An efficient interpolation for calculation of the response of composite layered material and its implementation in MUSIC imaging

Giacomo Rodeghiero, Yu Zhong, Marc Lambert, Dominique Lesselier, Xudong Chen

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G. Rodeghiero[1], Y. Zhong[2], M. Lambert[1], D. Lesselier[1], and X. Chen[2]

[1]: *Département de Recherche en Electromagnétisme - Laboratoire des Signaux et Systèmes, UMR8506 (CNRS-SUPELEC-Univ. Paris-Sud)*
[2]: *Department of Electrical and Computer Engineering, National University of Singapore*
contact: rodeghiero@lss.supelec.fr

Summary

Focus is on computing the response of a layered composite anisotropic material to a source not close to the domain of interest. A novel interpolation and integration method taking care of fast oscillating spectral response is proposed. The implementation into the multiple signal classification (MUSIC) imaging method is presented also.

1. Introduction

Availability of accurate computational models of complex multi-layer composite materials and robust, fast imaging procedures for problems of quality, viability, and safety of complex systems are getting quite essential nowadays. From eddy currents to microwaves and beyond, an example is the non-destructive testing-evaluation (often referred with acronym NdT-E) of manufactured parts in industry [1], [2].

The structures under investigation here can be considered as a succession of planar slabs (panels) one over the other; each slab is usually formed from a bundle of fibers within some polymer matrix, the orientation of those being parallel with the interfaces and usually differing from one slab to the next [3]. The fibers themselves or the way they are organized might lead to either electromagnetic isotropy or electromagnetic anisotropy of the layered material, bringing to either scalar or tensor quantities while describing the material parameters.

Internal cracks and voids, delaminations, fiber breakings, etc. might affect these structures with impact on their electromagnetic behaviors. So, making images in order to indicate the presence, position, and geometric and electromagnetic parameters of a defect is needed. Imaging techniques are then involved, those being usually tailored to the expected electric size of the defect (to be appraised vs. the local wavelengths or skin-depths) and, in most cases, the response of the background medium is needed.

A fast algorithm to construct the impedance matrix used in the method of moments is proposed in [4], where the response of layered anisotropic media on a rectilinear mesh to a current dipole source with arbitrary orientation can be efficiently computed. A sophisticated numerical interpolation and integration method is adopted, based on the recently proposed Padua points [5].

Yet, directly using the fast algorithm in the imaging problem may not be so smart. Indeed, if in [4], since the challenge is to construct the impedance matrix, the current source can be set at the origin of the $x-y$ plane (z as vertical coordinate, all slabs' interfaces parallel to $x-y$), and the smoothly-varying spectrum of the response can be interpolated with a small number of Chebyshev polynomials [5]. However, in imaging, the measurement could be a few wavelengths away from the origin. This means a fast oscillating spectrum of the response, and the interpolation and integration method of [5] is not efficient anymore. If the source is shifted away from the origin of the $x-y$ plane, the spectrum is simply the multiplication of the original spectrum for the source at the origin and a sinusoidal term. Using [6], the method of [5] is extended to deal with the oscillating spectrum, yielding an accurate computation of the response. The technique is implemented in multiple signal classification (MUSIC) imaging to locate small defects affecting the anisotropic layering.

2. Methodology

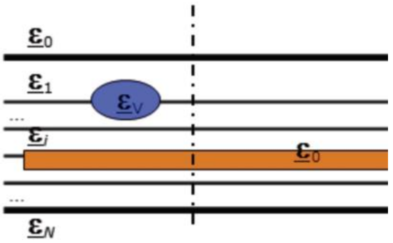

Fig. 1: Planar structure of anisotropic media

From now, one considers the physical scenario as sketched in Fig. 1, where each anisotropic slab is characterized by a permittivity tensor. If one shifts the source from the origin of the $x-y$ plane to a new position (x_s, y_s), the spatial response of the layering is not changed referring to the source, and so it is just laterally shifted by (x_s, y_s) due to the invariance of the system along the $x-y$ plane.

The change in the spectral domain of the response is a phase shift according to the Fourier transform property $\tilde{\eta}(k_x, k_y) = \tilde{\eta}_0(k_x, k_y)\exp(-\mathrm{i}k_x x_s - \mathrm{i}k_y y_s)$, where the spectral response of the layering after shifting the source is $\tilde{\eta}(k_x, k_y) = \mathrm{FT}\{\eta_0(x - x_s, y - y_s)\}$, and $\tilde{\eta}_0(k_x, k_y) = \mathrm{FT}\{\eta_0(x, y)\}$ is its spectral response when the source is located at the origin of the $x-y$ plane, with $\mathrm{FT}\{.\}$ the Fourier transform. The new spectrum can be factorized into two terms: the original spectrum and an oscillating term. In [5], the entire integrand of the integral is interpolated by means of the Chebyshev polynomials and the weights are calculated based on the samplings at Padua points. This facilitates the integration since integrals of the scaled Chebyshev polynomials can be obtained in closed form.

But, once the integrand is fast oscillating, one needs to separate the oscillating term from other smooth functions, such that the interpolation with the smooth functions can still be efficient. To deal with the integration itself, one needs to manage the scaled Chebyshev polynomials as $\int_{-1}^{1}\hat{T}(x)\exp(-\mathrm{i}xa)\mathrm{d}x$. Such integrations can be carried out by using the recipe given in [6], and the response of the background layered medium follows.

3. Implementation into MUSIC

Among the imaging methods developed in the literature, the MUSIC algorithm [8] [9] is a good imaging method to locate small inclusions, i.e., whose dimensions are much smaller than the local wavelength (in the undamaged material), or skin depth if this medium is essentially conductive. With MUSIC, one needs to have at hand the dyadic Green's functions that take from the domain of interest (the one in which the defect is sought) to the domain of measurements (where a multistatic response matrix is to be collected). To obtain such functions, the reciprocity theorem means that one can assume the dipole sources being set within the domain of measurements, and then calculate the responses within the domain of interest. The method proposed above could be implemented in such a MUSIC imaging to generate a reliable result when the spectrum of the response is fast oscillating.

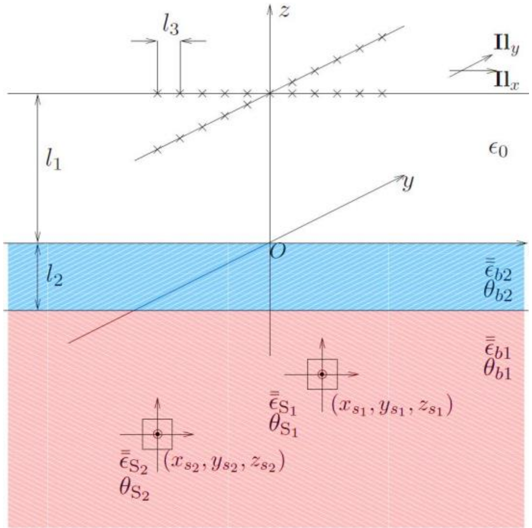

Fig. 2: Sketch of the configuration

4. Numerical results

MUSIC imaging method is employed in a fully anisotropic configuration, as sketched in Fig. 2. Typical results are shown in Fig. 3 [7]. A 5 by 5 cross-type transceivers array with x and y polarizations is employed as measurement set-up. Two small cubic inclusions with side $0.1\lambda_0$ are located in the bottom layer. The MUSIC method, to summarize, yields (in its easiest implementation) a map of the amplitude of the function $\varphi(\bar{r}) = 1/\sum_{\sigma_j<\sigma_L}\sum_{v=1}^{3}\left|\bar{u}_j^* \cdot \bar{G}_v(\bar{r})\right|^2$, summing over the vectorial components v and those singular values σ_j (with corresponding singular vectors u_j) smaller than σ_L associated to an appropriately defined noise subspace ($\bar{G}$ is as usual made of the Green dyad).

$\bar{\bar{\varepsilon}}_{b1} = \mathrm{diag}(4.5 + \mathrm{i}0.2, 6 + \mathrm{i}0.05, 6 + \mathrm{i}0.05)$ $\theta_{b1} = 45°$
$\bar{\bar{\varepsilon}}_{b2} = \mathrm{diag}(2 + \mathrm{i}0.3, 3 + \mathrm{i}0.1, 3 + \mathrm{i}0.1)$ $\theta_{b2} = 60°$
$\bar{\bar{\varepsilon}}_{S1} = \bar{\bar{I}}\varepsilon_0$ $\theta_{S1} = 0°$ $\theta_{S2} = 120°$
$\bar{\bar{\varepsilon}}_{S2} = \mathrm{diag}(4.5 + \mathrm{i}0.2, 6 + \mathrm{i}0.05, 6 + \mathrm{i}0.05)$ $l1 = 0.5\lambda_0$
$(x_{s2}, y_{s2}, z_{s2}) = (-0.2\lambda_0, -0.1\lambda_0, -0.55\lambda_0)$ $l2 = 0.2\lambda_0$
$(x_{s1}, y_{s1}, z_{s1}) = (0.15\lambda_0, 0.2\lambda_0, -0.35\lambda_0)$ $l3 = 0.2\lambda_0$

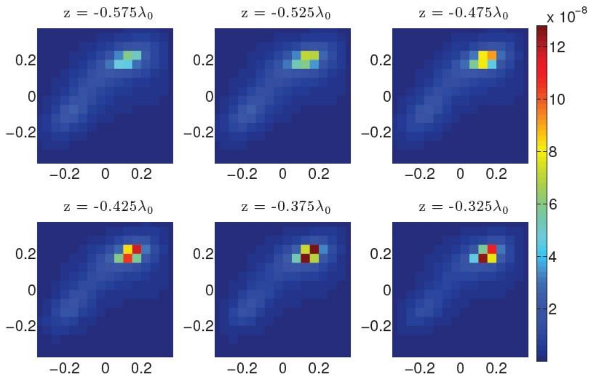

Fig. 3: Typical MUSIC images in horizontal planes at various depths

5. Acknowledgements

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