

Dynamic formation of self-organizing maps

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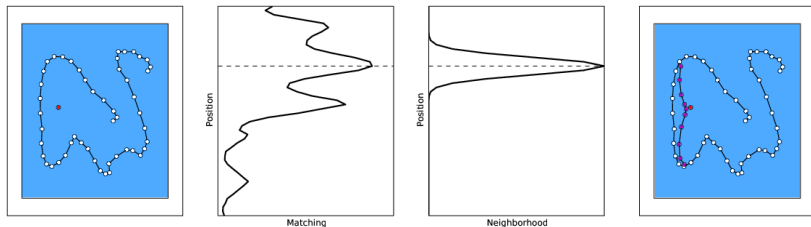
July 2nd 2014



Introduction and motivations

Kohonen Self Organizing maps

Given a sample $\zeta \sim \mathcal{I}$, a distance function $d_{\mathcal{I}}$:



The SOM algorithm :

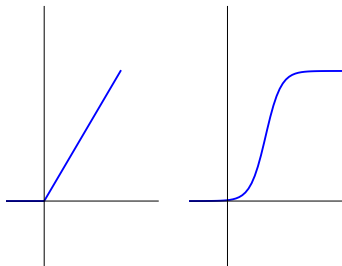
- ▶ is **synchronized** to the input distribution
- ▶ relies on a **global supervisor** for the competition
- ▶ computes a **neighborhood unaware** winner

Dynamic neural fields

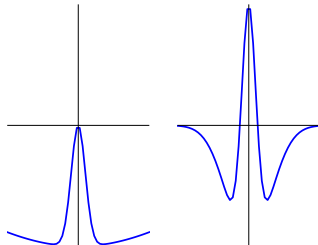
The dynamic neural field equation introduced by Amari (1977) is :

$$\tau \frac{\partial v}{\partial t}(x, t) = -v(x, t) + \int_y w(x, y) f(v(y, t)) + I(x, t)$$

Transfer function $f(x)$



Weights $w(x, y)$

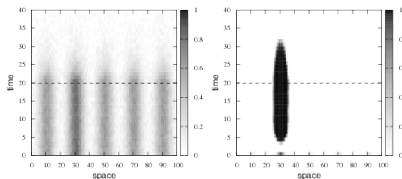


Typical behaviors

Depending on the parameters, dynamic neural fields can have different behaviors, e.g. :

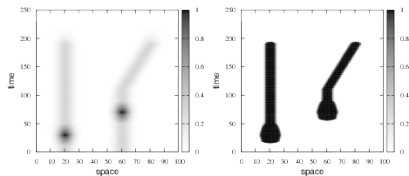
Competition

(local excitation),
global inhibition for competing



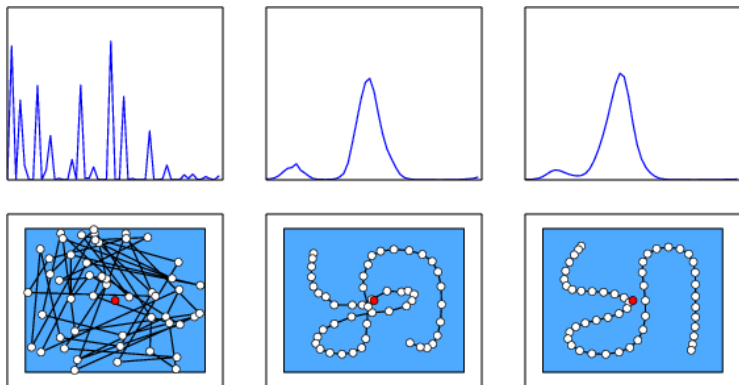
Working memory

local excitation for holding
local inhibition for bounding



The Amari equation is not sufficient for SOMs

Given a sample $\zeta \sim \mathcal{I}$, and a collection of prototypes w_i , the matching $g(d_{\mathcal{I}}(\epsilon, w_i))^1$ has different shapes as the map learns :

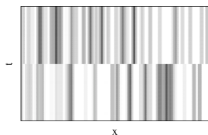


1. with g a gaussian, $d_{\mathcal{I}}$ the distance in the input space

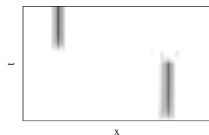
The Amari equation is not sufficient for SOMs

Experimentally, the Amari equation is **not an adequate competition mechanism** in the context of self-organizing maps; the field is whether **not responsive enough** or **blind to input changes**.

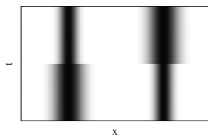
Random Input $I(x, t)$



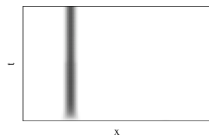
Decision $f(v(x, t))$



Structured Input $I(x, t)$



Decision $f(v(x, t))$



In the literature, it has been suggested to :

- ▶ learn the lat. weights with norm. Hebb rule LISSOM [Mikkulainen(1994)]
- ▶ learn the lat. weights with homeostatic plasticity rules [Moldakarimov(2006)]

A new equation

We introduce an intermediate layer between the input $I(x, t)$ and the decision layer $V(x, t)$:

$$\begin{aligned}\tau \frac{\partial u}{\partial t}(x, t) &= -u(x, t) + \beta \left(\overbrace{\int_y g(x, y) I(y, t)}^{\text{local input}} - \overbrace{\int_y f(v(y, t)) I(y, t)}^{\text{input below the decision}} \right) \\ \tau \frac{\partial v}{\partial t}(x, t) &= -v(x, t) + \int_y w(x, y) f(v(y, t)) + f(u(x, t))\end{aligned}$$

with :

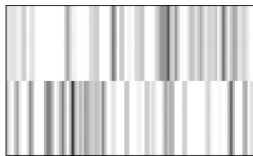
- ▶ g a gaussian, w a mexican hat,
- ▶ 7 parameters $(\tau, \beta, \sigma_g, A_+, \sigma_+, A_-, \sigma_-)$
- ▶ evaluation in $O(N \log(N))$, with the FFT

The lateral connections are not plastic \rightarrow we keep the same plasticity all the time

Qualitative behavior of the new equation

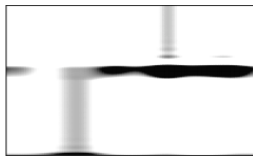
Local **raise of activity** in regions where the input is **locally higher** compared to the **input supporting the current decision**.

Input $I(x, t)$



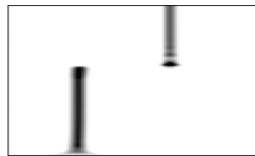
x

Intermediate $f(u(x, t))$



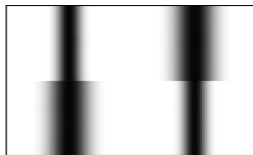
x

Decision $f(v(x, t))$



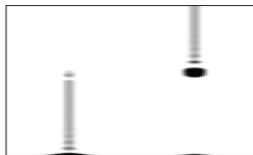
x

Input $I(x, t)$



x

Intermediate $f(u(x, t))$



x

Decision $f(v(x, t))$



x

Experiments

Discrete self-organizing dynamic neural field

Given a sample $\epsilon(t)$, initial prototypes $p(\cdot, 0)$, activities $u(\cdot, 0), v(\cdot, 0)$:

$$\Delta u(x, t) = \frac{\Delta t}{\tau} (-u(x, t) + \beta (\sum_y g(x, y) I(y, t) - \sum_y f(v(y, t)) I(y, t)))$$

$$\Delta v(x, t) = \frac{\Delta t}{\tau} (-v(x, t) + \sum_y w(x, y) f(v(y, t)) + f(u(x, t)))$$

$$I(x, t) = \exp\left(-\frac{|p(x, t) - \epsilon(t)|}{2\sigma_p^2}\right)$$

$$\Delta p(x, t) = \frac{\Delta t}{\tau_p} f(v(x, t)) (\epsilon(t) - p(x, t))$$

Parameters :

N	Δt	τ_p	σ_p	τ	β	σ_g	A_+	σ_+	A_-
50	0.01	10	0.2	0.05	2.6	4.7	1.2	4.6	1.08

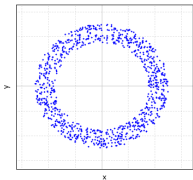
Ring experiment - 1D Neural Field

2D Distribution

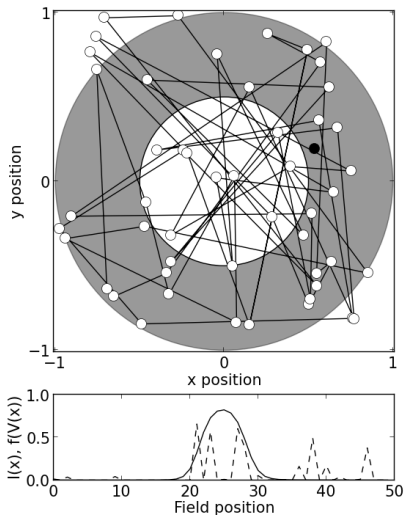
Samples are drawn in a ring

Domain $\mathcal{D} \subset [-1, 1]^2$

Prototypes : random in $[-1, 1]^2$



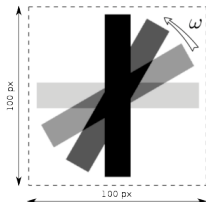
Simulation



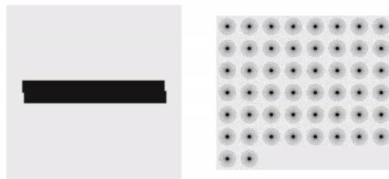
Rotating bar experiment - 1D Neural Field

100 × 100D Distribution

A bar is rotating at $\omega = 10 \text{ deg}\cdot\text{s}^{-1}$
Prototypes : mean of 50 stimuli + noise



Simulation



Without topology : dynamical online K-means

$$\tau \frac{\partial u}{\partial t}(x, t) = -u(x, t) + I(y, t) - \sum_y f(v(y, t))I(y, t)$$

$$\tau \frac{\partial v}{\partial t}(x, t) = -v(x, t) + w_e f(v(x, t)) - w_i \sum_{y \neq x} f(v(y, t)) + f(u(x, t))$$

$$I(x, t) = \exp\left(-\frac{|p(x, t) - \epsilon(t)|}{2\sigma_p^2}\right) + b(x, t)$$

$$\tau_p \frac{\partial p}{\partial t}(x, t) = f(v(x, t))(\epsilon(t) - p(x, t))$$

$$\tau_b \frac{\partial b}{\partial t}(x, t) = b^* - \overline{f(v)}(x, t)$$

$$\tau_b \frac{\partial \overline{f(v)}}{\partial t}(x, t) = -\overline{f(v)}(x, t) + f(v(x, t))$$

We remove all the dependencies on distance in the field ;

We add an adaptive baseline to avoid *dead units*, $b^* = 1/N$

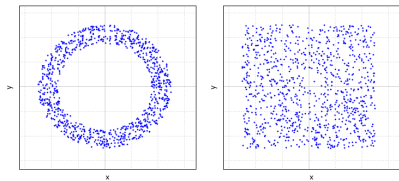
Without topology : dynamical online K-means

2D Distribution

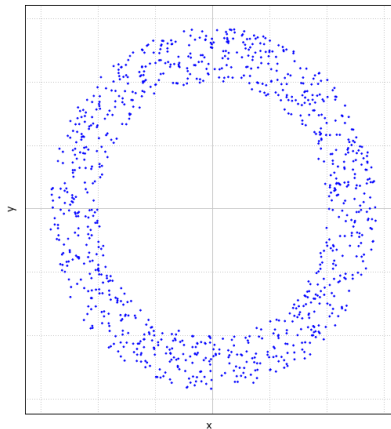
Samples are drawn in a ring or a square.

Domain $\mathcal{D} \subset [-1, 1]^2$

Prototypes : **outside** from \mathcal{D}



Simulation



Summary and perspectives

Summary

In the context of self-organizing maps :

- ▶ a static Amari equation is not sufficient
- ▶ an intermediate layer can be added to destabilize the output decision

Experiments conducted :

- ▶ with 1D populations, ring topology, with 2D or 100^2 prototypes
- ▶ without topology

Introducing an adaptive baseline avoids dead units.

Perspectives

Improving the performances of the self-organizing neural field :

- ▶ when the prototypes are too far from the samples
- ▶ without cyclic boundary conditions, the units on the border have less chance to win

Making use of dynamical systems for self-organizing maps may open the road toward :

- ▶ an easier analytic treatment of self-organizing maps, the **WTM** is a **result** of the interactions of the units,
- ▶ the study of exotic self-organizing structures with **multiple winning locations** by bounding the extent of lateral inhibition,
- ▶ suitable for **recurrent self-organizing maps**, where multiple interconnected maps require a relaxation