Time domain modeling of soft faults in wiring system by a nodal Discontinuous Galerkin Method with high-order hexahedral meshes

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A time domain nodal discontinuous Galerkin method is used to solve Maxwell equations and simulate reflectometry responses of soft faults. In this paper shielding defects of coaxial cables or other shielded lines are considered. Hexahedral high order elements are used for meshing. They allow to avoid bulky meshes compared to tetrahedral elements. A gaussian pulse is injected on the faulty line. The reflectogram of the line containing the chafing soft defect is obtained and parameters such as the reflection coefficient or the characteristic impedance of the fault are computed. These numerical values are compared to those obtained in experimental investigations. The experimental impedances estimated using a classical transmission matrix method are in very good agreement with those obtained by three-dimensional modeling.

***Index Terms*—Maxwell's equations, Discontinuous Galerkin Method, Hexahedral meshes, Wire fault modeling, Reflectometry diagnosis.**

I. INTRODUCTION

DETECTING and repairing partial and small faults in electrical wires before they become severe is a crucial issue in aerospace and automotive industries [1]. The analysis of electrical wires are performed by reflectometry [2]. This method consists in determining the characteristics of the wires from the measurement of the reflection pattern generated by high frequency electrical signal at each impedance discontinuities. The hard faults have been well studied and are efficiently characterized by traditional reflectometry techniques. Soft faults have been far less studied. They are defined as very small, not necessary localized, change of the wire characteristic impedance. The generated reflections are very small and hard to detect. Suitable methods for the recognition of soft fault signatures must be developed in order to characterize their type and prevently repair the defective harness before facing potentially severe safety issues. As a matter of fact, a soft fault such as a chafed wire may either cause dangerous arcing problems or evolve toward a hard fault causing partial or total malfunctioning of the system.

Numerical modeling of wiring system generally consist in combining two different approaches. Firstly, the voltage potential function of the faulty section is computed by solving a static $2D$ cross-sectional model based on Poisson equation. The finite element method or the finite difference method are widely used to perform calculations. After deducing electric field, the Gauss theorem allows to determine the cross section parameters such as, the resistance R, the inductance L, the capacitance C, the conductance G and the characteristic impedance Z. The return loss parameter S_{11} of the fault is then obtained by introducing these cross section parameters in a longitudinally model which can be based on different approaches. The transmission matrix method uses the impedance Z and consists in computing matrices to evaluate the linear networks for the simulation of the system's impulse response. The telegrapher's equations use the $RLCG$ parameters in two coupled equations of unknown the voltage V and the current I. This one-dimensional system is often solved by the finite difference method, both in frequency domain or in time domain, and allows to describe the voltage and the current at any point along the wire to obtain the reflectometry response. Unfortunately this modeling approach do not take into account the three-dimensional aspect of wave propagating in the wire.

In this paper a three-dimensional model of a coaxial cable with soft fault is presented. The presented work consists in solving time domain Maxwell's equations that describe the propagation phenomenon, and computes different parameters such as the reflection coefficient or the characteristic impedance of the faulty section. A nodal Discontinuous Galerkin method is adopted for the spatial discretization. This kind of approach has been recently introduced in the modeling of electromagnetic compatibility problems. Its discontinuous aspect allows to easily discretize objects of different sizes or shapes and provides a better consideration of discontinuous properties. These kind of methods are well adapted for parallel computing because the generated matrices are block diagonals. In this work, we adopt hexahedral spatial elements because they lead to less bulky meshes than with tetrahedral ones, and we consider high order elements in order to reduce numerical dispersion error. The time domain reflectogram obtained is treated through a fast fourrier transform algorithm to compute the frequency signature of the fault. The obtained values are compared to those issued from experimental measurements. The experimental impedances estimated using a classical transmission matrix method are then compared to the ones obtained by three dimensional modeling

II. Discontinuous Galerkin method

Let E, H and J, respectively, the electric field and the magnetic field and the current density. Time domain Maxwell's equations form a system (1) of 6 unknowns that are components of E and H:

$$\begin{cases} \epsilon\partial_t E - \nabla \times H = -J \\ \mu\partial_t H + \nabla \times E = 0 \end{cases} \tag{1}$$

where ϵ is the permittivity of the medium and μ its permeability and J is the current density. In a conductive medium, $J = \sigma E$, with σ the conductivity.

The Discontinuous Galerkin methods are introduced for solving the conservative form of partial differential equations. This method consists in discretizing the variational formulation of (1) on each mesh element T of the domain $\Omega = \cup T$.

$$\begin{cases} \displaystyle\int_T \epsilon\partial_t E\phi - \int_T H \times \nabla\phi - \int_{\partial T} (n \times H)^{num}\phi = -\int_T \sigma E\phi \\ \displaystyle\int_T \mu\partial_t H\psi + \int_T E \times \nabla\psi + \int_{\partial T} (n \times E)^{num}\psi = 0 \end{cases} \tag{2}$$

where ϕ and ψ are test functions. This approach is based on a classical finite element method on each T and flux expressions $(n \times H)^{num}$ and $(n \times E)^{num}$ are defined at the interfaces to connect the neighboring elements. The mapping technique is perfomed to increase the efficiency of the finite element method [3]. Different formulations of the flux expressions exist [4]. These following expressions resulting in different numerical schemes are implemented. For $\alpha = 0$, centred fluxes are obtained and numerical schemes are dispersive. For $\alpha = 1$, upwind fluxes are obtained and numerical schemes are dissipative.

$$\begin{cases} (n \times H)^{num} = n \times \dfrac{\{\sqrt{\frac{\mu}{\epsilon}}H\}}{\{\sqrt{\frac{\mu}{\epsilon}}\}} - \alpha\left(n \times \dfrac{(n \times [E])}{\{\sqrt{\frac{\mu}{\epsilon}}\}}\right) \\ (n \times E)^{num} = n \times \dfrac{\{\sqrt{\frac{\epsilon}{\mu}}E\}}{\{\sqrt{\frac{\epsilon}{\mu}}\}} + \alpha\left(n \times \dfrac{(n \times [H])}{\{\sqrt{\frac{\epsilon}{\mu}}\}}\right) \end{cases} \tag{3}$$

where $[u] = \dfrac{u^+ - u^-}{2}$ and $\{u\} = \dfrac{u^+ + u^-}{2}$. The subscript "-" denotes the values for filds in the current element, while "+" is for the adjacent element.

III. Numerical example

The chafing soft fault considered in this part is a small snatching of the dielectric health of length $l_f = 50mm$ and depth $h_f = 0.9mm$ as show in Fig.2. This kind of defect is very observed when a cable is not in its nominal position anymore and has moved against an edge. It is located in the middle of a damaged coaxial RG58 cable of length $L_c = 25cm$. The woven copper shield is of radius $R_e = 1.475mm$, the copper core radius is of radius $R_{in} = 0.425mm$. The inner dielectric insulator is of relative permitivity $\epsilon_r = 2.25$. Note that the outer plastic sheath is not modeled. The simulation consists in injecting an incident gaussian pulse in the cable. Hexahedral third order elements are used for meshing. The time integration is performed with a four stages explicit Runge-Kutta method. The reflected fields are recorded on a reflectogram. The treatment of this response allow to compute the parameters such as the characteristic impedance or the reflection coefficient of the soft fault. In a second time, the reflection coefficient of the line is experimentally measured using a Vector Network Analyzer and its impulse response computed. By adjusting a classical transmission matrix model, the cross sectionnal impedance of the defect is obtained. The results obtained by the two methods are in very good agreement and will be presented in an extended version of this paper.

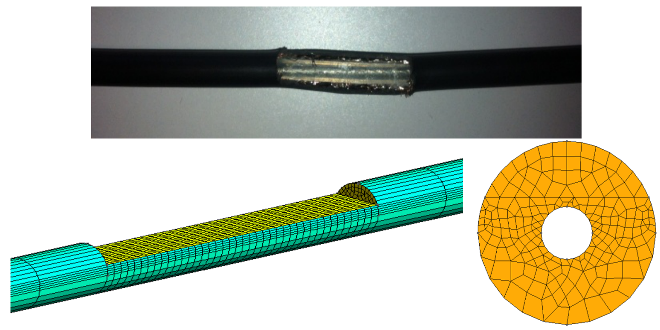

Fig. 1. Damaged coaxial cable of type dielectric sheath and its modeling.

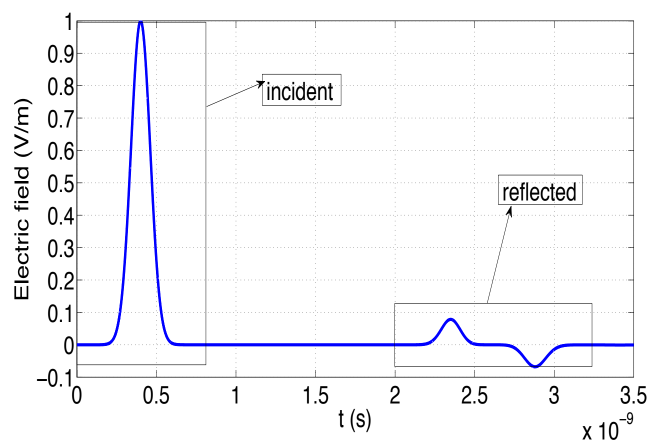

Fig. 2. The simulated reflectometry response of the chafing soft fault.

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