systems, especially for high-rate systems. The vast majority of technical contributions in the area of relaying assume ideal transceiver hardware. This paper quantifies the impact of transceiver hardware impairments in dual-hop Amplify-and-Forward (AF) relaying, both for fixed and variable gain relays. The outage probability (OP) in this practical scenario is a function of the instantaneous end-to-end signal-to-noise-and-distortion ratio (SNDR). This paper derives closed-form expressions for the exact and asymptotic OPs under Rayleigh fading, accounting for hardware impairments at both the transmitter and the relay. The performance loss is small at low spectral efficiency, but can otherwise be very substantial. In particular, it turns out that for high signal-to-noise ratio (SNR), the instantaneous end-to-end SNDR converges to a deterministic constant, called the SNDR ceiling, which is inversely proportional to the level of impairments. This stands in stark contrast to the ideal hardware case for which the end-to-end SNDR grows without bound in the high SNR regime.

*Index Terms*— Transceiver hardware impairments, AF relaying

1. INTRODUCTION

The use of relays for improving coverage, throughput, and quality-of-service in wireless systems has been a hot research topic over the past decade, both in academia [1,2] and in industry [3]. This is due to the fact that, unlike base stations, relays are low-cost nodes that can be easily deployed and, hence, enhance the network agility. The vast majority of works in the context of relaying networks make the assumption of ideal transceiver hardware. However, in practice, hardware suffers from impairments; for example, phase noise, IQ imbalance, and amplifier non-linearities [4–6]. These have a deleterious impact on the achievable performance [7–13]. This effect is more pronounced in high-rate systems, especially those employing inexpensive hardware [5]. For instance, some recent works have demonstrated that non-ideal hardware severely affects single-hop multi-antenna systems; [6, 8] proved that there is a finite capacity limit at high SNR, while [9, 13] showed that existing signal processing algorithms need to be re-designed to account for these impairments.

Despite the importance of transceiver impairments, their impact on relaying has only been partially investigated; in particular, bit
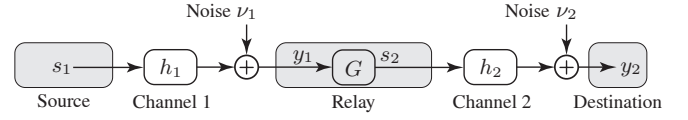
(a) Classical system model with ideal transceiver hardware.
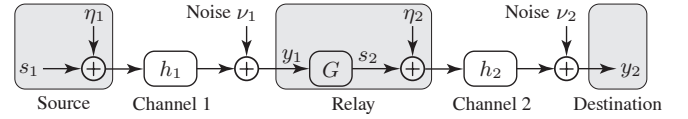
(b) Generalized system model with transceiver impairments.

Fig. 1: Block diagram of AF relaying with (a) ideal hardware or (b) transceiver impairments with aggregate distortion noise η_i, $i = 1, 2$.

error rate simulations were conducted in [10], while [11, 12] derived expressions for the bit/symbol error rates, considering only non-linearities or IQ imbalance, respectively. In this paper, we try to bridge this gap by analytically evaluating the performance of dual-hop AF relaying networks in the presence of aggregate transceiver impairments, both for fixed and variable gain relaying. In particular, we obtain the SNDRs and tractable closed-form expressions for the exact and asymptotic OPs of the system. This enables us to characterize the impact of impairments. To the best of our knowledge, this is the first paper presenting an analytical study of relaying with transceiver impairments under the generalized system model of [5].

One important observation is that for both types of relaying, an increase in the SNRs lead to an "SNDR ceiling" effect, which is explicitly quantified. The value of the SNDR ceiling is inversely proportional to the level of impairments. This manifests that transceiver impairments have a significant impact on AF relaying—especially at high SNRs and when high spectral efficiencies are desirable.

2. SIGNAL AND SYSTEM MODEL

This paper revisits the classical dual-hop relaying in Fig. 1a, where a source communicates with a destination through a relay using AF.

2.1. Ideal Hardware

In the classical complex baseband system model (see Fig. 1a), the received signals at the relay and destination are

$$y_i = h_i s_i + \nu_i, \qquad i = 1, 2, \tag{1}$$

where $s_1, s_2 \in \mathbb{C}$ are the transmitted signals from the source and relay, respectively, with average signal power $P_i = \mathbb{E}_{s_i}\{|s_i|^2\}$, for

$i = 1, 2$. Observe that $\mathbb{E}\{\cdot\}$ denotes the expectation operator. In addition, $\nu_i \sim \mathcal{CN}(0, N_i)$ represents the additive circularly-symmetric complex Gaussian noise terms for $i = 1, 2$. The random channel fading coefficients are modeled as Rayleigh fading[1] such that $h_i \sim \mathcal{CN}(0, \Omega_i)$, where $\Omega_i \triangleq \mathbb{E}_{h_i}\{|h_i|^2\}$ is the average power of the fading channel. Recall that, in this case, the cumulative distribution functions of the channel gains, $\rho_i = |h_i|^2$, are given by

$$F_{\rho_i}(x) = 1 - e^{-\frac{x}{\Omega_i}}, \quad x \geq 0, \qquad i = 1, 2. \tag{2}$$

The quantity $\frac{P_i \Omega_i}{N_i}$ will be referred to as the *average SNR*, for $i = 1, 2$. A high SNR is achieved by high signal and/or fading power.

The transmitted signal s_2 at the relay is simply an amplified version of the signal y_1 received at the relay: $s_2 = G y_1$. If the relay has instantaneous knowledge of the fading channel, h_1, it can apply variable gain relaying with $G^{\mathrm{v}} \triangleq \sqrt{P_2/\mathbb{E}_{s_1, \nu_1}\{|y_1|^2\}}$ [14]. Otherwise, fixed gain relaying with $G^{\mathrm{f}} \triangleq \sqrt{P_2/\mathbb{E}_{s_1, \nu_1, h_1}\{|y_1|^2\}}$ can be applied using only statistical information [2].[2] With ideal (id) hardware, the amplification factors can be expressed as

$$G_{\mathrm{id}}^{\mathrm{f}} = \sqrt{\frac{P_2}{P_1 \Omega_1 + N_1}}, \quad G_{\mathrm{id}}^{\mathrm{v}} = \sqrt{\frac{P_2}{P_1 \rho_1 + N_1}}. \tag{3}$$

The received signal at the destination is

$$y_2 = h_2 G_{\mathrm{id}}(h_1 s_1 + \nu_1) + \nu_2. \tag{4}$$

After some simple algebraic manipulations, the end-to-end SNDRs of (1) for variable and fixed gain relays are obtained as

$$\gamma_{\mathrm{id}}^{\mathrm{f}} = \frac{P_1 \rho_1 \rho_2}{\rho_2 N_1 + N_2/(G_{\mathrm{id}}^{\mathrm{f}})^2}, \quad \gamma_{\mathrm{id}}^{\mathrm{v}} = \frac{P_1 P_2 \rho_1 \rho_2}{P_2 \rho_2 N_1 + (P_1 \rho_1 + N_1) N_2}, \tag{5}$$

respectively. Similar expressions were previously derived in [2, 14].

2.2. Non-Ideal Hardware: Transceiver Impairments

The model in (1) implicitly assumes *ideal transceiver hardware*. However, physical radio-frequency (RF) transceivers suffer from a variety of impairments that create a mismatch between the intended signal for transmission, s_i, and the signal that is actually generated and emitted. The impact of each impairment (e.g., phase-noise, IQ imbalance, and amplifier non-linearities) can be modeled separately for a given hardware setup [5, 10]. Interestingly, the combined influence is well-modeled by the generalized system model

$$y_i = h_i(s_i + \eta_i^{\mathrm{tx}}) + \eta_i^{\mathrm{rx}} + \nu_i, \qquad i = 1, 2, \tag{6}$$

where $\eta_i^{\mathrm{tx}}, \eta_i^{\mathrm{rx}}$ are *distortion noises* in the transmitter and the receiver, respectively [5]. Experiments and theoretical investigations (e.g., [4, 6, 7]) have verified that these are well-modeled as

$$\eta_i^{\mathrm{tx}} \sim \mathcal{CN}\left(0, (\kappa_i^{\mathrm{tx}})^2 \mathcal{P}\right), \quad \eta_i^{\mathrm{rx}} \sim \mathcal{CN}\left(0, (\kappa_i^{\mathrm{rx}})^2 |h_i|^2 \mathcal{P}\right), \tag{7}$$

where the Gaussianity can be explained by the aggregate effect of many impairments.[3] Note that the distortion depends on the intended signal for transmission/reception, in the sense that the variance of η_i is proportional to its average power, here denoted as $\mathcal{P}$.

For convenience, we define $\eta_i \triangleq \eta_i^{\mathrm{tx}} + \eta_i^{\mathrm{rx}}/h_i \sim \mathcal{CN}(0, \kappa_i^2 \mathcal{P})$ as the *aggregate distortion noise*, where $\kappa_i^2 = (\kappa_i^{\mathrm{tx}})^2 + (\kappa_i^{\mathrm{rx}})^2$. The

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parameter $\kappa_i > 0$ describes the *level of impairments* on the ith channel and is an important parameter in this paper. Note that κ_i can be interpreted as the total error vector magnitude (EVM)—a common measure for quantifying the quality of RF transceivers.[4] This parameter is essentially constant while operating in the dynamic range of the power amplifier, but can otherwise increase very rapidly [7, 9]. The generalized system model of (6) is illustrated in Fig. 1b.

For the non-ideal (ni) hardware counterpart of (4), the received signal at the destination is now obtained as

$$y_2 = h_2 G_{\mathrm{ni}}\left(h_1(s_1 + \eta_1) + \nu_1\right) + h_2 \eta_2 + \nu_2 \tag{8}$$

$$= G_{\mathrm{ni}} h_1 h_2 s_1 + G_{\mathrm{ni}} h_1 h_2 \eta_1 + G_{\mathrm{ni}} h_2 \nu_1 + h_2 \eta_2 + \nu_2.$$

For fixed and variable gain relaying, G_{ni} reads respectively as

$$G_{\mathrm{ni}}^{\mathrm{f}} \triangleq \sqrt{\frac{P_2}{P_1 \Omega_1(1 + \kappa_1^2) + N_1}}, \quad G_{\mathrm{ni}}^{\mathrm{v}} \triangleq \sqrt{\frac{P_2}{P_1 \rho_1(1 + \kappa_1^2) + N_1}}. \tag{9}$$

The aggregate distortion noise is equal to $\eta_1 \sim \mathcal{CN}\left(0, \kappa_1^2 P_1\right)$ for the source and $\eta_2 \sim \mathcal{CN}\left(0, \kappa_2^2 G_{\mathrm{ni}}^2 \mathbb{E}_{s_1, \nu_1}\{|y_1|^2\}\right)$ for the relay. The latter becomes $\eta_2^{\mathrm{f}} \sim \mathcal{CN}\left(0, \left(G_{\mathrm{ni}}^{\mathrm{f}}\right)^2 \kappa_2^2 \left(P_1 \rho_1(1 + \kappa_1^2) + N_1\right)\right)$ and $\eta_2^{\mathrm{v}} \sim \mathcal{CN}\left(0, \kappa_2^2 P_2\right)$ for fixed and variable gain relays, respectively. After some algebraic manipulations (using the expressions for $G_{\mathrm{ni}}^{\mathrm{v}}$), the end-to-end SNDRs for fixed and variable gain relaying are

$$\gamma_{\mathrm{ni}}^{\mathrm{f}} = \frac{P_1 \rho_1 \rho_2}{P_1 \rho_1 \rho_2 d + \rho_2 N_1(1 + \kappa_2^2) + \frac{N_2}{\left(G_{\mathrm{ni}}^{\mathrm{f}}\right)^2}}, \tag{10}$$

$$\gamma_{\mathrm{ni}}^{\mathrm{v}} = \frac{P_1 P_2 \rho_1 \rho_2}{P_1 P_2 \rho_1 \rho_2 d + \rho_1 P_1 N_2(1 + \kappa_1^2) + \rho_2 P_2 N_1(1 + \kappa_2^2) + N_1 N_2}, \tag{11}$$

respectively, where $d \triangleq \kappa_1^2 + \kappa_2^2 + \kappa_1^2 \kappa_2^2$ will appear repeatedly in the paper. These SNDR expressions reduce to (5) for $\kappa_1 = \kappa_2 = 0$.

3. OUTAGE PROBABILITY ANALYSIS

This section derives new analytical expressions for the exact and asymptotic OPs with AF under the presence of transceiver impairments, for fixed and variable gain relaying. These results are compared with the standard results in the literature based on the assumption of ideal hardware. The OP, $P_{\mathrm{out}}(x)$, is defined as the probability that the channel fading makes the SNDR fall below a certain threshold x; that is,

$$P_{\mathrm{out}}(x) = \Pr\{\gamma \leq x\}, \tag{12}$$

where γ is the end-to-end SNDR of the system.

3.1. Exact SNR Analysis: Arbitrary Fading Distributions

This subsection considers the OP with arbitrary channel fading. The following lemma plays an important role when deriving $P_{\mathrm{out}}(x)$.

Lemma 1. *Let c_1, c_2, c_3 be strictly positive constants and let ρ be a non-negative random variable with cdf $F_\rho(\cdot)$. Then,*

$$\Pr\left\{\frac{c_1 \rho}{c_2 \rho + c_3} \leq x\right\} = \begin{cases} F_\rho\left(\frac{c_3 x}{c_1 - c_2 x}\right), & 0 \leq x < \frac{c_1}{c_2}, \\ 1, & x \geq \frac{c_1}{c_2}. \end{cases} \tag{13}$$

Suppose $c_2 = 0$ instead, then (13) simplifies to

$$\Pr\left\{\frac{c_1 \rho}{c_3} \leq x\right\} = F_\rho\left(\frac{c_3 x}{c_1}\right). \tag{14}$$

Proof. The left-hand side of (13) is equal to

$$\Pr\left\{c_1\rho \le (c_2\rho + c_3)x\right\} = \Pr\left\{\rho \le \frac{c_3 x}{(c_1 - c_2 x)}\right\} \quad (15)$$

after some algebra. The last expression is exactly $F_\rho(\frac{c_3 x}{c_1 - c_2 x})$. Note that if $(c_1 - c_2 x) \le 0$, then the outage event always occurs. □

The effective SNDRs under AF relaying are given in (5) and (10)–(11) for ideal and non-ideal transceiver hardware, respectively. The OPs can be computed using Lemma 1.

Theorem 1. *Suppose ρ_i is an independent non-negative random variable with cdf $F_{\rho_i}(\cdot)$ and pdf $f_{\rho_i}(\cdot)$ for $i = 1, 2$. For fixed and variable gain relaying, the OPs with ideal hardware are*

$$P_{\text{out}}^{\text{id,f}}(x) = \int_0^\infty F_{\rho_1}\left(\frac{x\left(N_1\rho_2 + \frac{N_2}{(G_{\text{id}}^{\text{f}})^2}\right)}{P_1\rho_2}\right) f_{\rho_2}(\rho_2)d\rho_2, \quad (16)$$

$$P_{\text{out}}^{\text{id,v}}(x) = 1 - \int_0^\infty \left(1 - F_{\rho_1}\left(\frac{(x+x^2) + \frac{P_2 x}{N_2}z}{\frac{P_1 P_2}{N_1 N_2}z}\right)\right) f_{\rho_2}\left(z + \frac{N_2 x}{P_2}\right)dz, \quad (17)$$

respectively. The corresponding OPs with transceiver hardware impairments satisfy $P_{\text{out}}^{\text{ni,f}}(x) = P_{\text{out}}^{\text{ni,v}}(x) = 1$ for $x \ge \frac{1}{d}$ and

$$P_{\text{out}}^{\text{ni,f}}(x) = \int_0^\infty F_{\rho_1}\left(\frac{(b_1\rho_2 + a_1)x}{\rho_2(1 - xd)}\right) f_{\rho_2}(\rho_2)d\rho_2, \quad (18)$$

$$P_{\text{out}}^{\text{ni,v}}(x) = 1 - \int_0^\infty \left(1 - F_{\rho_1}\left(\frac{\frac{x+x^2}{1-xd} + \frac{b_1 xz}{a_2}}{\frac{(1-xd)}{a_2}z}\right)\right) f_{\rho_2}\left(z + \frac{xb_2}{1-xd}\right)dz, \quad (19)$$

for $x < \frac{1}{d}$. In these expressions, we have $a_1 \triangleq \frac{N_2}{P_1(G_{\text{ni}}^{\text{f}})^2}$, $a_2 \triangleq \frac{N_1 N_2}{P_1 P_2}$, $b_1 \triangleq \frac{N_1(1+\kappa_2^2)}{P_1}$, and $b_2 \triangleq \frac{N_2(1+\kappa_1^2)}{P_2}$.

Proof. The OP in (12) depends on the end-to-end SNDR γ, which is a function of both ρ_1 and ρ_2. Using the law of total probability to condition on ρ_2, we can write $\Pr\{\gamma \le x\} = \int_0^\infty \Pr\{\gamma \le x|\rho_2\}f_{\rho_2}(\rho_2)d\rho_2$. The conditional probability $\Pr\{\gamma \le x|\rho_2\}$ can then be evaluated using Lemma 1, which immediately gives (16) and (18). Finally, (17) is obtained by the additional change of variable $z = \rho_2 - \frac{N_2 x}{P_2}$, while (19) is achieved by setting $z = \rho_2 - \frac{xb_2}{1-xd}$. □

This theorem provides integral expressions for the OPs. In the following, we particularize these expressions to the case of Rayleigh fading. Interestingly, the OP with transceiver impairments always equals 1 when $x \ge \frac{1}{d}$. This has an intuitive explanation since $\gamma \le \frac{1}{d}$ for any transmit power and fading distribution; see Section 3.3.

3.2. Exact SNR Analysis: Rayleigh Fading Channels

Under Rayleigh fading and ideal hardware, the OPs with fixed and variable gain relaying were obtained in [2, Eq. (9)] and [14, Eq. (14)], respectively. Using our notation, we get

$$P_{\text{out}}^{\text{id,f}}(x) = 1 - 2e^{-\frac{N_1 x}{P_1 \Omega_1}}\sqrt{c_{\text{id}}^{\text{f}}x}\, K_1\left(2\sqrt{c_{\text{id}}^{\text{f}}x}\right), \quad (20)$$

$$P_{\text{out}}^{\text{id,v}}(x) = 1 - 2e^{-x\left(\frac{N_1}{P_1 \Omega_1} + \frac{N_2}{P_2 \Omega_2}\right)}\sqrt{\frac{x+x^2}{c_{\text{id}}^{\text{v}}}}\, K_1\left(2\sqrt{\frac{x+x^2}{c_{\text{id}}^{\text{v}}}}\right), \quad (21)$$

where $c_{\text{id}}^{\text{f}} \triangleq \frac{N_2}{P_1(G_{\text{id}}^{\text{f}})^2\Omega_1\Omega_2}$, $c_{\text{id}}^{\text{v}} \triangleq \frac{P_1 P_2\Omega_1\Omega_2}{N_1 N_2}$, and $K_1(\cdot)$ denotes the first-order modified Bessel function of the second kind.

The non-ideal case with transceiver hardware impairments is more involved, as can be seen from the following corollary.

Corollary 1. *Suppose h_1, h_2 are independent and $h_i \sim \mathcal{CN}(0, \Omega_i)$ for $i = 1, 2$. The OPs under transceiver hardware impairments are*

$$P_{\text{out}}^{\text{ni,f}}(x) = \begin{cases} 1 - 2e^{-\frac{B_{\text{ni}}^{\text{f}}x}{1-xd}}\sqrt{\frac{A_{\text{ni}}^{\text{f}}x}{1-xd}}\, K_1\left(2\sqrt{\frac{A_{\text{ni}}^{\text{f}}x}{1-xd}}\right), & x < \frac{1}{d}, \\ 1, & x \ge \frac{1}{d}, \end{cases} \quad (22)$$

$$P_{\text{out}}^{\text{ni,v}}(x) = \begin{cases} 1 - 2e^{-\frac{B_{\text{ni}}^{\text{v}}x}{1-xd}}\frac{\sqrt{A_{\text{ni}}^{\text{v}}(x+x^2)}}{1-xd}\, K_1\left(2\frac{\sqrt{A_{\text{ni}}^{\text{v}}(x+x^2)}}{1-xd}\right), & x < \frac{1}{d}, \\ 1, & x \ge \frac{1}{d}, \end{cases} \quad (23)$$

for fixed and variable gain relaying, respectively. In these expressions, we have $A_{\text{ni}}^{\text{f}} \triangleq \frac{N_2}{P_1(G_{\text{ni}}^{\text{f}})^2\Omega_1\Omega_2}$, $B_{\text{ni}}^{\text{f}} \triangleq \frac{N_1(1+\kappa_2^2)}{\Omega_1 P_1}$, $A_{\text{ni}}^{\text{v}} \triangleq \frac{N_1 N_2}{P_1 P_2\Omega_1\Omega_2}$, and $B_{\text{ni}}^{\text{v}} \triangleq \frac{N_1(1+\kappa_2^2)}{\Omega_1 P_1} + \frac{N_2(1+\kappa_1^2)}{\Omega_2 P_2}$.

Proof. The proof consists of evaluating (18) and (19) using the fading distribution in (2). For fixed gain relaying, $P_{\text{out}}^{\text{ni,f}}(x)$ becomes

$$\int_0^\infty \left(1 - e^{-\frac{(b_1\rho_2 + a_1)x}{\Omega_1\rho_2(1-xd)}}\right)e^{-\frac{\rho_2}{\Omega_2}}d\rho_2 \\ = 1 - e^{-\frac{B_{\text{ni}}^{\text{f}}x}{1-xd}}\int_0^\infty e^{-\frac{A_{\text{ni}}^{\text{f}}x}{\rho_2(1-xd)}}e^{-\frac{\rho_2}{\Omega_2}}d\rho_2 \quad (24)$$

for $x < \frac{1}{d}$ and is equal to 1 otherwise. The remaining integral is evaluated using [16, Eq. (3.324.1)], leading to (22). The expression for $P_{\text{out}}^{\text{ni,v}}(x)$ is obtained in the same way. □

This corollary provides new and tractable closed-form expressions for the OPs in case of transceiver impairments.

3.3. Asymptotic SNR Analysis

To obtain some insights on the fundamental impact of impairments, we now elaborate on the high-SNR regime. For the ease of presentation, we assume that P_1, P_2 grow large with $P_1 = \beta P_2$ for some $\beta > 0$, such that the relay gain remains finite and strictly positive.

Corollary 2. *Suppose P_1, P_2 grow large with a finite non-zero ratio and consider any random distributions on ρ_1, ρ_2 that are strictly positive (with probability one). The asymptotic OP with transceiver impairments and either fixed or variable gain relaying satisfies*

$$\lim_{P_1, P_2 \to \infty} P_{\text{out}}(x) = \begin{cases} 0, & x \le \frac{1}{\kappa_1^2 + \kappa_2^2 + \kappa_1^2\kappa_2^2}, \\ 1, & x > \frac{1}{\kappa_1^2 + \kappa_2^2 + \kappa_1^2\kappa_2^2}. \end{cases} \quad (25)$$

Proof. Referring back to (10) and by taking the limit $P_1, P_2 \to \infty$ (with $P_1 = \beta P_2$), we can easily see that the instantaneous end-to-end SNDR, for fixed gain relaying, converges to

$$\lim_{P_1, P_2 \to \infty} \gamma_{\text{ni}}^{\text{f}} = \gamma^* = \frac{1}{d} = \frac{1}{\kappa_1^2 + \kappa_2^2 + \kappa_1^2\kappa_2^2}. \quad (26)$$

Therefore, (25) is trivially obtained. The proof for the case of variable gain relaying follows a similar line of reasoning. □

A number of conclusions can be drawn from Corollary 2. First, an SNDR ceiling effect appears in the high-SNR regime, which significantly limits the performance of AF relaying systems. This phenomenon is fundamentally different from the ideal hardware case, in
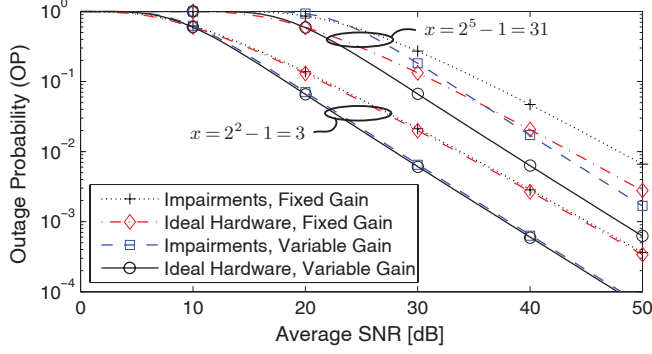
Fig. 2: Outage probability $P_{\text{out}}(x)$ with AF relaying with ideal and non-ideal transceiver hardware.

Fig. 3: Maximum goodput $R_{\text{goodput}}(x)$ with AF relaying with ideal and non-ideal transceiver hardware.

Fig. 4: Value of the SDNR ceiling in (26) for different levels of impairments κ_1, κ_2 for which $\kappa_1 + \kappa_2 = 0.15$.

which an increasing SNR makes the end-to-end SNDR grow without bound and $P_{\text{out}}(x) \to 0$ for any x. Note that this ceiling effect is independent of the fading distributions of the two hops.

Moreover, the SNDR ceiling γ^* in (26) is inversely proportional to the squares of κ_1, κ_2. We also note that the impact of κ_1 and κ_2 on the value of γ^* is symmetric. Recall that these parameters describe the level of transceiver impairments and equal the EVMs of the source and relay hardware, respectively. This validates that transceiver impairments dramatically affect performance of AF relaying, and, hence, they should be taken into account when designing and evaluating relaying networks. To this end, we now assume that we have a fixed total EVM constraint κ_{tot}, such that $\kappa_1 + \kappa_2 = \kappa_{\text{tot}}$, and present the following insightful corollary.

Corollary 3. *Suppose* $\kappa_1 + \kappa_2 = \kappa_{\text{tot}}$ *for some constant* $0 < \kappa_{\text{tot}} < 2$. *The SNDR ceiling in* (26) *is maximized if* $\kappa_1 = \kappa_2 = \frac{\kappa_{\text{tot}}}{2}$.

Proof. The maximization of (26) is equivalent to a minimization of $f(\kappa_1) = \kappa_1^2 + (\kappa_{\text{tot}} - \kappa_1)^2 + \kappa_1^2 (\kappa_{\text{tot}} - \kappa_1)^2$. Simple differentiation reveals that the function is convex for any $0 < \kappa_{\text{tot}} < 2$ and that $\kappa_1 = \frac{\kappa_{\text{tot}}}{2}$ is the only real-valued solution to $\frac{\partial}{\partial \kappa_1} f(\kappa_1) = 0$. $\square$

Corollary 3 shows that it is better to have the same hardware quality on the two hops of the relay channel, than having one high-quality node and one low-quality node. We finally point out that the SNDR ceiling can be trivially bounded as $\frac{1}{(\kappa_1 + \kappa_2)^2} \leq \gamma^* \leq \frac{1}{\kappa_1^2 + \kappa_2^2}$.

4. NUMERICAL ILLUSTRATIONS

This section provides numerical illustrations of our analytical results. We consider a symmetric Rayleigh fading scenario where the average SNR, $\frac{P_i \Omega_i}{N_i}$, is the same for $i = 1, 2$. This enables us to focus on the impact of transceiver hardware impairments.

The OP, $P_{\text{out}}(x)$, with fixed and variable gain relaying is shown in Fig. 2 as a function of the average SNR. We compare the system performance with ideal transceivers and non-ideal transceivers with $\kappa_1 = \kappa_2 = 0.1$. Two thresholds are considered: $x = 2^2 - 1 = 3$ and $x = 2^5 - 1 = 31$. These correspond to spectral efficiencies of 2 and 5 bits/s/Hz, respectively. The curves were generated by the analytical expressions in (20)–(23), while the marker symbols show the results of Monte-Carlo simulations. As seen in Fig. 2, there is a minor performance loss caused by transceiver impairments in the low threshold case of $x = 3$. However, there is a substantial performance loss of around 5 dB in SNR when the threshold is increased to $x = 31$. This effect applies to both variable and fixed gain relaying. Note that variable gain relaying outperforms the fixed one in most scenarios of interest, which is in line with [2].
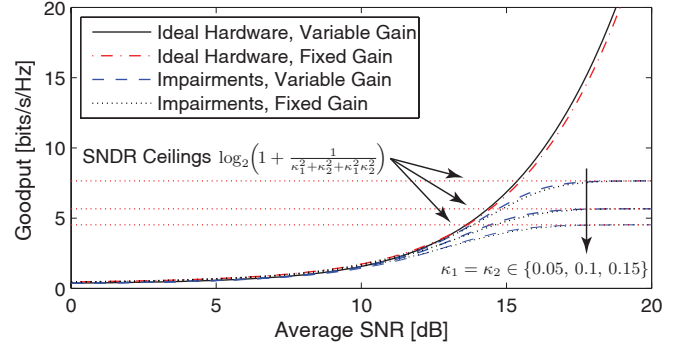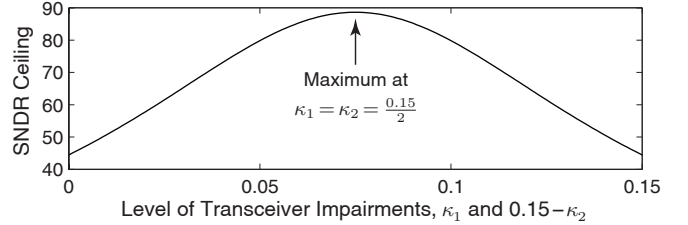
Next, we consider the average rate of data correctly received under fading (in bits/s/Hz) [17, Chapter 4.2.3], or the so-called *goodput* [18]. This is defined as the product between the gross spectral efficiency and the corresponding probability of successful communication. For any given setup, the maximum goodput is defined as

$$R_{\text{goodput}} = \max_{x \geq 0} \; \log_2(1 + x)(1 - P_{\text{out}}(x)). \tag{27}$$

The analytical expressions for $P_{\text{out}}(x)$ in (20)–(23) allow simple numerical computation of the maximum goodput in (27). This is illustrated in Fig. 3 for different levels of impairments: $\kappa_1 = \kappa_2 \in \{0.05, 0.1, 0.15\}$. This figure demonstrates the deleterious impact of non-ideal hardware; the goodput with transceiver impairments saturates at high SNR and approaches $\log_2(1 + \gamma^*)$, where $\gamma^* = \frac{1}{\kappa_1^2 + \kappa_2^2 + \kappa_1^2 \kappa_2^2}$ is the SNDR ceiling. This is consistent with Corollary 2, because γ^* is the highest SNDR for which the OP approaches zero as $P_1, P_2 \to \infty$. The saturation level increases when κ_1, κ_2 are decreased. Finally, Fig. 4 shows the value of the SNDR ceiling for different sets of κ_1, κ_2 for which $\kappa_1 + \kappa_2 = 0.15$. As stated in Corollary 3, the maximum is achieved by $\kappa_i = \frac{0.15}{2}$ for $i = 1, 2$.

5. CONCLUSIONS

Physical transceiver hardware introduces impairments that distort the emitted signals. Despite the practical importance of these impairments, little was known about their impact on the achievable performance of relaying systems. This paper has analytically shown that the performance of dual-hop AF relaying is highly affected by these impairments, particularly when high spectral efficiencies are required. Closed-form expressions for the exact and asymptotic OPs were derived for fixed and variable gain relaying. These expressions effectively characterize the impact of impairments and manifest the existence of an SNDR ceiling that cannot be crossed by increasing transmit powers or improving fading conditions. The engineering insight is that both hops should have the same hardware quality.

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