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# MODELISATION DES MULTIPLES DEGRADATIONS DEPENDANTES EN CONCURRENCE SOUS INCERTITUDE EPISTEMIQUE PAR PDMP

## MODELING MULTIPLE DEPENDENT COMPETING DEGRADATIONS UNDER EPISTEMIC UNCERTAINTY VIA PDMP

Y.H.LIN, Y.F.LI et E.ZIO.

Chair on Systems Science and the Energetic Challenge, European Foundation for New Energy-Electricite' de France, at EcoleCentrale Paris - Supelec, France  
Grande Voie des Vignes, 92295 CHATENAY-MALABRY  
3 Rue Joliot Curie Plateau de Moulon, 91190 GIF-SUR-YVETTE

### Résumé

Le cadre de modélisation de processus de Markov déterministes par morceaux (PDMP) peut gérer les dépendances entre les modèles basés sur la physique, entre les modèles multi-états et entre ces deux types de modèles. L'incertitude épistémique peut se produire à cause de la connaissance incomplète ou imprécise sur les processus de dégradation et sur les paramètres de contrôle: pour tenir compte de cette incertitude, nous décrivons les paramètres du modèle PDMP comme des nombres flous. Dans cet article, nous étendons la méthode des volumes finis (FV) pour quantifier la fiabilité (floue) du système. La méthode proposée a été testée sur un sous-système du système du refroidissement du réacteur à l'arrêt (RRA) d'une centrale nucléaire, une comparaison avec la solution de la simulation de Monte Carlo (MC) est offerte.

### Summary

Piecewise-deterministic Markov process (PDMP) modeling framework can handle the dependencies between physics-based models, between multi-state models and between these two types of models. Epistemic uncertainty can arise due to the incomplete or imprecise knowledge about the degradation processes and the governing parameters: to take into account this, we describe the parameters of the PDMP model as fuzzy numbers. In this paper, we extend the finite-volume (FV) method to quantify the (fuzzy) reliability of the system. The proposed method is tested on one subsystem of the residual heat removal system (RHRS) of a nuclear power plant, and a comparison is offered with a Monte Carlo (MC) simulation solution.

### Introduction

In real applications, industrial components always undergo degradation processes until they become completely failure. For multi-component systems, the degradation dependency within one component or/and among different components need to be considered under certain circumstances, e.g. the wear of rubbing surfaces can be influenced by the environmental stress shocks within the micro-engine (Lei et al., 2012) and the vibration of the pump due to its degradation can aggravate the degradation of the valve in RRA system (Lin et al.). This leads to a challenging problem to model the degradation processes of such systems.

Piecewise-deterministic Markov process (PDMP) approach was employed in our previous work (Lin et al.) to handle the dependencies between physics-based models, between multi-state models and between these two types of models. To solve this model, Monte Carlo (MC) simulation (Marseguerra and Zio, 1996) was used since the analytical solution is difficult to obtain due to the system complex behaviors e.g. stochasticities of multi-state models and time-dependent evolutions of physics-based models. However, the major shortcoming is that MC can be quite time-consuming.

Epistemic uncertainty can arise due to the incomplete or imprecise knowledge about the degradation processes, for examples the limited amount of available information/data and the human errors attributed to insufficient inspection (Liu et al., 2008). For physics-based models, the parameters (e.g. wear coefficient) and environmental factors (e.g. temperature and pressure) may be unknown or elicited from expert judgment; for multi-state models, the state performances may be fuzzy due to the imprecise discretization of the underlying continuous degradation processes and the transition rates between states may be difficult to estimate statistically due to insufficient data, especially for those highly reliable critical components.

Efficient numerical scheme is needed for the multiple dependent degradations (modeled by PDMP) associated with epistemic uncertainty to avoid the computational burden of MC simulation. In this paper, we extend a finite-volume (FV) method (Cocozza-Thivent et al., 2006) to quantify the (fuzzy) reliability of the system. A comparison is offered with a Monte Carlo (MC) simulation solution to show the efficiency of the proposed method. The rest of the paper is organized as follows. Section 2 introduces the PDMP for multiple dependent competing degradation processes. Section 3 presents the FV scheme for PDMP. Section 4 presents the PDMP under uncertainty and the extended FV scheme for system reliability quantification. Section 5 presents a case study on one subsystem of the residual heat removal system (RHRS) (Coudray and Mattei, 1984) of a nuclear power plant and numerical results and analysis. Section 6 concludes the work.

### PDMP for systems degradation considering dependency

Physics-based models (PBMs) (Daigle and Goebel, 2011) and multi-state models (MSMs) (Lisnianski and Levitin, 2003) are two modeling frameworks that can be used for describing the evolution of degradation in structures and components. The former aims to develop an integrated mechanistic description of the component/system life consistent with the underlying degradation mechanisms (e.g. wear, stress corrosion, shocks, cracking, fatigue, etc.) using physics knowledge and equations, while the latter

can be built upon material science knowledge, or degradation data from historical collection or degradation tests, to model the degradation processes in a discrete way.

The following assumptions are made on the multiple dependent competing degradation processes of a system:

1. The system consists of two groups of components: the first group contains  $M$  components,  $\vec{L} = (L_1, L_2, \dots, L_M)$ , whose degradation processes are modeled by PBMs; the second group contains  $N$  components,  $\vec{K} = (K_1, K_2, \dots, K_N)$ , whose degradation processes are modeled by MSMs.
2. All degradation processes of the system follow the PDMP, taking into account the degradation dependency of components within each group and between the groups.
3. For component  $L_m, m = 1, 2, \dots, M$ ,  $d_{L_m}$  time-dependent continuous variables are used to describe the degradation processes denoted by vector  $\vec{X}_{L_m}(t)$ : their development in time is described by a set of first-order differential equations, i.e. physics equations.
4. For component  $K_n, n = 1, 2, \dots, N$ , its degradation state space is finite, denoted by  $S_{K_n} = \{0, 1, \dots, d_{K_n}\}$ , ranging from perfect functioning state ' $d_{K_n}$ ' to complete failure state '0'. The component is functioning or partially functioning in all generic intermediate states. The transition rates between two different degradation states are used to describe the speed of reaching another degradation state. The performance level of one component (e.g. vibration of the valve due to degradation) at each degradation state and the impact on the other components are considered as deterministic.

The degradation condition of the whole system is, then, represented as follows:

$$\vec{Z}(t) = \begin{pmatrix} \begin{pmatrix} \vec{X}_{L_1}(t) \\ \vec{X}_{L_2}(t) \\ \vdots \\ \vec{X}_{L_M}(t) \end{pmatrix} = \vec{X}(t) \\ (Y_{K_1}(t), Y_{K_2}(t), \dots, Y_{K_N}(t)) = \vec{Y}(t) \end{pmatrix} \in E = \mathbb{R}^{d_L} \times S\{1\}$$

where  $Y_{K_n}(t), n = 1, 2, \dots, N$  denotes the degradation state of component  $K_n$  at time  $t$ ,  $E$  is a hybrid space of  $\mathbb{R}^{d_L}$  ( $d_L = d_{L_1} + d_{L_2} + \dots + d_{L_M}$ ) and  $S$  ( $S = S_{K_1} \times S_{K_2} \times \dots \times S_{K_N}$ ).

The evolution of the degradation processes  $\vec{Z}(t)$  involves the stochastic behavior of  $\vec{Y}(t)$  and the deterministic behavior of  $\vec{X}(t)$ , between two consecutive jumps of  $\vec{Y}(t)$ , given  $\vec{Y}(t)$ . Let  $\vec{Y}_k \in S, k \in \mathbb{N}$  denote the state of the  $N$  components in the second group after  $k$  transitions (a transition occurs as long as any one of the  $N$  components changes its state) and  $T_k \in \mathbb{R}^+, k \in \mathbb{N}$  denote the time of arrival at state  $\vec{Y}_k$ .  $\vec{Y}(t)$  is written as follows:

$$\vec{Y}(t) = \vec{Y}_k, \forall t \in [T_k, T_{k+1}] \{2\}$$

The probability that  $\vec{Y}(t)$  will step to state  $\vec{j}$  from state  $\vec{i}$  in the next infinitesimal time interval  $[T_n, T_n + \Delta t]$ , given  $(\vec{Z}(t))_{0 \leq t \leq T_n}$ , is as follows:

$$P[\vec{Y}_{n+1} = \vec{j}, T_{n+1} \in [T_n, T_n + \Delta t] | (\vec{Z}(t))_{0 \leq t \leq T_n}, \vec{\theta}_k] = P[\vec{Y}_{n+1} = \vec{j}, T_{n+1} \in [T_n, T_n + \Delta t] | \vec{Z}(T_n) = (\vec{X}(T_n), \vec{Y}), \vec{\theta}_k] \\ = \lambda_{\vec{j}}(\vec{j}, \vec{X}(T_n) | \vec{\theta}_k) \Delta t \forall n \geq 0, \vec{i}, \vec{j} \in S, \vec{i} \neq \vec{j} \{3\}$$

where  $\vec{\theta}_k$  represents the external influencing factors of the components in the second group and the related coefficients to the transition rates,  $\lambda_{\vec{j}}(\vec{j}, \vec{X}(T_n) | \vec{\theta}_k)$  represents the corresponding transition rate. The evolution of  $\vec{X}(t)$ , when  $t \in [T_k, T_{k+1}]$ ,  $k \in \mathbb{N}$ , is deterministically described by a set of differential equations as follows:

$$\vec{X}(t) = \begin{pmatrix} \vec{X}_{L_1}(t) \\ \vec{X}_{L_2}(t) \\ \vdots \\ \vec{X}_{L_M}(t) \end{pmatrix} = \begin{pmatrix} f_{L_1}^{\vec{Y}_k}(\vec{X}(t), t | \vec{\theta}_{L_1}) \\ f_{L_2}^{\vec{Y}_k}(\vec{X}(t), t | \vec{\theta}_{L_2}) \\ \vdots \\ f_{L_M}^{\vec{Y}_k}(\vec{X}(t), t | \vec{\theta}_{L_M}) \end{pmatrix} = f_{L_n}^{\vec{Y}_k}(\vec{X}(t), t | \vec{\theta}_{L_n}) \{4\}$$

where  $f_{L_n}^{\vec{Y}_k}, n = 1, 2, \dots, M$  are the set of physics equations, given the influence of the degradation state  $\vec{Y}_k$  of the second group components,  $\vec{\theta}_{L_n}, n = 1, 2, \dots, M$  represents the external influencing factors of the component  $L_n$  and the physical parameters used in the physics equations.

The reliability of the system at time  $t$  is defined as follows:

$$R(t) = P[\vec{Z}(s) \notin \mathcal{F}, \forall s \leq t] \{5\}$$

where  $\mathcal{F} = \mathcal{F}_{\vec{X}} \times \mathcal{F}_{\vec{Y}} \subset E$  denotes the space of the failure states of  $\vec{Z}(t)$ , where  $\mathcal{F}_{\vec{X}}$  denotes the sub-space of the failure states for  $\vec{X}(t)$  and  $\mathcal{F}_{\vec{Y}}$  denotes the sub-space of the failure states for  $\vec{Y}(t)$ . Let  $p_t(\vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K), \vec{x} \in \mathbb{R}^{d_L}, \vec{i} \in S$  denote the probability density function (PDF) of processes  $(\vec{X}(t), \vec{Y}(t))_{t \geq 0}$  being in state  $(\vec{x}, \vec{i})$  at time  $t$ , which satisfies:

$$\int_{\mathbb{R}^{d_L}} \sum_{\vec{i} \in S} p_t(\vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K) d\vec{x} = 1 \{6\}$$

The reliability of the system can be calculated as:

$$R(t) = \int_{\vec{x} \notin \mathcal{F}_{\vec{X}}} \sum_{\vec{i} \notin \mathcal{F}_{\vec{Y}}} p_t(\vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K) d\vec{x} \{7\}$$

The PDF  $p_t(\vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$  obeys the Chapman-Kolmogorov equation (Devooght and Smidts, 1996) as follows:

$$\frac{\partial}{\partial t} p_t(\vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K) = \sum_{\vec{j} \neq \vec{i}} \lambda_{\vec{j}}(\vec{j}, \vec{x} | \vec{\theta}_K) p_t(\vec{x}, \vec{j} | \vec{\theta}_L, \vec{\theta}_K) - \lambda_{\vec{i}}(\vec{x} | \vec{\theta}_K) p_t(\vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K) - \text{div} \left( \vec{f}_{L_n}^{\vec{i}}(\vec{x}, t | \vec{\theta}_{L_n}) p_t(\vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K) \right) \{8\}$$

where  $\lambda_{\vec{i}}(\vec{x} | \vec{\theta}_K) = \sum_{\vec{j} \neq \vec{i}} \lambda_{\vec{j}}(\vec{j}, \vec{x} | \vec{\theta}_K)$  is the transition rate departing from the state  $\vec{i}$ . Among the right-hand parts of equation {8}, the first two terms are due to the stochastic behavior of processes  $\vec{Y}(t)$ : the first term accounts for the transition of processes  $\vec{Z}(t)$  into state  $(\vec{i}, \vec{x})$ , the second term accounts for the transition of processes  $\vec{Z}(t)$  out of state  $(\vec{i}, \vec{x})$ ; the last term is due to the deterministic behavior of processes  $\vec{X}(t)$ , which represents the volume density of the outward flux of the probability field around the point  $(\vec{i}, \vec{x})$ . Given the initial probability distribution of the system  $p_0(\vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$ , its evolution in time and that of the system reliability can be obtained solving equations {8} and {7}, respectively.

A challenging problem is to calculate the probability density function  $p_t(\vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$ , because the analytical solution is difficult to obtain due to the complex behavior of the processes (Labeau, 1996). MC simulation methods can be applied for such numerical computations, but the major shortcoming is that they are typically time-consuming (Eymard and Mercier, 2008). FV methods is an alternative that can lead to comparable results as MC simulation, but within a more acceptable computing time (Eymard and Mercier, 2008).

### **Finite-volume scheme for PDMP**

Instead of directly solving the probability density function  $p_t(\vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$  through the Chapman-Kolmogorov equation {8}, an approximate solution can be obtained by the FV scheme by discretizing the state space of the continuous variables and the time space of PDMP. The approximated solution converges towards the accurate solution under certain conditions. Here, we employ an explicit FV scheme to PDMP, developed by Coccozza-Thivent et al. (Coccozza-Thivent et al., 2006).

#### **3.1 Assumptions**

This approach can be applied under the following assumptions:

1. The transition rates  $\lambda_i(\vec{j}, \cdot | \vec{\theta}_K)$ ,  $\forall \vec{i}, \vec{j} \in S$  are continuous and bounded functions from  $\mathbb{R}^{d_L}$  to  $\mathbb{R}^+$ .
2. The physics equations  $f_L^i(\cdot, \cdot | \vec{\theta}_L)$ ,  $\forall \vec{i} \in S$  are continuous functions from  $\mathbb{R}^{d_L} \times \mathbb{R}^+$  to  $\mathbb{R}^{d_L}$  and locally Lipschitz continuous.
3. The physics equations  $f_L^i(\cdot, \cdot | \vec{\theta}_L)$ ,  $\forall \vec{i} \in S$  are sub-linear, i.e. there are some  $V_1 > 0$  and  $V_2 > 0$  such that
$$\forall \vec{x} \in \mathbb{R}^{d_L}, t \in \mathbb{R}^+ \left| f_L^i(\vec{x}, t | \vec{\theta}_L) \right| \leq V_1(\|\vec{x}\| + |t|) + V_2$$
4. The functions  $\text{div}(f_L^i(\cdot, \cdot | \vec{\theta}_L))$ ,  $\forall \vec{i} \in S$  are almost everywhere bounded in absolute value by some real value  $D > 0$ .

#### **3.2 Numerical scheme**

For the ease of notation, first we let  $\vec{g}^i(\cdot, \cdot): \mathbb{R}^{d_L} \times \mathbb{R} \rightarrow \mathbb{R}^{d_L}$  denote the solution of:

$$\frac{\partial}{\partial t} \vec{g}^i(\vec{x}, t | \vec{\theta}_L) = f_L^i(\vec{g}^i(\vec{x}, t | \vec{\theta}_L), t | \vec{\theta}_L), \forall \vec{i} \in S, \vec{x} \in \mathbb{R}^{d_L}, t \in \mathbb{R} \quad \{9\}$$

with

$$\vec{g}^i(\vec{x}, 0 | \vec{\theta}_L) = \vec{x}, \forall \vec{i} \in S, \vec{x} \in \mathbb{R}^{d_L} \quad \{10\}$$

and  $\vec{g}^i(\vec{x}, t | \vec{\theta}_L)$  is the result of the deterministic behavior of  $\vec{X}(t)$  after time  $t$ , starting from the point  $\vec{x}$  while the processes  $\vec{Y}(t)$  hold on state  $\vec{i}$ .

The state space  $\mathbb{R}^{d_L}$  of continuous variables  $\vec{X}(t)$  is divided into an admissible mesh  $\mathcal{M}$ , which is a family of measurable subsets of  $\mathbb{R}^{d_L}$  ( $\mathcal{M}$  is a partition of  $\mathbb{R}^{d_L}$ ) such that (Coccozza-Thivent et al., 2006):

- (1)  $\bigcup_{A \in \mathcal{M}} A = \mathbb{R}^{d_L}$ .
- (2)  $\forall A, B \in \mathcal{M}, A \neq B \Rightarrow A \cap B = \emptyset$ .
- (3)  $m_A = \int_A d\vec{x} > 0, \forall A \in \mathcal{M}$ , where  $m_A$  is the volume of grid  $A$ .
- (4)  $\sup_{A \in \mathcal{M}} \text{diam}(A) < +\infty$  where  $\text{diam}(A) = \sup_{\vec{x}, \vec{y} \in A} |\vec{x} - \vec{y}|$ .

Additionally, the time space  $\mathbb{R}^+$  is divided into small intervals  $\mathbb{R}^+ = \bigcup_{n=0,1,2,\dots} [n\Delta t, (n+1)\Delta t[$  by setting the time step  $\Delta t > 0$  (the length of each interval).

The numerical scheme aims at giving an approximate value for the probability density function  $p_t(\vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$  on each  $\{i\} \times [n\Delta t, (n+1)\Delta t[ \times A, \forall \vec{i} \in S, n \in \mathbb{N}, A \in \mathcal{M}$  denoted by  $p_n(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$ , by assuming that:

$$p_t(\vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K) = p_n(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K), \forall \vec{i} \in S, \vec{x} \in A, t \in [n\Delta t, (n+1)\Delta t[ \quad \{11\}$$

Given the initial probability density function  $p_0(\vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$  of the system at time  $t = 0$ ,  $p_0(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$ ,  $\forall \vec{i} \in S, A \in \mathcal{M}$  can be obtained as:

$$p_0(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K) = \int_A p_0(\vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K) d\vec{x} / m_A \quad \{12\}$$

Then,  $p_{n+1}(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$ ,  $\forall \vec{i} \in S, A \in \mathcal{M}, n \in \mathbb{N}$  can be calculated considering the deterministic evaluation of  $\vec{X}(t)$  and the stochastic evolution of  $\vec{Y}(t)$  based on  $p_n(\mathcal{M}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$  by the Chapman-Kolmogorov forward equation (Davis, 1993), as follows:

$$p_{n+1}(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K) = \frac{1}{1 + \Delta t b_A^i} \widehat{p}_{n+1}(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K) + \Delta t \sum_{\vec{j} \in S} \frac{a_{ij}^{\vec{i}}}{1 + \Delta t b_A^i} \widehat{p}_{n+1}(A, \vec{j} | \vec{\theta}_L, \vec{\theta}_K) \quad \{13\}$$

where

$$a_{ij}^{\vec{i}} = \int_A \lambda_j(\vec{i}, \vec{x} | \vec{\theta}_K) d\vec{x} / m_A, \forall \vec{i} \in S, A \in \mathcal{M} \quad \{14\}$$

is the average transition rate from state  $\vec{j}$  to state  $\vec{i}$  for grid  $A$ ,

$$b_A^i = \sum_{\vec{j} \neq \vec{i}} a_{ij}^{\vec{i}}, \forall \vec{i} \in S, A \in \mathcal{M} \quad \{15\}$$

is the average transition rate out of state  $\vec{i}$  for grid  $A$ ,

$$\widehat{p}_{n+1}(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K) = \sum_{B \in \mathcal{M}} m_{BA}^i p_n(B, \vec{i} | \vec{\theta}_L, \vec{\theta}_K) / m_A, \forall \vec{i} \in S, A \in \mathcal{M} \quad \{16\}$$

is the approximate value for probability density function on  $\{i\} \times [(n+1)\Delta t, (n+2)\Delta t[ \times A$  according to the deterministic evaluation of  $\vec{X}(t)$ ,

$$m_{BA}^i = \int_{\{\vec{y} \in B | \vec{g}^i(\vec{y}, \Delta t | \vec{\theta}_L) \in A\}} d\vec{y}, \forall \vec{i} \in S, A, B \in \mathcal{M} \quad \{17\}$$

is the volume of the part of grid  $B$ , which will enter grid  $A$  after time  $\Delta t$  according to the deterministic evaluation of  $\vec{X}(t)$ .

The approximated solution  $p_n(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$  weakly converges towards the unique solution of equation {8} when  $\Delta t \rightarrow 0$  and  $|\mathcal{M}|/\Delta t \rightarrow 0$  where  $|\mathcal{M}| = \sup_{A \in \mathcal{M}} \text{diam}(A)$  (Coccozza-Thivent et al., 2006).

### **PDMP under uncertainty**

Fuzzy set theories and techniques introduced by Zadeh(Zadeh, 1999)have been employed in reliability models under epistemic uncertainty when the crisp values are insufficient to capture the actual behavior of components. In this work, the following assumptions are madetoextend the previous PDMP model with the consideration of epistemic uncertainty:

1. The values of the external influencing factors and physical parameters  $\vec{\theta}_L$  in the physics equations  $\vec{f}_L^T(\vec{x}, t | \vec{\theta}_L)$ ,  $\forall \vec{i} \in S, \vec{x} \in \mathbb{R}^{d_L}$  and equations  $g^i(\vec{x}, t | \vec{\theta}_L)$ ,  $\forall \vec{i} \in S, \vec{x} \in \mathbb{R}^{d_L}, t \in \mathbb{R}$ for the deterministic processes  $\vec{X}(t)$  can be fuzzy numbers, denoted by  $\vec{\theta}_L$ .
2. The values of the external influencing factors and the related coefficients  $\vec{\theta}_K$  in the transition rates for the stochastic processes  $\vec{Y}(t)$  between two different states  $\lambda_{ij}(\vec{x} | \vec{\theta}_K)$ ,  $\forall t \in \mathbb{R}^+, \vec{x} \in \mathbb{R}^{d_L}, \vec{i}, \vec{j} \in S, \vec{i} \neq \vec{j}$  can be fuzzy numbers, denoted by  $\vec{\theta}_K$ .

The values of the probability density function  $p(t, \vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$  and reliability function  $R(t)$  have, therefore, changed from crisp values to fuzzy numbers, denoted by  $\tilde{p}(t, \vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$  and  $\tilde{R}(t)$  respectively. In the next section, we extend the approach presented in Section 2 to quantify the dependent degradation processes modeled by PDMP under uncertainty.

#### 4.1 Quantification of PDMP under uncertainty

Let  $[\tilde{a}]_{\alpha} \equiv [a_{\alpha}, \bar{a}_{\alpha}]$  denote the  $\alpha$ -cut of a fuzzy number  $\tilde{a}$ , where  $a_{\alpha}$  and  $\bar{a}_{\alpha}$  are the boundaries; then, the  $\alpha$ -cut of  $\tilde{p}(t, \vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$ ,  $\forall \vec{i} \in S, \vec{x} \in \mathbb{R}^{d_L}, t \in \mathbb{R}$  can be obtained based on the extension principle (Zadeh, 1999) as:

$$[\tilde{p}(t, \vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)]_{\alpha} = [\min_{\substack{\vec{\theta}_L \in [\vec{\theta}_L]_{\alpha} \\ \vec{\theta}_K \in [\vec{\theta}_K]_{\alpha}}} p(t, \vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K), \max_{\substack{\vec{\theta}_L \in [\vec{\theta}_L]_{\alpha} \\ \vec{\theta}_K \in [\vec{\theta}_K]_{\alpha}}} p(t, \vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)] \quad (18)$$

The approximate solution for  $[\tilde{p}(t, \vec{x}, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)]_{\alpha}$ ,  $\forall \vec{i} \in S, \vec{x} \in A, t \in [n\Delta t, (n+1)\Delta t[$  denoted by  $\tilde{p}_n(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$  then can be obtained by varying  $\vec{\theta}_L$  in  $[\vec{\theta}_L]_{\alpha}$  and  $\vec{\theta}_K$  in  $[\vec{\theta}_K]_{\alpha}$  as follows

$$[\tilde{p}_n(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)]_{\alpha} = [\min_{\substack{\vec{\theta}_L \in [\vec{\theta}_L]_{\alpha} \\ \vec{\theta}_K \in [\vec{\theta}_K]_{\alpha}}} p_n(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K), \max_{\substack{\vec{\theta}_L \in [\vec{\theta}_L]_{\alpha} \\ \vec{\theta}_K \in [\vec{\theta}_K]_{\alpha}}} p_n(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)] \quad (19)$$

where  $p_n(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)$  is obtained by equation{13}through FV scheme. The parametric programming algorithms can be applied to find the solutions (Liu et al., 2008).

The approximate solution for the  $\alpha$ -cut of fuzzy reliability  $\tilde{R}(t)$  of the system at time  $t \in [n\Delta t, (n+1)\Delta t[$  can, then, be obtained as follows:

$$[\tilde{R}(t)]_{\alpha} = \sum_{A \in \mathcal{M}} \sum_{\vec{i} \in \mathcal{F}_{\vec{y}}} [\tilde{p}_n(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K)]_{\alpha} \int_{\{\vec{x} \in A | \vec{x} \notin \mathcal{F}_{\vec{x}}\}} \vec{d}\vec{x} \quad (20)$$

In most cases, the original  $R(t)$  is monotonic with  $\vec{\theta}_L$  and  $\vec{\theta}_K$ ; then, we can directly obtain that instead of using equation{19}:

$$[\tilde{R}(t)]_{\alpha} = \left[ \sum_{A \in \mathcal{M}} \sum_{\vec{i} \in \mathcal{F}_{\vec{y}}} p_n(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K) \int_{\{\vec{x} \in A | \vec{x} \notin \mathcal{F}_{\vec{x}}\}} \vec{d}\vec{x}, \sum_{A \in \mathcal{M}} \sum_{\vec{i} \in \mathcal{F}_{\vec{y}}} p_n(A, \vec{i} | \vec{\theta}_L, \vec{\theta}_K) \int_{\{\vec{x} \in A | \vec{x} \notin \mathcal{F}_{\vec{x}}\}} \vec{d}\vec{x} \right] \quad (21)$$

### Illustrative case and results

The illustrative case refers to one important subsystem of a residual heat removal system (RHRS) consisting of a centrifugal pump and a pneumatic valve. The definition of the system has been provided by Électricité de France (EDF). Upon discussion with the experts, a degradation dependency between the two components has been considered, as follows: the degradation of the pump will cause it to vibrate (Zhang et al., 2006) which, in turn, will lead the valve to vibrate and therefore aggravate the degradation processes of the latter (Moussou et al., 2001).

Given its series logic structure, the subsystem is considered failed when one of the two components is failed.

#### 5.1 Centrifugal pump

The degradation model of the centrifugal pump is a modified multi-state model from the one originally supplied by EDF. It is a continuous-time homogeneous Markov chain with constant transition rates as shown in Figure1:

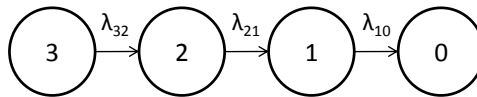
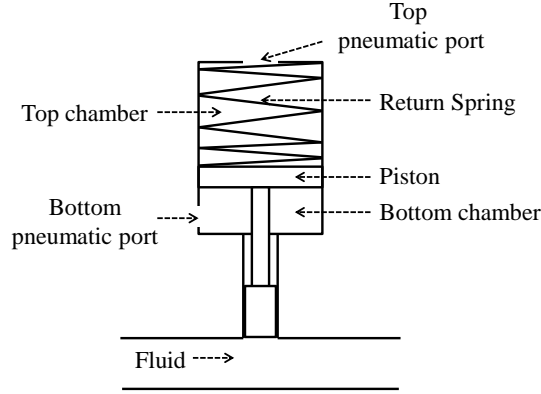


Figure 1. Degradation processes of the pump.

There are four degradation states for the pump to represent its different degradation conditions, from the perfect functioning state '3' to the complete failure state '0'. Due to the degradation, the pump can vibrate when it reaches the degradation states '2' and '1'. The intensity of the vibration of the state '2' is assigned as 'smooth' and that of the state '1' is assigned as 'rough' by the experts. Let  $Y_p(t)$  denote the degradation state of the pump at time  $t$  and  $S_p = \{0, 1, 2, 3\}$  denote the degradation states set. The pump is functioning until it reaches the complete failure state '0';  $\lambda_{32}$ ,  $\lambda_{21}$  and  $\lambda_{10}$  are the transition rates of the degradation process.

#### 5.2 Pneumatic valve

The degradation model of the valve is a physics-based model developed by Daigle and Goebel (Daigle and Goebel, 2011). The simplified scheme of the pneumatic valve is shown in Figure2.



**Figure 2.** Simplified scheme of the pneumatic valve (Daigle and Goebel, 2011).

The pneumatic valve is a normally-closed and gas-actuated valve with a linear cylinder actuator. Top chamber and bottom chamber are separated by the piston, and are connected to a top pneumatic port and a bottom pneumatic port, respectively. The position of the piston between fully closed position '0' and fully open position ' $x_s$ ', can be controlled by regulating the pressure of the pneumatic ports to fill or evacuate the two chambers. A return spring is linked with the piston to ensure that the valve will close when pressure is lost, due to the spring force.

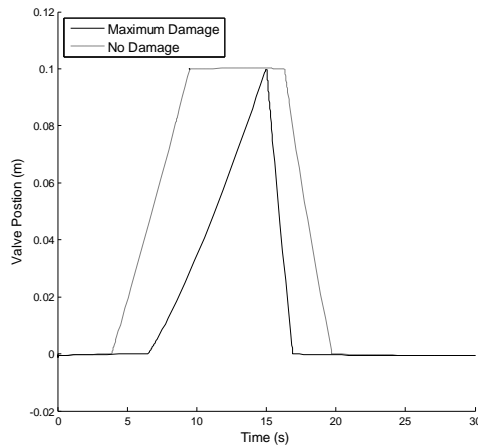
There are several common degradation mechanisms of the valve (e.g. sliding wear, internal leaks, external leaks, etc.). In this case study, as degradation mechanism we have chosen the external leak at the actuator connections to the bottom pneumatic port due to corrosion and other environmental factors, for two reasons: 1) it is more significant than the other degradation mechanisms according to the results shown in (Daigle and Goebel, 2011); 2) the uncertainty associated with the wear coefficient estimated from a limited amount of data should be taken into account. The leak will lead the valve to be more difficult to open but easier to close. The threshold of the area of leak hole  $D_b^*$  is defined as the value above which ( $D_b(t) > D_b^*$ ) the valve cannot reach the fully open position within the 15s time limit from the fully closed position, after an opening command is executed.

Let  $D_b(t)$  denote the area of the leak hole at the bottom pneumatic port at time  $t$ , the development of the leak size is described by:

$$\dot{D}_b(t) = \omega_b(1 + \beta_{y_p(t)}) \quad \{22\}$$

where  $\omega_b$  is the original wear coefficient and where  $\beta_{y_p(t)}$  is the relative increment of the developing rate of the external leak at the bottom pneumatic port caused by the vibration of the pump at the degradation state '2' or '1' (if we ignore the degradation dependency, then  $\beta_{y_p(t)} = 0$ ). All the other physics equations of the valve and the parameter definitions and values can be founded in (Daigle and Goebel, 2010).

The threshold of the area of leak hole  $D_b^* = 1.06e - 5 \text{ m}^2$  (maximum damage) can be calculated: once exceeded, the valve will not reach the fully open position within the 15s limit, as shown in Figure3.



**Figure 3.** Valve behavior with different sizes of the external leak.

### 5.3 PDMP for the system under uncertainty

The degradation processes of the whole system are modeled by PDMP as follows:

$$\vec{Z}(t) = \begin{pmatrix} D_b(t) \\ Y_p(t) \end{pmatrix} \in \mathbb{R}^+ \times S_p \quad \{23\}$$

The space of the failure states of  $\vec{Z}(t)$  is  $\mathcal{F} = \mathcal{F}_{D_b} \times \mathcal{F}_{Y_p} = [D_b^*, +\infty) \times \{0\}$ . We have  $\vec{\theta}_L = (\omega_b, \beta_{y_p(t)})$  and  $\vec{\theta}_K = (\lambda_{32}, \lambda_{21}, \lambda_{10})$  which are the uncertain parameters. They are assumed to be triangle fuzzy numbers (TFNs) represented by a triplet  $(a_1, a_2, a_3)$ . TFNs are widely used to represent uncertain parameters in reliability engineering (Ding et al., 2008), because they are easier to be manipulated and to be elicited from the experts. However, the proposed framework is generally suitable for fuzzy numbers with

other types of membership functions. The values of  $\widetilde{\omega}_b$ ,  $\widetilde{\beta}_{Y_p(t)}$ ,  $\widetilde{\lambda}_{32}$ ,  $\widetilde{\lambda}_{21}$  and  $\widetilde{\lambda}_{10}$  are shown in Table 1. The fuzzy numbers are assigned by considering a relative uncertainty of  $\pm 10\%$  of the original parameters values.

**Table 1.** The values of the fuzzy parameters in PDMP.

| Parameter                  | Value                      |
|----------------------------|----------------------------|
| $\widetilde{\omega}_b$     | (9e-9, 1e-8, 1.1e-8) m2/s  |
| $\widetilde{\beta}_2$      | (9%, 10%, 11%)             |
| $\widetilde{\beta}_1$      | (18%, 20%, 22%)            |
| $\widetilde{\lambda}_{32}$ | (2.7e-3, 3e-3, 3.3e-3) s-1 |
| $\widetilde{\lambda}_{21}$ | (2.7e-3, 3e-3, 3.3e-3) s-1 |
| $\widetilde{\lambda}_{10}$ | (2.7e-3, 3e-3, 3.3e-3) s-1 |

The initial state of the system is assumed as follows:

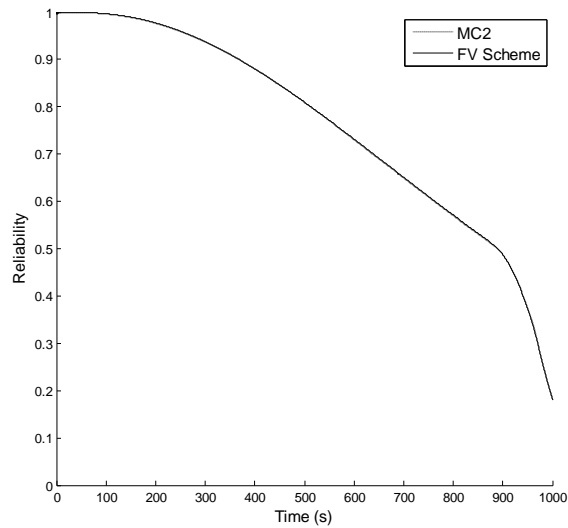
$$\vec{z}_0 = \begin{pmatrix} D_b(0) \\ Y_p(0) \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix} \quad (24)$$

which means that the two components are both in their perfect state. The initial PDF of the processes  $(D_b(t), Y_p(t))_{t \geq 0}$ ,  $p_0(x, i | \widetilde{\theta}_l, \widetilde{\theta}_k)$ , hence equals to 1 if  $(x, i) = (0, '3')$  and to 0 otherwise.

#### 5.4 Results

The comparisons between the results of the reliability of the system at cut level  $\alpha = 1$ , i.e. without fuzziness in the parameters values, over a time horizon 1000s calculated by MC simulation and the FV scheme are shown in Figure4. For the FV scheme, the state space  $\mathbb{R}^+$  of  $D_b(t)$  has been divided into an admissible mesh  $\mathcal{M} = \cup_{n=0,1,2,\dots} [n\Delta x, (n+1)\Delta x[$  where  $\Delta x = 1e-8 \text{ m}^2/\text{s}$  and the time space  $\mathbb{R}^+$  into small intervals  $\mathbb{R}^+ = \cup_{n=0,1,2,\dots} [n\Delta t, (n+1)\Delta t[$  by setting the time step  $\Delta t = 1 \text{ s}$ . All the experiments were carried out in MATLAB on a PC with an Intel Core 2 Duo CPU at 1.97 GHz and a RAM of 1.95 GB. The MC simulation method (Baudrit et al., 2008) with  $10^6$  replications (named MC2), and the proposed FV scheme are applied for the fuzzy reliability assessment of the system. The average computation time of MC2 is 9.40 s, while that of the FV scheme is 0.20 s. The system reliability decreases more rapidly after around 885 s, because at that time the valve could fail, corresponding to the situation when the pump steps to the state '1' very quickly and stays there until the valve fails.

The quantitative comparison of the results over a time horizon 1000 s is shown in Table 2. Compared with the results of MC2, the mean absolute relative difference (MARD) of the results of the FV scheme is 0.17%.



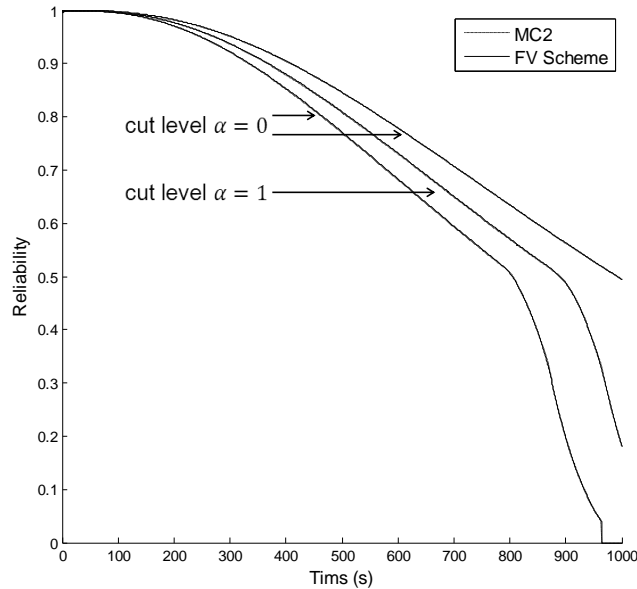
**Figure 4.** Fuzzy reliability at cut level  $\alpha = 1$  (no fuzziness) obtained by MC2 and FV scheme.

**Table 2.** Comparison of the fuzzy reliability of the system at cut level  $\alpha = 1$  (no fuzziness) between MC2 and FV scheme at different times.

| Time | Method | MC2           | FV scheme | Relative difference |
|------|--------|---------------|-----------|---------------------|
| 100s |        | <b>0.9965</b> | 0.9964    | -0.01%              |
| 200s |        | <b>0.9769</b> | 0.9773    | 0.04%               |
| 300s |        | <b>0.9372</b> | 0.9379    | 0.07%               |
| 400s |        | <b>0.8799</b> | 0.8805    | 0.07%               |
| 500s |        | <b>0.8094</b> | 0.8102    | 0.10%               |
| 600s |        | <b>0.7305</b> | 0.7321    | 0.22%               |
| 700s |        | <b>0.6496</b> | 0.6513    | 0.26%               |

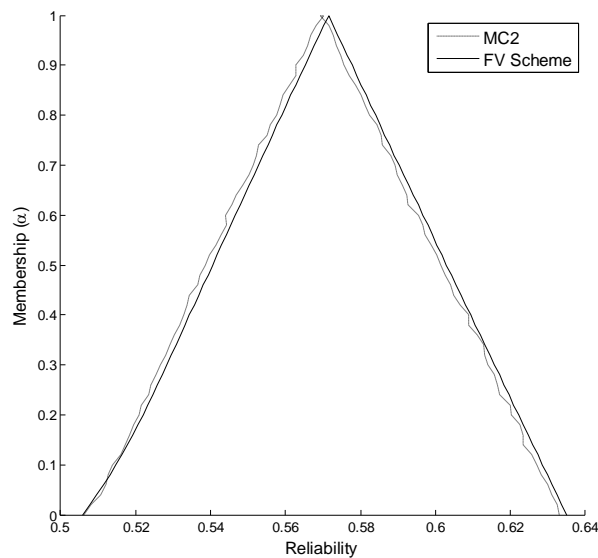
|       |               |        |       |
|-------|---------------|--------|-------|
| 800s  | <b>0.5696</b> | 0.5714 | 0.32% |
| 900s  | <b>0.4873</b> | 0.4874 | 0.02% |
| 1000s | <b>0.1801</b> | 0.1811 | 0.56% |

The results of the fuzzy reliability of the system at cut levels  $\alpha = 0$  and  $\alpha = 1$  over a time horizon 1000 s obtained by MC2 and FV scheme are shown in Figure 5. The lower boundary of the fuzzy reliability of the system at cut level  $\alpha = 0$  decreases more sharply after around 790 s, earlier than the fuzzy reliability at  $\alpha = 1$ . It is seen that the system fails after around 964 s, because at that time the valve is completely failed. The upper boundary of the fuzzy reliability at  $\alpha = 0$  does not experience a rapid decrease because the valve is mostly functioning over the time horizon.



**Figure 5.** Fuzzy reliability at cut levels  $\alpha = 0$  and  $\alpha = 1$  obtained by MC2 and FV scheme.

The membership function of fuzzy reliability  $\tilde{R}(t)$  at mission time  $t = 800$  s at different cut levels  $\alpha \in [0, 1]$  obtained by MC2 and FV scheme are illustrated in Figure 6 (we have uniformly chosen 51 points in  $[0, 1]$  with a step equal to 0.02 assigned to  $\alpha$ ). The average computation time of MC2 is 201.94 s respectively, while that of FV scheme is 15.91 s.



**Figure 6.** Membership function of fuzzy reliability  $\tilde{R}(t)$  at mission time  $t = 800$  s obtained by MC2 and FV scheme.

The quantitative comparison of the results of the membership functions obtained by the MC simulation methods and FV scheme is shown in Table 3. Compared with the results of MC2, the MARD of the results the FV scheme is 0.27%.

**Table 3.** Comparison of the results of the membership function obtained by MC2 and FV scheme.

| Method<br>Cut level | MC2              | FV<br>scheme     | Relative difference<br>(Minimum/Maximum) |
|---------------------|------------------|------------------|--|
| $\alpha = 0$        | [0.5062, 0.6330] | [0.5057, 0.6350] | -0.10% / 0.32%                           |
| $\alpha = 0.1$      | [0.5137, 0.6271] | [0.5148, 0.6285] | 0.21% / 0.22%                            |
| $\alpha = 0.2$      | [0.5209, 0.6203] | [0.5220, 0.6221] | 0.21% / 0.29%                            |
| $\alpha = 0.3$      | [0.5266, 0.6141] | [0.5283, 0.6157] | 0.32% / 0.26%                            |
| $\alpha = 0.4$      | [0.5329, 0.6088] | [0.5344, 0.6093] | 0.28% / 0.08%                            |
| $\alpha = 0.5$      | [0.5386, 0.6015] | [0.5405, 0.6030] | 0.35% / 0.25%                            |
| $\alpha = 0.6$      | [0.5440, 0.5955] | [0.5466, 0.5966] | 0.48% / 0.18%                            |
| $\alpha = 0.7$      | [0.5513, 0.5892] | [0.5528, 0.5903] | 0.27% / 0.19%                            |
| $\alpha = 0.8$      | [0.5577, 0.5825] | [0.5590, 0.5840] | 0.23% / 0.26%                            |
| $\alpha = 0.9$      | [0.5626, 0.5756] | [0.5652, 0.5777] | 0.46% / 0.36%                            |

The above results show that the FV scheme achieves comparable results as MC2, with less computational burden.

### **Conclusion**

In this work, we have considered multiple dependent competing degradation processes in system components modeled by PDMP. Uncertainty is included by describing the model parameters as fuzzy numbers. For the calculation of the system (fuzzy) reliability, the FV method has been extended and shown to lead to comparable results as MC simulation, but with reduced computing time.

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