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# An efficient interpolation for calculation of the response of composite layered material and its implementation in MUSIC imaging

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## Research context

## Main institutions involved

- Département de Recherche en Électromagnétisme, Laboratoire des Signaux et Systèmes UMR8506
- Department of Electrical and Computer Engineering, National University of Singapore



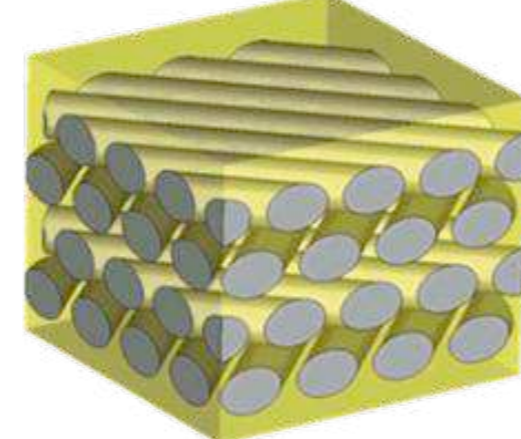
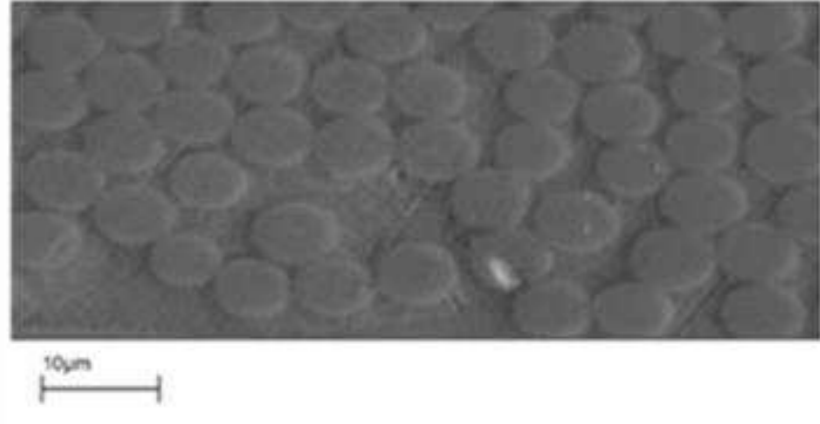
## Main financial support

- Bilateral Merlion Project (2011–2012), projet N° 8.14.10 "Fast 3-D electromagnetic imaging of anisotropic media and non-destructive evaluation"
- DIGITEO : PhD Scholarship jointly involving L2S and CEA-LIST (Département Imagerie et Simulation pour le Contrôle)
- Univ Paris-Sud : invited assistant-professor
- Internal ressources (DRÉ & NUS)



## To build

- Accurate computational models of complex anisotropic multi-layer composite panels
- Robust, fast, end-user's friendly imaging procedures

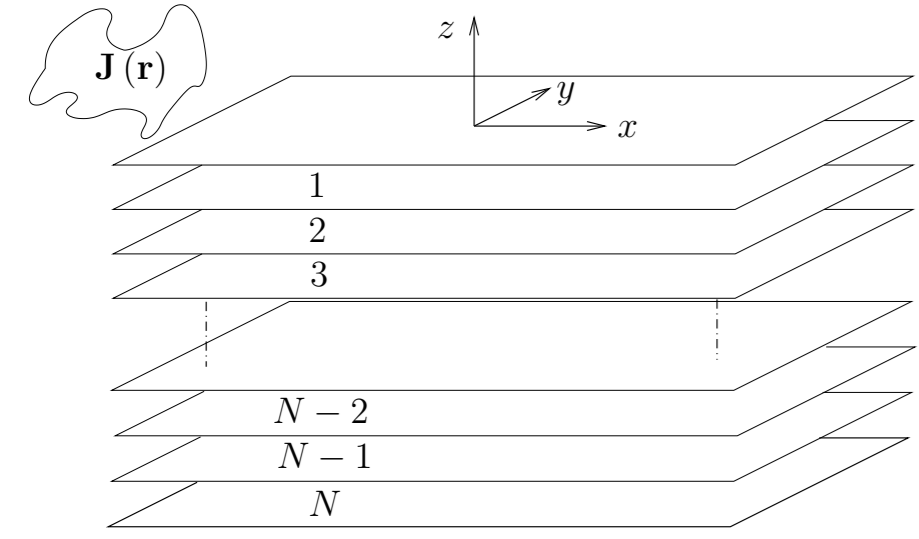


## NdT-E from eddy-currents to microwaves in aeronautics and automotive industries



## Electromagnetic modeling and preliminary numerical results

## Undamaged structure

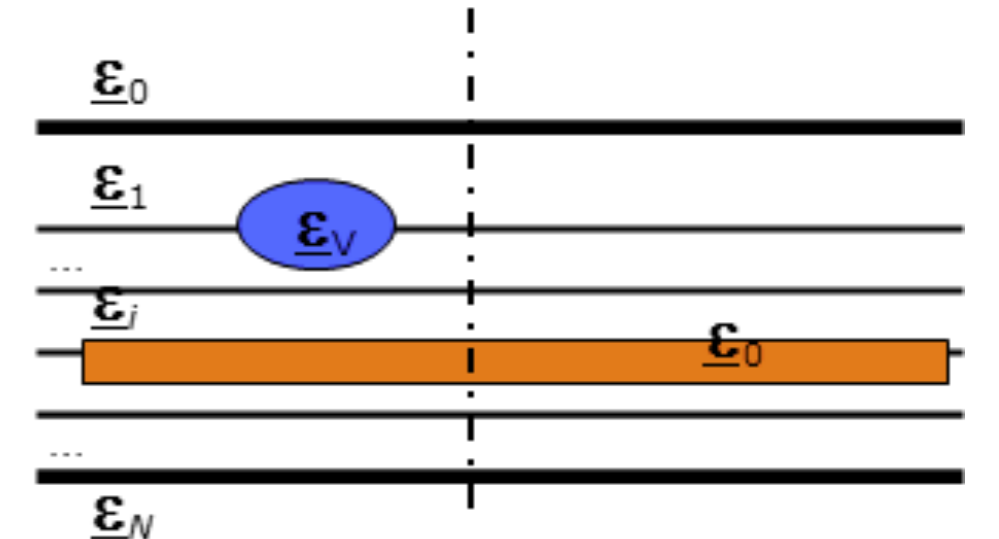


- Each layer: homogeneous *anisotropic* (different e.m. properties from layer to next)
- Uniaxial dielectric (glass-based) or conductive (graphite-based) effective media *large-scale hypothesis*
- *Diagonal* tensor in eigenframes along and orthogonal to fibers' axes
- *Dyadic Green* function in need as well as response to known distributed electric source anywhere in the structure

$$\bar{\epsilon}(\vec{r}) = \text{diag}[\epsilon_{n,11}; \epsilon_{n,22}(\vec{r}); \epsilon_{n,22}(\vec{r})] \text{ in local coordinate system}$$

## Damaged structure

- E.M. parameters differ from background stratified panel within layers or at interfaces
- 3-D volumetric defects (voids, fluid-filled cavities, localized damaged zones, etc.), or delaminations (thin, air-type slabs)
- Method of Moments upon vector contrast-source integral formulations, or change of dyads via supplementary reflection/transmission



## Constructing the spectral and spatial response of the laminate (forward modeling)

New recurrence relations based on the propagator matrix method [8]

- To efficiently calculate the spectral response of the laminate
- Capable of stably dealing with distributed source along  $z$
- More efficient compared to the traditional Green's function method
- To numerically solve the state equation

$\frac{d}{dz}\vec{\psi}(z) = \bar{A}_n \cdot \vec{\psi}(z) + \vec{f}(z)$  with  $\vec{\psi}(z)$  containing the tangential components of the fields and  $\vec{f}(z)$  being the source term

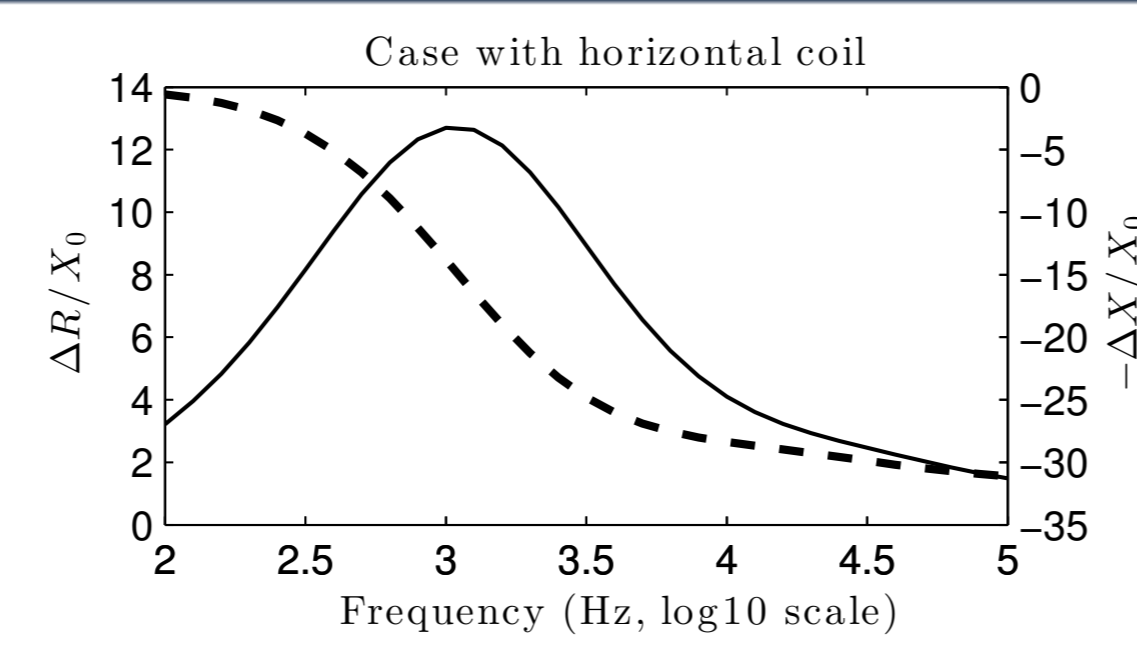


Figure : Reproduction of the results found in [2]

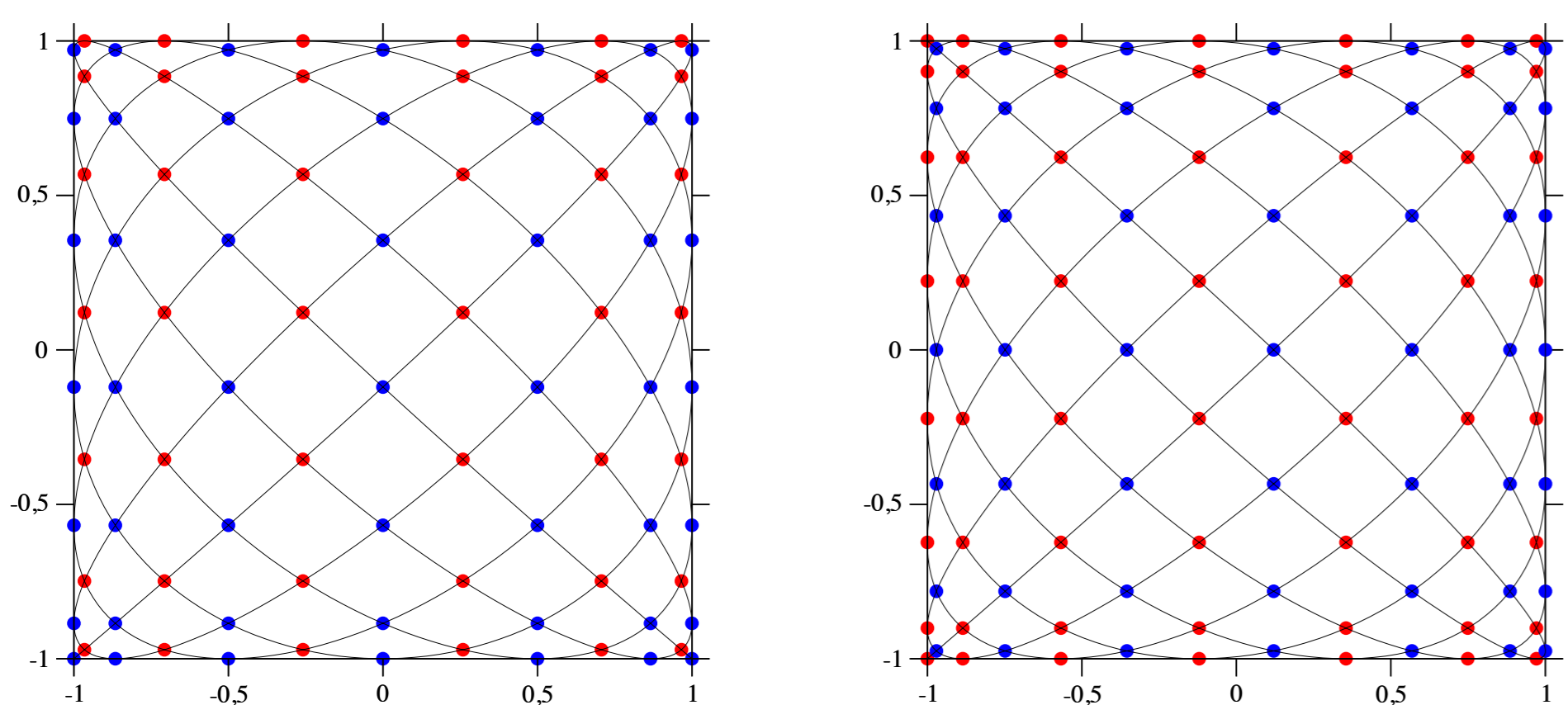
Fast algorithm to calculate the spatial response of the laminate [7]

- To efficiently calculate the spatial response on the rectilinear mesh
- No inverse Fourier transform involved
- Much faster than the inverse Fourier transform
- Incorporating the fast numerical interpolation and integration based on Padua points
- Especially useful when constructing the impedance matrix of the MoM

## Multiple Signal Classification (MUSIC) imaging with anisotropic layered media

## Interpolation and integration using the Padua points

- Alternative representation as self intersections and boundary contacts of the generating curve
- $\gamma(t) = (-\cos((n+1)t), -\cos(nt), t \in [0, \pi])$

Figure : The Padua points with their generating curve for  $n = 12$  (left, 91 points) and  $n = 13$  (right, 105 points), also as union of two Chebyshev-Lobatto sub-grids (red and blue bullets). Image taken from [3].

## Dealing with fast oscillating integrals

- Goal is to compute the I-FT of fast oscillating spectrum in the  $k_x - k_y$  plane

$$G(x, y) = \frac{1}{4\pi^2} \iint_{-\infty}^{\infty} \tilde{G}_0(k_x, k_y) e^{i(k_x x + k_y y)} dk_x dk_y \quad (1)$$

- Interpolation of the non-oscillating part at the Padua points with Chebyshev's polynomial interpolant

$$\mathcal{L}_n \tilde{G}_0(k_x, k_y) = \sum_{k=0}^n \sum_{j=0}^k c_{j, k-j} \hat{T}_j(k_x) \hat{T}_{k-j}(k_y) - \frac{1}{2} c_{n,0} \hat{T}_n(k_x) T_0(k_y) \quad (2)$$

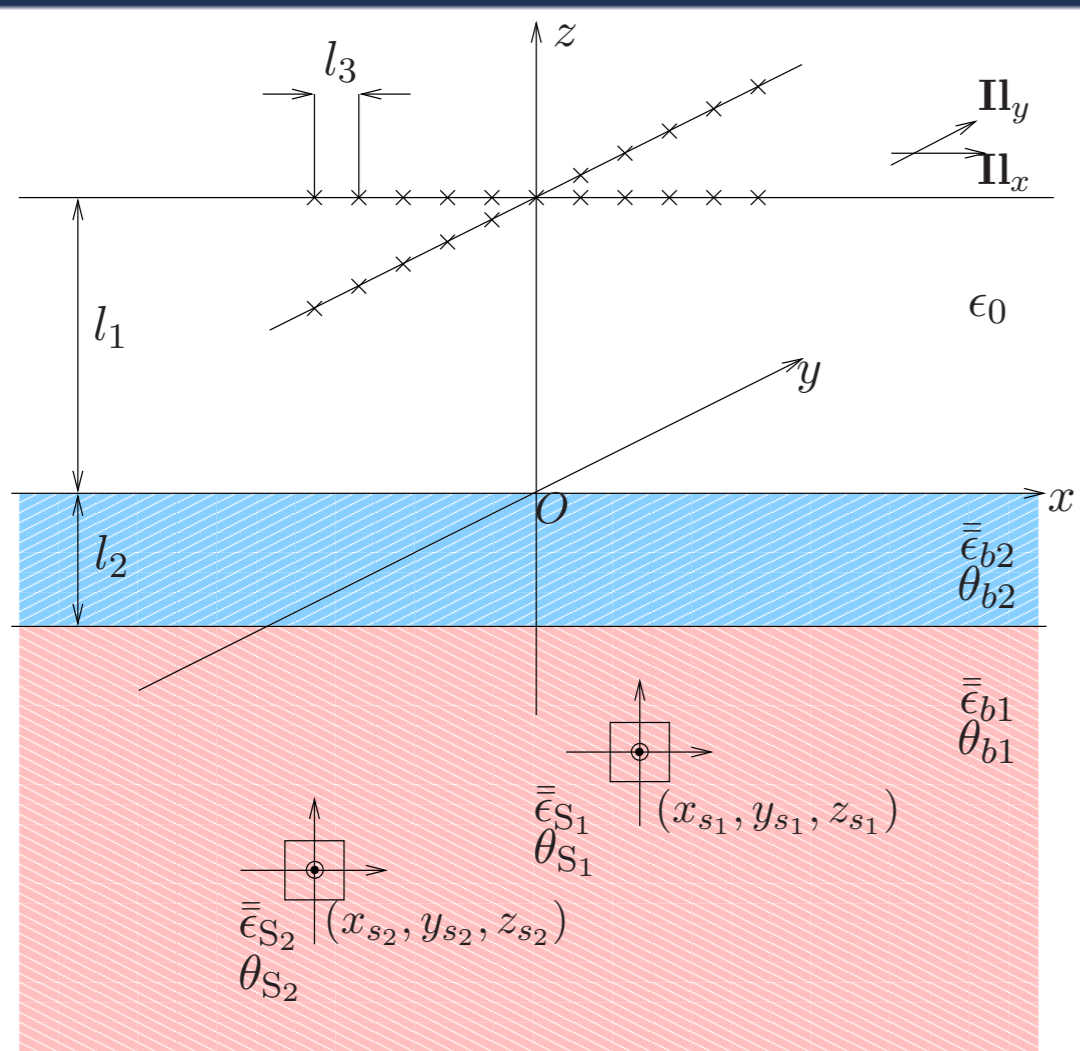
- with weights  $c_{j, k-j}$  computed using [4]

- Fourier transform of Chebyshev polynomials given by

$$\int_{-1}^1 \hat{T}_n(k_x) \exp(-ik_x x) dk_x \quad (3)$$

- are managed using [6] among other good options.

## MUSIC images of anisotropic layered media affected by two defects



- Standard MUSIC imaging method [1]

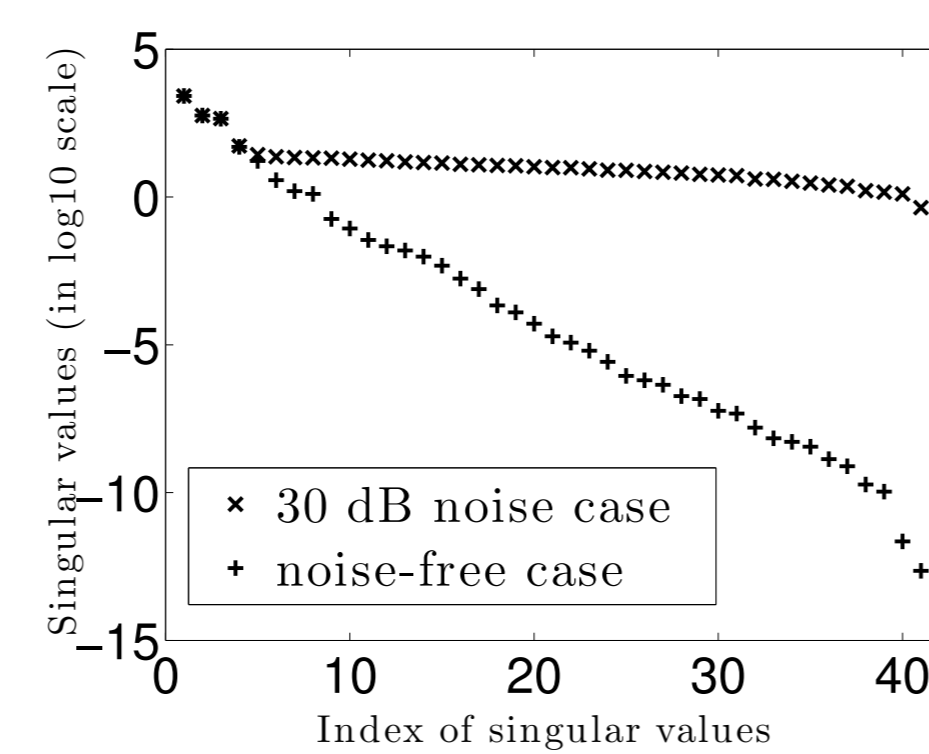
$$\phi(\vec{r}) = \frac{1}{\sum_{\sigma_j < \sigma_L} \sum_{v=1}^3 |\vec{u}_j^* \cdot \vec{G}_v(\vec{r})|^2}$$

- Enhanced MUSIC imaging method [5]

$$\phi(\vec{r}) = \frac{1}{1 - \sum_{\sigma_j > \sigma_L} |\vec{u}_j^* \cdot \vec{G}(\vec{r}) \cdot \vec{a}_{\text{test}}|^2}$$

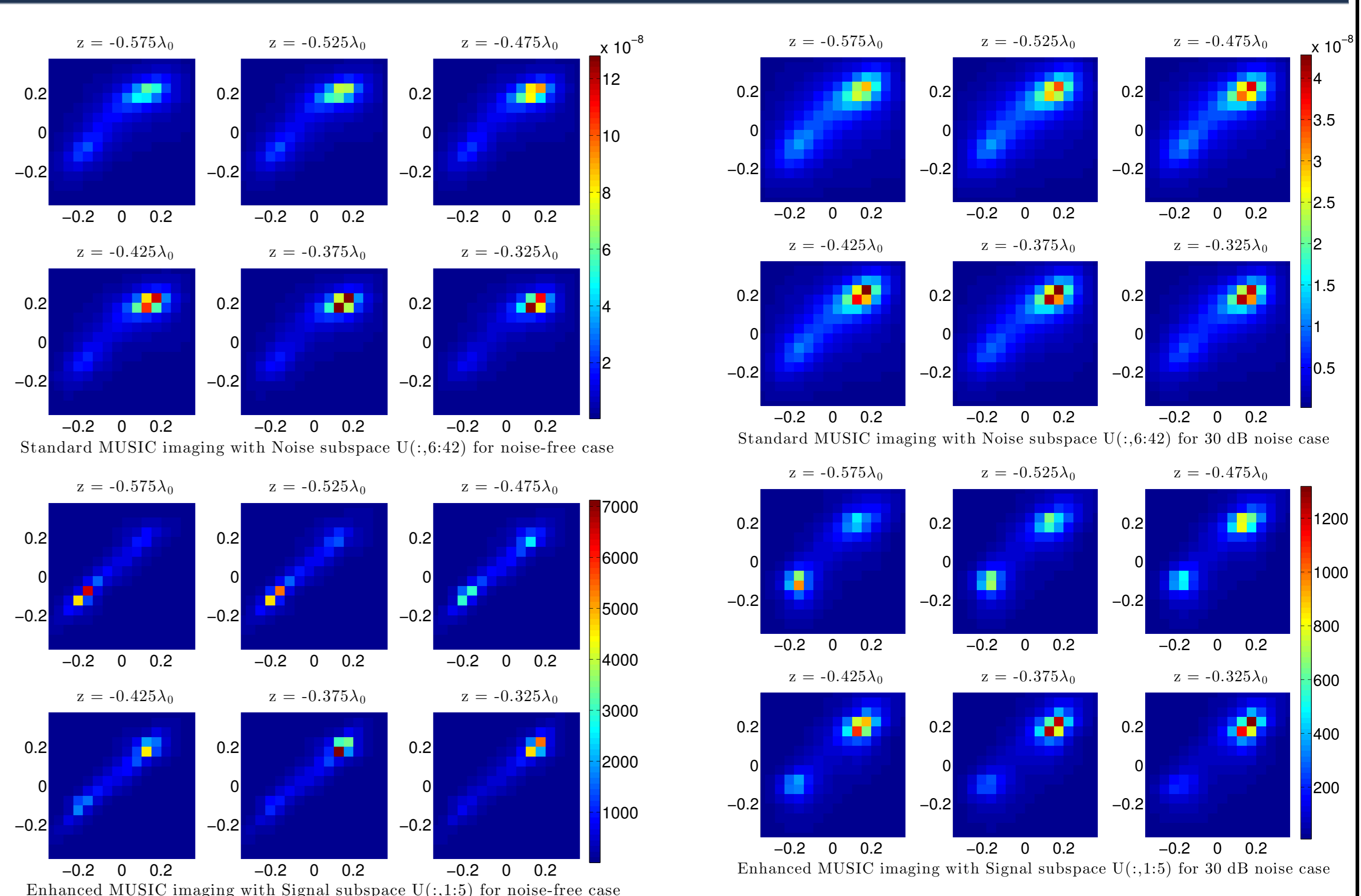
with

$$\vec{a}_{\text{test}} = \arg \max_{\vec{a}} \frac{\sum_{\sigma_j > \sigma_L} |\vec{u}_j^* \cdot \vec{G}(\vec{r}) \cdot \vec{a}|^2}{|\vec{G}(\vec{r}) \cdot \vec{a}|^2}$$



- A cross type antenna array with 5 antennas on each arm.
- Each antenna as transceivers with  $x$  and  $y$  polarizations.
- Two small inclusions with dimension of  $0.1\lambda_0 \times 0.1\lambda_0 \times 0.1\lambda_0$ .

$$\begin{aligned} \bar{\epsilon}_{b1} &= \text{diag}[4.5 + i0.2, 6 + i0.05, 6 + i0.05]; \theta_{b1} = 45^\circ \\ \bar{\epsilon}_{b2} &= \text{diag}[2 + i0.3, 3 + i0.1, 3 + i0.1]; \theta_{b2} = 60^\circ \\ \bar{\epsilon}_{s1} &= \bar{\epsilon}_0 \quad \theta_{s1} = 0^\circ \quad \theta_{s2} = 120^\circ \\ \bar{\epsilon}_{s2} &= \text{diag}[4.5 + i0.2, 6 + i0.05, 6 + i0.05] \quad l_1 = 0.5\lambda_0 \\ (x_{s2}, y_{s2}, z_{s2}) &= (-0.2\lambda_0, -0.1\lambda_0, -0.55\lambda_0) \quad l_2 = 0.2\lambda_0 \\ (x_{s1}, y_{s1}, z_{s1}) &= (0.15\lambda_0, 0.2\lambda_0, -0.35\lambda_0) \quad l_3 = 0.2\lambda_0 \end{aligned}$$



## Conclusions &amp; perspectives

- Numerical integration method based on Padua points is proposed to avoid directly interpolating on the fast oscillating function.
- The approach is validated by comparison with configurations found in the literature

- Green's function constructed by the proposed method is applied in MUSIC imaging.
- Preliminary numerical results show the efficiency of the proposed method in a fully complex anisotropic configuration

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