



**HAL**  
open science

# A Unified Explanation of gm /ID -Based Noise Analysis

Jack Ou, Pietro Maris Ferreira

► **To cite this version:**

Jack Ou, Pietro Maris Ferreira. A Unified Explanation of gm /ID -Based Noise Analysis. Journal of Circuits, Systems, and Computers, 2015, 24 (1), pp.1550010. 10.1142/S0218126615500103 . hal-01222077

**HAL Id: hal-01222077**

**<https://centralesupelec.hal.science/hal-01222077v1>**

Submitted on 5 Oct 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Journal of Circuits, Systems, and Computers

## A Unified Explanation of gm/ID Based Noise Analysis

--Manuscript Draft--

<b>Manuscript Number:</b>	
<b>Full Title:</b>	A Unified Explanation of gm/ID Based Noise Analysis
<b>Article Type:</b>	Letter
<b>Section/Category:</b>	Emre Salman (America, Integrated circuits and VLSI systems, Electronic design automation)
<b>Keywords:</b>	noise analysis; gm/ID; design methodology
<b>Corresponding Author:</b>	Jack Ou, Ph.D. Sonoma State University UNITED STATES
<b>Corresponding Author Secondary Information:</b>	
<b>Corresponding Author's Institution:</b>	Sonoma State University
<b>Corresponding Author's Secondary Institution:</b>	
<b>First Author:</b>	Jack Ou, Ph.D.
<b>First Author Secondary Information:</b>	
<b>Order of Authors:</b>	Jack Ou, Ph.D. Pietro M. Ferreira, Ph.D.
<b>Order of Authors Secondary Information:</b>	
<b>Abstract:</b>	<p>There are currently two competing transconductance-to-drain current based noise analysis techniques in the literature: one based on the thermal coefficient and device corner frequency and another based on the normalized noise power.</p> <p>This letter presents a unified explanation of transconductance-to-drain current based noise analysis. Analytical expressions are presented to clearly show the dependence of thermal noise coefficient and device corner frequency on the transconductance-to-drain current ratio of a transistor, as well as their relationship to the normalized noise power technique. Finally, this letter highlights a potential advantage of the thermal noise coefficient and device corner frequency technique in circuit noise optimization.</p>
<b>Suggested Reviewers:</b>	<p>Fernando Silveira, Ph.D. Associate Professor, Universidad de la República silveira@fing.edu.uy Dr. Silveira is an expert in the field of gm/ID based design methodology.</p> <p>Jui-Chu Lee, Ph.D. Staff Engineer, IBM juichu@us.ibm.com Dr. Lee is a well-regarded expert in device noise.</p>

Journal of Circuits, Systems, and Computers  
© World Scientific Publishing Company

## A Unified Explanation of $g_m/I_D$ Based Noise Analysis

Jack Ou

*Engineering Science, Sonoma State University, 1801 E. Cotati Ave,  
Rohnert Park, California 94928, United States  
jack.ou@sonoma.edu*

Pietro M. Ferreira

*DHS Department, IEMN UMR CNRS  
Universite de Lille  
Lille, France  
maris@ieee.org*

Received (Day Month Year)  
Revised (Day Month Year)  
Accepted (Day Month Year)

There are currently two competing transconductance-to-drain current ( $g_m/I_D$ ) based noise analysis techniques in the literature: one based on the thermal coefficient ( $\gamma$ ) and device corner frequency ( $f_{co}$ ) and another based on the normalized noise power. This letter presents a unified explanation of transconductance-to-drain current ( $g_m/I_D$ ) based noise analysis. Analytical expressions are presented to clearly show the dependence of thermal noise coefficient ( $\gamma$ ) and device corner frequency ( $f_{co}$ ) on the  $g_m/I_D$  of a transistor, as well as their relationship to the normalized noise power technique. Finally, this letter highlights a potential advantage of the thermal noise coefficient ( $\gamma$ ) and device corner frequency ( $f_{co}$ ) technique in circuit noise optimization.

*Keywords:* noise analysis;  $g_m/I_D$ ; design methodology.

### 1. Introduction

Nanoscale Metal Oxide Field Effect Transistor (MOSFET) circuit design is driven by power consumption constraints. The minimal power consumption is achieved when transistors are operated in the weak inversion region.<sup>1</sup> In the absence of a model suitable for back of envelope calculations, designers often explore design space using arduous circuit simulations. Over-reliance on circuit simulator can be problematic, potentially luring inexperienced designers to dive into simulation without understanding basic trade-offs in a properly optimized circuits.

In 1996, Silveira *et al.* proposed a powerful transconductance-to-drain current ( $g_m/I_D$ ) technique to help designers size up transistors quickly.<sup>1</sup> The so called “ $g_m/I_D$  design approach” was originally developed to calculate parameters such as small signal gain and bandwidth,<sup>1</sup> and later extended to noise.<sup>2</sup> In the formulation that was published by Ou<sup>2</sup> in 2011, bias dependent thermal noise coefficient ( $\gamma$ ) and

device noise corner frequency ( $f_{co}$ ) were introduced. A procedure for determining  $\gamma$  and  $f_{co}$  from simulation was described. In 2012, Alvarez *et al.* published an alternative formulation of  $g_m/I_D$  based noise analysis which used normalized noise power.<sup>3,4</sup> The normalized power formulation was first confirmed using simple noise equations,<sup>3</sup> and later with BSIM noise equations.<sup>4</sup>

*What remains unknown at the present* are: first, a unified explanation of two seemingly unrelated  $g_m/I_D$  noise analysis techniques presented by Ou<sup>2</sup> and Alvarez *et al.*,<sup>3</sup> second, a physical explanation that clearly shows the dependence of  $\gamma$  and  $f_{co}$  on  $g_m/I_D$ ,<sup>2</sup> and third, circumstances under which each technique can be used.

In this letter, starting from basic principles in device physics, we provide a unified explanation of  $g_m/I_D$  noise analysis. We provide analytical expressions to clearly show the  $g_m/I_D$  dependence of  $f_{co}$  and  $\gamma$ , and in doing so addressing the concern raised by Alvarez *et al.*<sup>3</sup> Finally, we provide insights on the application of each noise analysis technique.

## 2. Theory

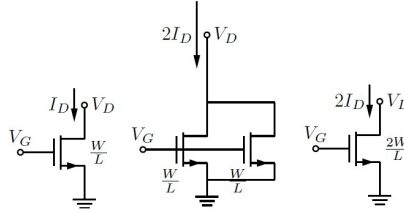
The  $g_m/I_D$  principle is applicable to parameters which are independent of the width ( $W$ ) of a transistor. Figure 1 shows a transistor with a transconductance ( $g_m$ ), a drain-to-source conductance  $g_{ds}$ , and a current ( $I_D$ ) biased at a gate-to-source voltage ( $V_{GS}$ ) and a drain-to-source ( $V_{DS}$ ). If an identical device is connected in parallel so both devices are biased at the same  $V_{GS}$  and  $V_{DS}$ , both devices have the same  $g_m$ ,  $g_{ds}$  and the same  $I_D$ . Since the devices are connected in parallel, they can be treated as one device with an aspect ratio of  $2W/L$ . The effective transconductance over current ratio is  $g_m/I_D$  for both the merged device and the stand alone device because  $g_m$  and  $I_D$  are doubled. The drain-to-source conductance is also doubled for the merged device. Therefore the intrinsic gain ( $g_m/g_{ds}$ ) is identical for both the stand alone device and the merged device. As long as transistors are biased at the same  $g_m/I_D$ , they will have the same  $g_m/g_{ds}$ . This observation is true for any two parameters whose ratio depend solely on the  $g_m/I_D$  and not on the width of a transistor. Once a transistor of a given  $W$  is characterized over a range of  $g_m/I_D$ , the  $g_m/I_D$  based parameters can be generalized to a transistor of an arbitrary  $W$ , *assuming that  $L$  remains constant*. The  $g_m/I_D$  methodology will hold as long as the parameter of interest scales with  $W$ .

## 3. $g_m/I_D$ Dependent Thermal Noise

We present in this section a unified explanation of  $g_m/I_D$  based thermal noise analysis techniques presented by Ou<sup>2</sup> and Alvarez *et al.*<sup>3</sup> Starting from basic device noise equation, we show the dependence of  $\gamma$  on  $g_m/I_D$ . We conclude the section by showing the link between the method proposed by Ou<sup>2</sup> and Alvarez *et al.*<sup>3</sup>

The MOSFET noise arises from thermal noise fluctuations in the channel. It can be shown that the thermal noise at the drain terminal is,<sup>5</sup>

$$\overline{i_{n,id}^2} = 4kT\gamma g_{do}, \quad (1)$$


 Fig. 1. Transistors biased at the same  $g_m/I_D$ .

where  $k$  is the Boltzmann constant,  $T$  is the temperature,  $g_{do}$  is the drain conductance at zero drain-to-source voltage, and  $\gamma$  is the bias-dependent noise parameter. According to this model, the  $\gamma$  approaches 1 when the drain-to-source voltage ( $V_{DS}$ ) approaches zero and decreases to  $2/3$  when the device enters the saturation regime. The saturation value of  $\gamma = 2/3$  is valid for long-channel MOS devices built on lightly doped substrates. Early studies have reported  $\gamma$  values between  $2/3$  and 4,<sup>6</sup> but recent studies have shown that by accounting for parasitic resistances,  $\gamma$  is approximately  $2/3$  for channel lengths equal or greater than 100 nm.<sup>7</sup>

At a more fundamental level, the thermal noise ( $\overline{i_{n,id}^2}$ ) at the drain terminal of a MOS transistor is

$$\overline{i_{n,id}^2} = 4kT \frac{\mu}{L^2} (-Q_I), \quad (2)$$

where  $Q_I$  is the total inversion layer charge underneath the gate oxide,  $L$  is the length of the transistor, and  $\mu$  is the mobility. Equation 2 is valid for all regions of operation<sup>5</sup>. The total inversion layer charge ( $Q_I$ ) is obtained by integrating the inversion charge per unit length ( $Q'_I(x)$ ) over the length of the channel,

$$Q_I = \int_0^L Q'_I(x) W dx, \quad (3)$$

where  $Q'_I(x)$  is a function of  $V_{GS}$  and  $V_{DS}$ , as well as  $V_{SB}$ , and consequently a function of transistor's  $g_m/I_D$ . Since  $Q_I$  is proportional to  $W$  (see Eq. (3)),  $\overline{i_{n,id}^2}$  is proportional to  $W$  according Eq. (2). Since  $g_m$  is proportional to  $W$ ,  $\overline{i_{n,id}^2}/g_m$  becomes independent of  $W$ . Equation (2) also shows that  $\overline{i_{n,id}^2}$  is inversely proportional to  $L^2$ . Even though the transconductance of a transistor also depends on the  $L$ , it is not inversely proportional to  $L^2$ . Therefore,  $\overline{i_{n,id}^2}/g_m$  remains a function of  $L$ . Once  $\overline{i_{n,id}^2}/g_m$  of a transistor for a given  $(W/L)$  is characterized over a range of  $g_m/I_D$ ;  $\overline{i_{n,id}^2}/g_m$  of the transistor can be generalized to a transistor of arbitrary  $W$  as long as  $L$  and the  $g_m/I_D$  are constant. This width-independent property is the crucial link to the  $g_m/I_D$  design methodology described earlier.

The  $\overline{i_{n,id}^2}/g_m$  is directly related to  $\gamma$  of a transistor since the thermal noise of an MOS transistor is modeled by a current source ( $\overline{i_{n,id}^2}$ ) connected between the drain

4 *J. Ou and P. M. Ferreira*

terminal and the source terminal with a spectral density of

$$\overline{i_{n,id}^2} = 4kT\gamma g_m, \quad (4)$$

where  $k$  is the Boltzmann constant, and  $T$  is the temperature.

If we divide  $\overline{i_{n,id}^2}$  by  $4kTg_m$ , we obtain a closed expression for  $\gamma$ .

$$\frac{\overline{i_{n,id}^2}}{g_m} = 4kT\gamma, \quad (5)$$

The right side of the Eq.(5) represents the  $g_m/I_D$  dependent  $\gamma$  used in the noise analysis technique proposed by Ou.<sup>2</sup> If we divide  $\overline{i_{n,id}^2}$  by  $I_D$ , instead of  $g_m$ , we have the normalized noise power formulation proposed by Alvarez et al.<sup>3</sup> Similar to  $\overline{i_{n,id}^2}/g_m$ ,  $\overline{i_{n,id}^2}/I_D$  is also width-independent since both  $g_m$  and  $I_D$  are proportional to  $W$ .

Figure 2 illustrates  $\gamma$  as a function of  $g_m/I_D$  for an NMOS transistor with a  $V_{DS} = 0.6V, V_{SB} = 0.2V$ , a bias current of  $10 \mu A$ , and a length of  $0.18 \mu m$ .<sup>2</sup> The  $\overline{i_{n,id}^2}$  is estimated by noise simulation in Cadence Spectre with transistors modeled using BSIM 4.4. The unified physical model is used to model the flicker noise (`fnoimod=1`) and the charge based model is used to model thermal noise (`toimod=0`).  $g_m$  is determined by DC simulation. No curve fitting technique is used in our analysis (see Fig. 2).

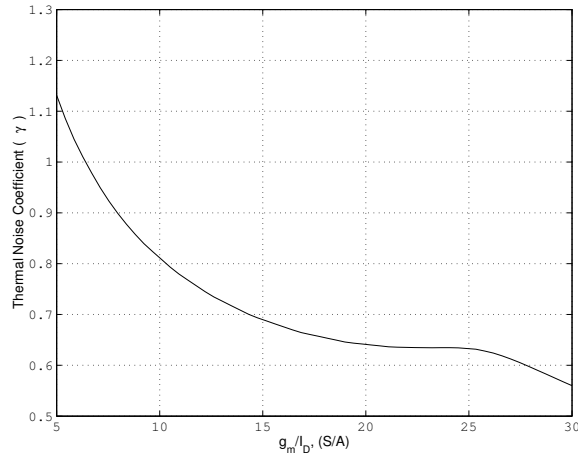


Fig. 2. Simulated thermal noise coefficient ( $\gamma$ ) of NMOS transistor with a  $V_{DS} = 0.6V, V_{SB} = 0.2V$ , a bias current of  $10 \mu A$ , and a length of  $0.18 \mu m$ .

#### 4. Flicker Noise

Since MOSFETs are surface-conduction devices, flicker noise is important at low frequencies. The flicker noise at the drain terminal of a transistor is

$$\overline{i_{nd,fn}^2} = \frac{K_f}{C_{ox}WL} \frac{g_m^2}{f}, \quad (6)$$

where  $K_f$  is a process dependent parameter,  $C_{ox}$  is the oxide capacitance,  $W$  is the width and  $L$  is the length. If we divide both sides of Eqn.(6) by  $g_m$ , we have

$$\frac{\overline{i_{nd,fn}^2}}{g_m} = \frac{K_f}{C_{ox}WL} \frac{g_m}{f}, \quad (7)$$

We see that  $\overline{i_{nd,fn}^2}/g_m$  is also width-independent since  $g_m$  is proportional to  $W$ . Both Eqn. (7) and Eqn. (5) are instances of normalized noise power proposed by Alvarez *et al.*<sup>3</sup>

Rather than introducing an all inclusive  $g_m/I_D$  dependent normalized noise parameter as was done by Alvarez *et al.*,<sup>3</sup> the  $g_m/I_D$  dependent flicker noise was separated from the  $g_m/I_D$  dependent thermal noise in order to identify transistor specific noise bottleneck at the circuit level.<sup>2</sup> Device noise corner frequency was chosen to represent the flicker noise. At the device noise corner frequency ( $f_{co}$ ), the thermal noise is equal to the flicker noise (i.e.  $\overline{i_{nd,th}^2} = \overline{i_{nd,fn}^2}$ ). Setting Eq.(4) equal to Eq.(6), we have

$$4kT\gamma g_m = \frac{K_f}{C_{ox}WL} \frac{g_m^2}{f_{co}} \quad (8)$$

Using Eqn. (8),  $f_{co}$  can be solved in order to explicitly show the dependence on  $g_m/I_D$  and current density ( $I_D/W$ ), which also depends on  $g_m/I_D$ .

$$f_{co} = \frac{K_f}{C_{ox}L} \frac{g_m}{I_D} \frac{I_D}{W} \frac{1}{4KT\gamma} \quad (9)$$

We wish to point that  $K_f$  is a geometry dependent coefficient.<sup>8</sup> At  $f = f_{co}$ , the Eq.(6) becomes

$$\overline{i_{nd,fn}^2} = \frac{K_f}{C_{ox}WL} \frac{g_m^2}{f_{co}} = \overline{i_{nd,th}^2}, \quad (10)$$

Multiplying Eq.(10) by  $f_{co}$ , we get,

$$\overline{i_{nd,th}^2} f_{co} = \frac{K_f}{C_{ox}WL} g_m^2. \quad (11)$$

Using Eq.(11) and Eq.(6), we have a simple equation that relates thermal noise current to flicker noise current,

$$\overline{i_{nd,fn}^2} = \overline{i_{nd,th}^2} \frac{f_{co}}{f}. \quad (12)$$

We showed in that similar to  $\gamma$ ,  $f_{co}$  is also a function of  $g_m/I_D$ , as well as  $V_{SB}$ , and  $L$ .<sup>2</sup> Figure 3 shows the dependence of  $f_{co}$  as a function of length at  $V_{DS} = 0.2V$  and  $V_{SB} = 0.2V$ .

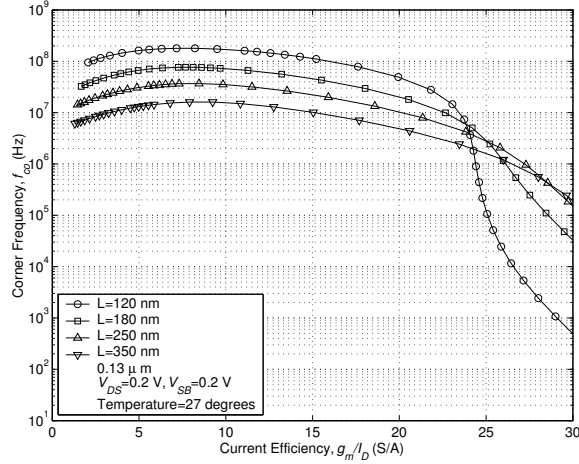


Fig. 3. Corner frequency as a function of device length and current efficiency.

The total noise at the output of the drain terminal of an MOS transistor includes both the thermal noise as well as flicker noise.

$$\overline{i_{nd}^2} = \overline{i_{nd,th}^2} + \overline{i_{nd,fn}^2} = \overline{i_{nd,th}^2} \left(1 + \frac{f_{co}}{f}\right). \quad (13)$$

We can gate-referred the drain noise current  $\overline{i_{nd}^2}$  by dividing  $\overline{i_{nd}^2}$  by  $g_m^2$ .

$$\overline{v_{ng}^2} = \frac{\overline{i_{nd,th}^2}}{g_m^2} = \frac{4kT\gamma}{g_m I_D} \left(1 + \frac{f_{co}}{f}\right). \quad (14)$$

Equation 14 shows that once  $I_D$  is fixed, the gate-referred noise  $\overline{v_{ng}^2}$  depend on  $\gamma$  and  $f_{co}$ , both of which are also dependent on  $g_m/I_D$ . It was shown in <sup>2</sup> that the input-referred noise can be contributed to the gate-referred noise of each transistor, and therefore the  $\gamma$  and  $f_{co}$  of each transistor.

## 5. Applications

Both noise analysis techniques proposed by Ou<sup>2</sup> and by Alvarez *et al.*<sup>3</sup> can be applied to analyze noise (e.g. the folded-cascode amplifier<sup>2</sup> and the charge amplifier<sup>4</sup>). There are, however, two **additional** benefits of using  $\gamma$  and  $f_{co}$  approach in an analysis. *First*, by choosing the  $g_m/I_D$  of each transistor carefully, noise at the circuit level can be optimized by tracing the transistor-noise constraint at the circuit level (e.g. input-referred noise) to  $\gamma$  and  $f_{co}$  of each transistor in the schematic. *This unique ability to express noise as a fundamental quantity such as  $f_{co}$  facilitates noise analysis well as enhance designers' understanding of the fundamental noise limitation in a circuit.* *Second*, different from the normalized power approach, which expresses the sum of thermal and flicker noise as one parameter, *the  $\gamma$  and*



$f_{co}$  approach keeps flicker noise separate from thermal noise, and thereby enabling thermal-noise/flicker-noise specific sizing optimization in circuits where one noise is more dominant than the other. For example, in biomedical applications,  $f_{co}$  is often larger than  $f$ , the first term in the parenthesis of Eqn. (14) can be neglected in the analysis in order to perform a flicker noise specific sizing optimization.

## 6. Conclusion

Starting from basic principles in device physics, this letter provides a unified explanation of  $g_m/I_D$  based noise analysis. Normalized noise power analysis was shown to be equivalent to the noise analysis based on  $\gamma$  and  $f_{co}$ . The analytical expressions in this letter clearly show the dependence of  $\gamma$  and  $f_{co}$  on the  $g_m/I_D$  of a transistor. The use of two noise parameters ( $\gamma$  and  $f_{co}$ ) allows designers to trace the transistor-noise constraint at the circuit level to the noise parameters of each transistor and perform thermal noise/flicker noise specific sizing optimization.

## References

1. F. Silveira, D. Flandre, and P. G. A. Jespers, A  $g_m/I_D$  based methodology for the design of CMOS analog circuits and its application to the synthesis of a Silicon-on-Insulator micropower OTA, *IEEE Journal of Solid-State Circuits* **31** (1996) 1314–1319.
2. J. Ou,  $g_m/I_D$  based noise analysis for CMOS analog circuits, *IEEE 54th International Midwest Symposium on Circuits and Systems*, Seoul, Korea, August 2011, pp. 1–4.
3. E. Alvarez and A. Abusleme, Noise power normalisation: extension of  $g_m/I_D$  technique for noise analysis, *Electronic Letters* **48** (2012) 430–432.
4. E. Alvarez, D. Avila, H. Campillo, A. Dragone, and A. Abusleme, Noise in charge amplifier—a  $g_m/I_D$  approach, *IEEE Trans. On Nuclear Science* **59** (2012).
5. Y. Tsidividis, *Operational and modeling of the MOS transistor* (McGraw-Hill, New York, 1999).
6. S. Dronavalli and R. P. Jindal, CMOS device noise considerations for terabit lightwave systems, *IEEE Trans. Electron Devices*, **53** (2006) 623–630.
7. L. F. Tiemeijer, R. J. Havens, R. D. Kort, and A. J. Scholten, Improved y-factor method for wide-band on-wafer noise-parameter measurements, *IEEE Trans. Microwave Theory and Techniques*, **53** (2005) 2917–2925.
8. P. K. Chan, L. S. Ng, L. Siek, and K. T. Lau, Designing CMOS folded-cascode operational amplifier with flicker noise minimisation, *Microelectronics Journal*, **32** 69–73.