Electromagnetic modeling of anisotropic medium and applications

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ELECTROMAGNETIC MODELING OF ANISOTROPIC MEDIUM AND APPLICATIONS

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ABSTRACT

A stable method of calculation of the electromagnetic response of planar anisotropic laminates to an active source with limited distribution along the strata direction is developed. The laminates can be sandwiched between isotropic, anisotropic, or perfectly conducting covers/substrates. The source is either inside the laminates or in the cover or substrate. Based on the propagator matrix method, the proposed method relies on downward- and upward-going recurrence relations which transfer the tangential electric and magnetic fields from one interface to the next in accord with the boundary conditions, even when an distributed active source is embedded between the two interfaces. The electromagnetic response of general anisotropic laminates to active sources within or above them can thus be efficiently and accurately computed without further consideration of the conventional numerical instability issue. Some focus is put also on the highly practical case of electrically uniaxial materials with anisotropy axes parallel with the strata (fiber-reinforced composite panels). The method works for conductive and dielectric materials, and from eddy-currents to microwaves, without specific tuning, as it will be illustrated in the presentation by a number of examples in comparison to the literature.

Once the incident field in a non-damaged structure has been obtained, then the case of a damaged layered structure is handled via a method of moments based on rectilinear discretization and windowing. The numerical approach is validated with comparisons to known results (isotropic case) and FEM computations (isotropic and anisotropic cases), illustrating efficiency and accuracy.

Index Terms— anisotropic planar laminates, electric sources, numerical simulations, dyadic Green's functions, method of moment

1. NON-DAMAGED ANISOTROPIC PLANAR LAMINATES

The availability of the electromagnetic response of anisotropic planar-layered media (laminates) is an important asset if one intends to appraise the effect of defects affecting them. This applies to the characterization of composite panels involving fiber-reinforced polymer matrices in aeronautics and in automotive industry. One might refer to [1] for a confrontation of SQUIDS and GMR on fiber/metal laminates, to [2] for in-line inspection of carbon-fiber materials via eddy-current arrays, to [3] for investigations of micro- and macroscopic, thermal and electric behaviors of carbon-fiber composites, to [4] and [5] for microwave testing of glass-fiber laminates. Yet, pioneering analyses of conductive composites [6] and [7] appear not developed further, like also those in [8] about metal-matrix composite slabs in the eddy-current regime.

There exist other interesting situations as well: microwave shielding [9], [10], marine controlled-source surveying of sea bottoms [11] (multi-layered anisotropic media in geophysics are studied before [12] with simpler models), complex antenna and guiding systems with planar-layered materials [13], possibly fibrous ones [14]. Much is also available on biaxial crystals and multi-layered film-like optical structures [15] and on magnetic cores laminated with steel sheets [16].

In brief, many domains are concerned with anisotropic laminates in electromagnetics. Still, to afford the least computational burden, achieve the best stability, and work in the broadest frequency band and for arbitrary numbers of strata is not easy. Isotropic structures are already challenging, due to complex surface and/or evanescent waves within, and slow decays and oscillatory behaviors of the spectral dyadic Green functions (DGF) involved and complexities of the Sommerfeld integrals required to go back to the spatial domain [17], [18].

The need to cleverly model scattering by anisotropic laminates is emphasized in [19], the decomposition into Transverse Electric (TE) and Magnetic (TM) modes in the spectral domain uncoupled at the planar boundaries being not true anymore.

Take $\mathbf{z}$ as vertical axis orthogonal to the laminate interfaces and $(\mathbf{x}, \mathbf{y})$ as horizontal axes parallel to them. Define (k_x, k_y) as the lateral wave-number plane. For electrical uniaxial media with anisotropy axes along $\mathbf{z}$, dispersion relationships for each plane-wave component are circular like in isotropic cases [12, 20, 28] and the boundary conditions are simply expressed in the radial wave-number k_ρ domain. But, for general anisotropy, or when the anisotropy axes of the uniaxial media are not along **z**, the dispersion relations could be elliptic (and no longer the same as with isotropy), and the boundary conditions should be matched in the whole (k_x, k_y) plane.

This applies to the aforementioned composite panels involving repeated stacks of planar strata, each stratum with orientation of the fibers parallel with the interfaces (and usually differing from one to the next). To match the boundary conditions, the field must be decomposed into four wave modes coupled at each interface. A first-order differential (state) equation is satisfied in each stratum (cover and substrate being accounted for). This leads to the well-known propagator matrix method [21, 23]. Yet this method suffers from numerical instability [24, 25] due to evanescent waves fast decaying along **z**.

To cope with this instability, recurrence relations (RR) have been proposed [25]. Magnitudes of evanescent components are normalized to avoid too large numbers in the computation. In addition, [26] and [27] introduced another RR involving impedance and hybrid matrices to describe the transformation of the tangential field components, an asymptotic approximation of the field relations with very thin layers being made to avoid the eigenproblem analysis. Also, a geometrical optics series method is underlined in [19] for source-free cases.

However, if an active current source bounded along **z** is continuously distributed within or outside the laminate, the RR in [25], [26] and [27] are not enough. As for the method in [19], the radiation pattern of the source is needed in the host anisotropic medium to use the reflection and transmission concepts required, and it may not be easy to calculate. Alternatively, given the Green's function, the spatial one or the spectral one, which has been well studied in the literature and can be calculated by several methods such as the RR given in [25], the response of the laminate to the distributed source can always be computed by integrating in the spatial or the spectral domain. Yet one would need to get the Green's function that covers the whole source, which would require many of them if it is not small and thus is not computationally efficient. Furthermore, by discretizing the source, inaccuracy is introduced, which can be cause of concern when the near-field response is needed.

In the present work, based on the eigen-analysis of the state equation and the RR given in [25], novel RR are introduced to handle the above scenario. The field vector is now not only decomposed into two wave modes as in [25]. An extra source term is introduced to consider the source effect and meanwhile to stabilize it. The only knowledge needed is the 3-D spatial Fourier transform of the current density source. In addition, for completeness and following the same scheme, we also extend the RR given in [25] to make them able to account for an active source.

To the best of our knowledge, these two sets of RR are the first ones capable of stably dealing with a distributed source for anisotropic laminates. Comparison of these two sets of new RR to traditional methods is given, the main advantage of using the proposed two sets of RR being that one can efficiently and accurately calculate the spectral response of the anisotropic laminate to the distributed source, even when the observation point is in the near-field zone of the source. This could dramatically facilitate the calculation in many scenarios, such as constructing the impedance matrix in the method of moments when dealing with a scattering problem with anisotropic laminates as background media.

2. DAMAGED ANISOTROPIC PLANAR LAMINATES

The so-called volume integral equation method, usually implemented in numerical practice via a method of moments (MoM), is a popular approach to many scattering problems, like when inhomogeneous bodies are embedded within planarly layered media (in particular, this is no way the only situation of interest) and made to interact with given sources.

As is well-known, when the layered media are isotropic, the construction of the impedance matrix of the MoM, the bottleneck of the method, can be performed in fast way by accelerating the calculation of the Sommerfeld integrals (SI) involved, e.g., by the discrete complex image method in [29] and references therein. Then, the accelerated SI is a one-dimensional (1D) integral coming from the two-dimensional (2D) inverse Fourier transform (IFT). As aforementioned, this 2D IFT can indeed be cast into a 1D integral in view of the dispersion relations of the isotropic media, enabling to calculate the integral along the azimuth direction in closed form.

The above fast methods apply to uniaxial layered media provided that optical axes are orthogonal to the planar interfaces. Yet, if they lie parallel with the interfaces, e.g., [8] [3], they do not anymore. It results in possibly high computational cost to ensure accuracy in dealing with the 2D IFT. However, one might still circumvent the 2D IFT, with the rectilinear mesh usual in volume integral equation methods, then achieving efficient construction of the impedance matrix.

In pioneering contribution [30], a method that uses the continuous Fourier transform and a windowing technique is proposed to analyze metal patches in an isotropic multilayered structure. Though this method may not be so competitive against fast algorithms dedicated to isotropic layered media, it has the quite unique advantage of avoiding the conventional 2D IFT. Yet the relations between the continuous Fourier spectrum and the sampled signals are not explicitly provided; this results in a erroneous statement on usage of the window function, i.e., the method is accurate only with a rectangular window, impairing efficiency.

Here, from generalized Poisson summation formula, we derive a new relation between the continuous Fourier spectrum of a continuous function and its sampled signals on a rectilinear mesh in the spatial domain. Using this relation and applying the windowing technique, with a properly chosen window function, a fast algorithm is proposed, which yields the discrete sampled signals on a rectilinear mesh from the continuous Fourier spectrum of the original function in both efficient and accurate manner. So, we are able to construct the impedance matrix in a MoM involving uniaxial layered media whose optical axes lie parallel to the interfaces.

Compared to [30], the use of the window function is different, and window functions other than the rectangular one can be used without compromising the accuracy, while the procedure is computationally efficient if they are properly chosen. In addition, new numerical interpolation and numerical integration methods, based on recently proposed Padua points to deal with the 2D continuous function, e.g., [31], are implemented. Numerical simulations, with comparisons to known results (isotropic case) and FEM computations (isotropic and anisotropic cases) then illustrate efficiency and accuracy.

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3. REFERENCES

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