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# Eddy-current NDE of combustion turbine blade coatings. Determination of conductivity profiles in the presence of a diffusion process

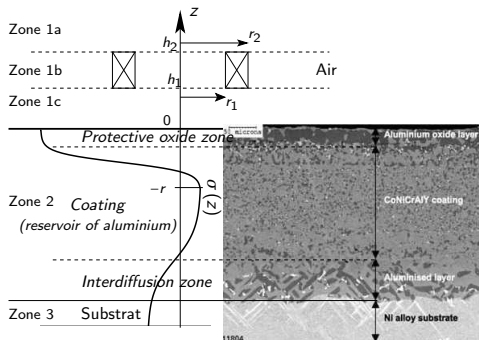
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ENDE 2008

# Context and configuration of the study

- Eddy-Current measurements over combustion turbine blade coatings affected by depletion of aluminium;
- Model taking inward and outward depletion of aluminum inside the coating into account;
- Conductivity profile follows a two-hyperbolic-tangent law;
- Analytical formulation of the variation of impedance obtained combining the approaches found in [1, 2, 3]



# General formulation

- $\mathbf{A}(r) = A(r, z) \mathbf{u}_\theta + A_{ec}(r, z) = R(r)W(z)$  - lead to

$$\frac{\partial^2}{\partial r^2} R(r) + \frac{1}{r} \frac{\partial}{\partial r} R(r) + \left( a^2 - \frac{1}{r^2} \right) R(r) = 0 \quad (1)$$

$$\frac{\partial^2}{\partial z^2} W(z) = \left[ a^2 + j\omega \mu \sigma(z) \right] W(z) \quad (2)$$

- Following [2] the general solution given by

$$W(z) = CF_1(f(z)) + BF_2(f(z)) \quad (3)$$

where  $F_1$  and  $F_2$  typical mathematical functions related to  $\sigma(z)$

- Expression of

$$A_{1c}(r, z) = \int_0^{+\infty} \frac{\mu N I I(r_1, r_2) e^{-az} J_1(ar) (e^{-ah_1} - e^{-ah_2})}{2a^3(h_2 - h_1)(r_2 - r_1)} da + \int_0^{+\infty} C_1 e^{-az} J_1(ar) da, \text{ with } I(r_1, r_2) = \int_{ar_1}^{ar_2} x J_1(x) dx \quad \forall z \in [0; +\infty[$$

$$A_2(r, z) = \int_0^{+\infty} [C_2 F_1(f(z)) + B_2 F_2(f(z))] J_1(ar) da \quad \forall z \in [-r; 0]$$

$$A_3(r, z) = \int_0^{+\infty} B_3 F_3(g(z)) J_1(ar) da \quad \forall z \in ]-\infty; -r]$$

- $A_{1c}(r, z)$  known, then  $Z$  deduced as

$$Z = K \int_0^{+\infty} \frac{I(r_1, r_2)^2}{a^6} \left[ 2 \left( e^{-a(h_2 - h_1)} - 1 + a(h_2 - h_1) \right) + \left( e^{-ah_2} - e^{-ah_1} \right)^2 \phi(a) \right] da \quad (4)$$

$$\text{with } \phi(a) = \frac{C_1}{\mathcal{X}}; \mathcal{X} = \frac{\mu N I I(r_1, r_2) (e^{-ah_1} - e^{-ah_2})}{2a^3 (h_2 - h_1) (r_2 - r_1)}; K = \frac{-j\omega\pi\mu N^2}{(h_2 - h_1)^2 (r_2 - r_1)^2}.$$

- Continuity conditions of the quantities and/or their derivatives with respect  $z$  and/or their cancellation at  $\pm\infty$  express  $\phi(a)$  as

$$\phi(a) = \frac{(aM - O)RT - (aL - N)ST + [a(LQ - MP) - NQ + OP]U}{(aM + O)RT - (aL + N)ST + [a(LQ - MP) + NQ - OP]U} \quad (5)$$

where

$$\begin{aligned} L &= F_1(f(z=0)); & M &= F_2(f(z=0)); & N &= F_1'(f(z))|_{z=0}; \\ O &= F_2'(f(z))|_{z=0}; & P &= F_1(f(z=-r)); & Q &= F_2(f(z=-r)); \\ R &= F_1'(f(z))|_{z=-r}; & S &= F_2'(f(z))|_{z=-r}; & T &= F_3(g(z=-r)); \\ U &= F_3'(g(z))|_{z=-r} \end{aligned} \quad (6)$$

' means derivative with respect to  $z$

# Formulation for a two-tanh profile

- Conductivity profile given by

$$\sigma(z) = \begin{cases} \sigma_{12} + \frac{\sigma_1 - \sigma_{12}}{2} \left[ 1 + \tanh\left(\frac{z + c_1}{2v_1}\right) \right] & \forall z \in [-r, 0] \\ \sigma_2 + \frac{\sigma_{12} - \sigma_2}{2} \left[ 1 + \tanh\left(\frac{z + c_2}{2v_2}\right) \right] & \forall z \in ]-\infty, -r] \end{cases} \quad (7)$$

- Particular functions  $F_1, F_2$  and  $F_3$  are

$$F_1(y_2(z)) = y_2^\mu(z) [1 - y_2(z)]^\nu F(\mu + \nu, \mu + \nu + 1, 2\mu + 1; y_2(z)) \quad (8)$$

$$F_2(y_2(z)) = y_2^{-\mu}(z) [1 - y_2(z)]^\nu F(\nu - \mu + 1, \nu - \mu, -2\mu + 1; y_2(z)) \quad (9)$$

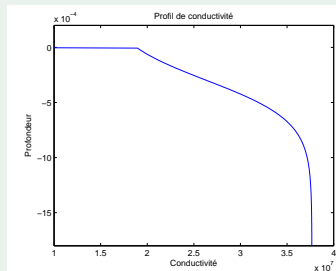
$$F_3(y_3(z)) = y_3^\lambda(z) [1 - y_3(z)]^\tau F(\lambda + \tau, \lambda + \tau + 1, 2\lambda + 1; y_3(z)) \quad (10)$$

$$\begin{aligned} \text{with } y_2(z) &= \left( 1 + e^{-\frac{z+c_1}{v_1}} \right)^{-1}, \quad y_3(z) = \left( 1 + e^{-\frac{z+c_2}{v_2}} \right)^{-1} \\ \mu &= v_1 \sqrt{a^2 + j\omega\mu_0\sigma_{12}}, \quad \nu = v_1 \sqrt{a^2 + j\omega\mu_0\sigma_1} \\ \lambda &= v_2 \sqrt{a^2 + j\omega\mu_0\sigma_2}, \quad \tau = v_2 \sqrt{a^2 + j\omega\mu_0\sigma_{12}} \end{aligned} \quad (11)$$

$F(\alpha, \beta, \gamma; x)$  is the hypergeometric function

## Description of the configuration

$r_1$	1.3 mm
$r_2$	3.3 mm
$h_1$	0.5 mm
$h_2$	7.8 mm
$N_{\text{turn}}$	580
$\sigma_1$	$1.883 \cdot 10^7 \text{ Sm}^{-1}$
$\sigma_{12}$	$3.766 \cdot 10^7 \text{ Sm}^{-1}$
$c_1$	0.3 mm
$v_1$	0.1857 mm

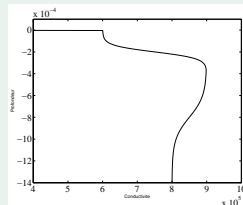


## Comparison with the results given in [1]

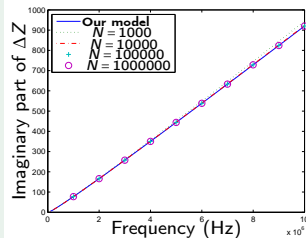
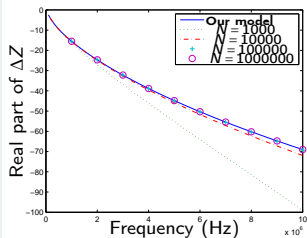
Frequency	Real part of $Z$ , ( $b = NR_2$ )			Imaginary part of $Z$ , ( $b = NR_2$ )		
	from [1]	$N = 10$	$N = 20$	from [1]	$N = 10$	$N = 20$
1 kHz	0.00817	0.008169	0.008165	-0.00828	-0.008267	-0.00828
10 kHz	0.02583	0.02585	2.5823	-0.22571	-0.22557	-0.22566
100 kHz	-0.68836	-0.68799	-0.6882	-1.49719	-1.49645	-1.496769

## Description of the configuration




$r_1 = 2.0 \text{ mm}$	$\sigma_1 = 6 \cdot 10^5 \text{ S m}^{-1}$
$r_2 = 4.0 \text{ mm}$	$\sigma_{12} = 9 \cdot 10^5 \text{ S m}^{-1}$
$h_1 = 0.5 \text{ mm}$	$\sigma_2 = 8 \cdot 10^5 \text{ S m}^{-1}$
$h_2 = 7.3 \text{ mm}$	$N_{\text{turn}} = 200$
$c_1 = 0.2 \text{ mm}$	$v_1 = 0.03 \text{ mm}$
$c_2 = 0.8 \text{ mm}$	$v_2 = 0.1 \text{ mm}$



## Comparison with the results obtained from a multi-layer model





-  E. Uzal and J. Rose, The impedance of eddy current probes above layered metals whose conductivity and permeability vary continuously, *IEEE Trans. Magn.* **29**, (1993), 1869–1873.
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-  T. Theodoulidis and E. Kriezis, Series expansions in eddy current nondestructive evaluation models, *J. Mater. Process. Technol.* **161**, (2005), 343–347.