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Abstract: This paper proposes the design of a robust predictive control strategy which guarantees robustness towards parameters mismatch for a simplified macroscopic continuous photobioreactor model, obtained from mass balance based modelling. Firstly, this work is focused on classical robust nonlinear model predictive control law under model parameters uncertainties implying solving min-max optimization problem for setpoint trajectory tracking. Secondly, a new approach is proposed, consisting in reducing the basic min-max problem into a regularized optimization problem based on the use of linearization techniques, to ensure a good trade-off between tracking accuracy and computation time. Finally, the developed control law is compared to classical and robust predictive controllers. Its efficiency is illustrated through numerical results and robustness against parameter uncertainties is discussed for the worst case model mismatch.

Keywords: Bioprocesses, Robust Nonlinear Model Predictive Control, Min-max optimization problem, Uncertain systems.

1. INTRODUCTION

Microalgae are cultivated nowadays for feed, food or cosmetics production, and have recently emerged as an interesting source for sustainable energy production at large scale which attracted the interest from large companies. For these reasons also, microalgae cultivation is becoming a key research topic since it received high attention from scientific community leading to many studies.

Biochemical processes are systems where nonlinear effects are significant enough to justify the use of nonlinear model to give a sufficiently adequate representation of the system behavior. In addition, generally the process model is identified and uncertain parameters are estimated with evaluated confidence intervals, which motivates the development of robust control laws in the presence of modelling uncertainties.

In the literature of microalgae cultivations, several nonlinear control strategies have been developed: optimization-based (Abdollahi and Dubljevic, 2012; Tebbani et al., 2014), adaptive (Mailleret et al., 2004), sliding mode (Selisteanu et al., 2007), input/output linearization (Ifrim et al., 2013; Tebbani et al., 2015), and backstepping (Toroghi et al., 2013) approaches. They however do not specifically focus on robustness features.

Our aim is therefore to design a robust predictive controller which would be able to find an optimal feeding strategy in order to guarantee that the process will yield the desired amount of biomass along the cultivation period under model parameter uncertainties. Here, the challenge is to lay down a stable real time operation, insensitive to various disturbances, then, close to a certain state or desired profile. This requires the application of advanced optimal control strategies to ensure the bioprocess efficiency.

This work is focused on Nonlinear Model Predictive Control (NMPC) strategy (Camacho and Bordons, 2004). The main advantage of NMPC law is that it allows the current control input to be optimized, while taking into account the future system behavior. This is achieved by optimizing the control profile over a finite time horizon, but applying only the current control input. However, the performances of the NMPC law usually decrease when the true plant evolution deviates significantly from the one predicted by the model. Robust variants of NMPC (Kerrigan and Maciejowski, 2004; Limon et al., 2004) are able to take into account set bounded disturbance and/or constraints. The RNMPC can be formulated as a nonlinear min-max optimization problem which tends to become too large to be solved online numerically. Moreover, estimators are generally needed to reconstruct the states due to the lack of reliable online measurements. Consequently, the total calculation time is an important factor that must be reduced as much as possible. The proposed solution aims at transforming the min-max problem into a robust regularized least squares problem. A similar dual problem for robust state estimation, consisting on the design of receding-horizon observer based on (Sayed et al., 2002) work was presented in Goffaux and Vande Wouwer (2008). Thus, in this paper, we propose to apply a similar approach, in the case of NMPC law design. The original problem is converted into a scalar minimization problem using a model linearization technique (first order Taylor series expansion) at each sampling time along the nominal trajectory which is defined by the nominal parameter values and the current operating point. The main advantage of this approach is to be computationally tractable in calculating the optimal control compared to a min-max robust approach, which makes it suitable for online application.

The paper is structured as follows. Section 2 describes the dynamical model of the considered system, which is based on Droop model. Section 3 addresses the characterization of the steady state of the system. Section 4 presents nonlinear predictive controllers (classical, robust and regularized formulations) in order to regulate the biomass concentration at a desired value, by manipulating the dilution rate chosen as a control variable. Linearized Robust MPC (LRMPC) is derived in Section 5, based on the linearization technique cited previously. Moreover, numerical results are provided in Section 6 to compare classical NMPC, Robust NMPC (RNMPC) and proposed LRMPC performances in case of model mismatch. Conclusions and perspectives end this paper in Section 7.

2. MODEL DESCRIPTION

The specificity of microalgae in comparison to other microorganisms is that inorganic substrate uptake and growth are decoupled thanks to an intracellular storage of nutrients (Bernard, 2011). In order to take into account this phenomenon, the growth of microalgae is represented by Droop model (Bernard and Gouzé, 1995), (Bernard, 2011) which involves three state variables: the biomass concentration (denoted X, in $\mu m^3 L^{-1}$), the internal quota (denoted Q, in $\mu mol \ \mu m^{-3}$), which is defined as the quantity of substrate per unit of biomass, and the substrate concentration (denoted S, in $\mu mol \ L^{-1}$).

The considered dynamic model assumes that the photobioreactor operates in continuous mode (medium withdrawal flow rate equals its supply one, leading to a constant effective volume), without any additional biomass in the feed, and neglecting the effect of gas exchanges. The time varying equations resulting from mass balances are given by (Masci et al., 2010):

$$\begin{cases} \dot{X}(t) = \mu(Q(t),I(t)) X(t) - DX(t) \\ \dot{Q}(t) = \rho(S(t)) - \mu(Q(t),I(t)) Q(t) \\ \dot{S}(t) = (S_{in} - S(t))D - \rho(S(t))X(t) \end{cases} \quad (1)$$

where D represents the dilution rate (d^{-1}, d: day) and S_{in} the inlet substrate concentration ($\mu mol \ L^{-1}$).

The specific uptake rate, $\rho(S)$, and the specific growth rate, $\mu(Q,I)$, are given by:

$$\begin{cases} \rho(S) = \rho_m \dfrac{S}{S + K_s} \\ \mu(Q,I) = \bar{\mu}(1 - \dfrac{K_Q}{Q})\mu_I(I) \end{cases} \quad (2)$$

Parameters K_s and ρ_m represent respectively the substrate half saturation constant and the maximal specific uptake rate. The theoretical maximal specific growth rate is denoted $\bar{\mu}$ and K_Q represents the minimal cell quota allowing growth.

The modelling of light effect consists in including the term μ_I which is represented by a Haldane kinetics to model the photoinhibition (Masci et al., 2010):

$$\mu_I(I) = \dfrac{I}{I + K_{sI} + \dfrac{I^2}{K_{iI}}} \quad (3)$$

where I is the light intensity ($\mu E \ m^{-2} \ s^{-1}$), K_{sI} and K_{iI} are light saturation and inhibition constants respectively. The optimal light intensity that maximises the function μ_I is given by $I_{opt} = \sqrt{K_{sI}K_{iI}}$. In the sequel, the light intensity is set at this optimal value I_{opt}. The parameters of the model used in this study are displayed in Table 1 (Goffaux and Vande Wouwer, 2008; Munoz-Tamayo et al., 2014).

Table 1. Model parameters.

Parameter	Value	Unit
$\bar{\mu}$	2	d^{-1}
ρ_m	9.3	$\mu mol \ \mu m^{-3} \ d^{-1}$
K_Q	1.8	$\mu mol \ \mu m^{-3}$
K_s	0.105	$\mu mol \ L^{-1}$
K_{sI}	150	$\mu E \ m^{-2} \ s^{-1}$
K_{iI}	2000	$\mu E \ m^{-2} \ s^{-1}$
S_{in}	100	$\mu mol \ L^{-1}$
I_{opt}	547	$\mu E \ m^{-2} \ s^{-1}$

3. STEADY STATE CHARACTERIZATION

The nonlinear model (1) is represented thereafter in the state-space formalism as follows:

$$\begin{cases} \dot{x}(t) = f(x(t),u(t),\theta), \ x(t_0 = 0) = x_0 \\ y(t) = X(t) \end{cases} \quad (4)$$

with:

$$\begin{cases} x = \begin{bmatrix} X \\ Q \\ S \end{bmatrix}, f = \begin{bmatrix} \mu(Q,I)X - DX \\ \rho(S) - \mu(Q,I)Q \\ (S_{in} - S)D - \rho(S)X \end{bmatrix} \\ \theta = [\rho_m \ K_s \ \bar{\mu} \ K_Q \ K_{sI} \ K_{iI}]^\top, \ u = D \end{cases} \quad (5)$$

where $x \in \mathbb{R}^{n_x}$ is the state vector and x_0 its initial value, f the nonlinear process dynamics, $u \in U \subset \mathbb{R}^{n_u}$ represents the control input with U the set of admissible control values and $\theta \in \mathbb{R}^{n_\theta}$ is the vector of uncertain parameters that are assumed to lie in the admissible region $\Theta = [\theta^-, \theta^+]$ ($n_x = 3$, $n_u = 1$ and $n_\theta = 6$). To simplify notations, the exogenous inputs (S_{in}, I) are omitted but are applied to the model.

The state and control variables are restricted to fulfill the following constraints (Bernard and Gouzé, 1995):

$$X > 0, \ K_Q \leq Q \leq \dfrac{\rho_m}{\mu_I \bar{\mu}} + K_Q, \ 0 \leq S \leq S_{in}, \ D \geq 0 \quad (6)$$

The steady states of the system are derived from three nonlinear equations, given by $f(x(t),u(t),\theta) = 0$. For a given value of X (denoted X^r), the goal here is to characterize the corresponding values for Q and S. Then, the equilibrium is defined by (X^r, Q^r, D^r) as follows:

$$\begin{cases} D^r = \mu(Q^r, I) \\ \left(S_{in} - \dfrac{D^r Q^r K_s}{\rho_m - D^r Q^r}\right)D^r - D^r Q^r X^r = 0 \end{cases} \quad (7)$$

After developments, taking Q^r as an unknown variable, for a given biomass concentration X^r, the only admissible solution is given by:

$$Q^r = \dfrac{\bar{\mu}\mu_I(S_{in} + K_s) + (\rho_m + \bar{\mu}\mu_I K_Q)X^r - \sqrt{\Delta}}{2\bar{\mu}\mu_I X^r} \quad (8)$$

with

$$\begin{aligned} \Delta = &((\rho_m + \bar{\mu}\mu_I K_Q)X^r - \bar{\mu}\mu_I(S_{in} + K_s))^2 \\ &+ 4\bar{\mu}\mu_I K_s \rho_m X^r > 0 \end{aligned} \quad (9)$$

4. NONLINEAR MODEL PREDICTIVE CONTROLLER

The main objective of this study is to regulate the biomass concentration X to a reference value X^r, while the dilution rate D is constrained to track the reference D^r. The dilution rate reference trajectory is computed from the knowledge of the targeted setpoint at each time instant as detailed in section 3. Thereafter, the notations introduced in (4-5) will be used.

Nonlinear model given by (4-5) can be replaced by a discrete-time model which is obtained from the discretization of the continuous time state space model (4) using the Runge-Kutta scheme. Considering a constant sampling time T_s, the discrete time prediction model over a time interval $[t_k, t_{k+1}] \triangleq [kT_s, (k+1)T_s)]$ is defined as follows:

$$\begin{cases} x_{k+1} = x_k + \int_{t_k}^{t_{k+1}} f(x(\tau), u_k, \theta)d\tau \\ y_k = Hx_k \end{cases} \qquad (10)$$

where x_{k+1} is the state at t_{k+1}, k is the time index, x_k and y_k are the discrete state vector and the sampled measurement at time k, respectively. The measurement matrix is given by $H = [1 \ 0 \ 0]$. The control input u is parametrized using a piecewise-constant approximation ($u(\tau) = u(k)$, $\tau \in [kT_s, (k+1)T_s])$.

Let us define the discrete state trajectory g by the solution at time t_{k+1} of system (4-5) with initial state x_0, and $u_{t_0}^{t_k}$ the control sequence from the initial time instant t_0 to the time instant t_k:

$$x_{k+1} = g(t_0, t_{k+1}, x_0, u_{t_0}^{t_k}, \theta) \qquad (11)$$

This solution may result from generalisation of (10).

The predictive controller takes measurements of the system at each sampling time, using a nonlinear dynamical model to predict the behavior of the plant over a finite time receding horizon of length $N_p T_s$. The optimal control sequence is computed minimizing a cost function expressed as a quadratic criterion based on the tracking error while making sure that all constraints are fulfilled. This optimal control sequence is implemented until the next measurement becomes available. The optimization problem is solved again at the next sampling time according to the receding horizon principle.

Assuming a perfect knowledge of the parameter vector θ, the formulation of the optimization problem is moved into a nonlinear programming problem over the prediction horizon $N_p T_s$ at each sampling time t_k. The optimal control sequence is obtained as follows:

$$\tilde{u}_k^{k+N_p-1} = arg \min_{u_k^{k+N_p-1}} \Pi(u_k^{k+N_p-1}, \theta) \qquad (12)$$

where the cost function is defined as

$$\Pi(u_k^{k+N_p-1}, \theta) = ||\hat{y}_{k+1}^{k+N_p} - \bar{y}_{k+1}^{k+N_p}||_P^2 + ||u_k^{k+N_p-1} - \bar{u}_k^{k+N_p-1}||_R^2 \qquad (13)$$

$||z||_P^2 = z^\top P z$, is the Euclidean norm weighted by P.

with

$$\bar{y}_{k+1}^{k+N_p} = \begin{bmatrix} X_{k+1}^r \\ \vdots \\ X_{k+N_p}^r \end{bmatrix} \text{ the setpoint values (i.e. } X^r \text{ values),}$$

$$u_k^{k+N_p-1} = \begin{bmatrix} u_k \\ \vdots \\ u_{k+N_p-1} \end{bmatrix} \text{ the optimization variable,}$$

$$\bar{u}_k^{k+N_p-1} = \begin{bmatrix} D_k^r \\ \vdots \\ D_{k+N_p-1}^r \end{bmatrix} \text{ the reference control sequence} \qquad (14)$$

(i.e. D^r values), and the predicted output

$$\hat{y}_{k+1}^{k+N_p} = \begin{bmatrix} Hg(t_k, t_{k+1}, x_k, u_k, \theta) \\ Hg(t_k, t_{k+2}, x_k, u_k^{k+1}, \theta) \\ \vdots \\ Hg(t_k, t_{k+N_p}, x_k, u_k^{k+N_p-1}, \theta) \end{bmatrix}$$

where, the subscript is related to the time instant.
$P \geq 0$ and $R > 0$ are tuning diagonal matrices.

This problem is solved using nonlinear least-square optimization techniques.

5. ROBUST NONLINEAR MODEL PREDICTIVE CONTROLLER

5.1 Min-max optimization problem

In practice the parameter vector θ is often uncertain. The parameters values are nevertheless assumed to belong to a known interval $\Theta = [\theta^-, \theta^+]$. In this case, robust predictive control strategy (RNMPC) implying a min-max optimization problem (Yu, 1998; Kerrigan and Maciejowski, 2004) can be defined as follows:

$$\tilde{u}_k^{k+N_p-1} = arg \min_{u_k^{k+N_p-1}} \max_{\hat{\theta} \in \Theta} \Pi(u_k^{k+N_p-1}, \hat{\theta}) \qquad (15)$$

Π and $\hat{y}$ same as in (13-14) respectively, with $\theta = \hat{\theta}$.

The optimal control sequence is determined so that the maximum deviation for all trajectories over all possible data scenarii is minimized. Nevertheless, the min-max optimization problem is time consuming. In the sequel, it will be simplified, in order to reduce the computational burden.

5.2 Linearization techniques

In this paper, we propose a new formulation of RNMPC law. The trajectory prediction is linearized around the reference trajectory given by the reference dilution rate sequence $\bar{u}_k^{k+N_p-1}$ and for the nominal parameters, θ_{nom}, using a first order Taylor series expansion for $j = \overline{1, N_p}$:

$$g(t_k, t_{k+j}, x_k, u_k^{k+j-1}, \hat{\theta}) \approx g_{nom}(t_{k+j}) + \nabla_u g(t_{k+j})(u_k^{k+j-1} - \bar{u}_k^{k+j-1}) + \nabla_\theta g(t_{k+j})(\hat{\theta} - \theta_{nom}) \qquad (16)$$

with the state trajectory for the nominal case:

$$g_{nom}(t_{k+j}) = g(t_k, t_{k+j}, x_k, \bar{u}_k^{k+j-1}, \theta_{nom}) \qquad (17)$$

The nominal parameters are chosen as the average parameters values in the uncertain interval $[\theta^-, \theta^+]$:

$$\theta_{nom} = \frac{\theta^+ + \theta^-}{2} \qquad (18)$$

The partial derivatives of the state vector x with respect to the parameter vector θ and the control input sequence u, so-called *sensitivity functions*, are defined as follows:

$$\nabla_\theta g(t_{k+j}) = \left. \frac{\partial g(t_k, t_{k+j}, x_k, u_k^{k+j-1}, \theta)}{\partial \theta} \right|_{\substack{u_k^{k+j-1} = \bar{u}_k^{k+j-1} \\ \theta = \theta_{nom}}} \qquad (19)$$

$$\nabla_u g(t_{k+j}) = \left. \frac{\partial g(t_k, t_{k+j}, x_k, u_k^{k+j-1}, \theta)}{\partial u_k^{k+j-1}} \right|_{\substack{u_k^{k+j-1} = \bar{u}_k^{k+j-1} \\ \theta = \theta_{nom}}} \qquad (20)$$

Different approaches may be considered for determining the sensitivity functions defined in (19). The most precise method involves analytical derivation (Dochain, 2008). In the latter approach, the dynamics of the sensitivity function with respect to θ can be computed for time $t \in [t_k, t_{k+N_p}]$ by solving numerically the following differential equation (from (4) and (11)):

$$\frac{d}{dt}(\nabla_\theta g) = \frac{\partial f(x, u, \theta_{nom})}{\partial x} \nabla_\theta g + \frac{\partial f(x, u, \theta)}{\partial \theta}\Big|_{\theta = \theta_{nom}} \qquad (21)$$

with as an initial condition:
$$\nabla g_\theta(t_k) = 0_{3\times 6} \tag{22}$$
where $0_{n\times m} \in \mathbb{R}^{n\times m}$ is the zero matrix.
In order to simplify the calculation of the gradient $\nabla_u g$, finite differences are used to approximate numerically the derivative $\nabla_u g(t_{k+j})$ for each control u_j, $j \in [k, k+N_p-1]$.

From (14) and (16), it comes:
$$\begin{aligned}\hat{y}^{k+N_p}_{k+1} &\approx \bar{G}^{k+N_p}_{nom,k+1} + \bar{G}^{k+N_p-1}_{u,k}(u^{k+N_p-1}_k - \bar{u}^{k+N_p-1}_k)\\ &\quad + \bar{G}^{k+N_p}_{\theta,k+1}(\hat{\theta} - \theta_{nom})\end{aligned} \tag{23}$$

with $\bar{G}^{k+N_p}_{\theta,k+1} = [H\nabla_\theta g(t_{k+1}),\dots,H\nabla_\theta g(t_{k+N_p})]^\top$, the vector of Jacobian matrices related to the parameters,
$\bar{G}^{k+N_p}_{u,k+1} = [H\nabla_u g(t_{k+1}),\dots,H\nabla_u g(t_{k+N_p})]^\top$, the vector of Jacobian matrices related to the control sequence,
$\bar{G}^{k+N_p}_{nom,k+1} = [Hg_{nom}(t_{k+1}),\dots,Hg_{nom}(t_{k+N_p})]^\top$, the vector containing the predicted output for the nominal case given by $(\bar{u}^{k+N_p-1}_k, \theta_{nom})$.

5.3 Proposed control strategy

Assuming that the uncertain parameters are uncorrelated, then the bounded parametric error can be expressed by:
$$\hat{\theta} - \theta_{nom} = \gamma \delta\theta_{max} \tag{24}$$
with
$$\delta\theta_{max} = \frac{\theta^+ - \theta^-}{2} \text{ and } ||\gamma|| \le 1 \tag{25}$$
The min-max optimization problem (15) but without the inequality constraints is now converted into a robust regularized least squares problem when applying (23-24) in the presence of uncertain data:
$$\tilde{z} = arg \min_z \max_{\delta b} ||Az - (b + \delta b)||^2_P + ||z||^2_R \tag{26}$$
with
$$\begin{cases} z = u^{k+N_p-1}_k - \bar{u}^{k+N_p-1}_k, \\ A = \bar{G}^{k+N_p-1}_{u,k}, \\ b = \bar{y}^{k+N_p}_{k+1} - \bar{G}^{k+N_p}_{nom,k+1}, \\ \delta b = -\bar{G}^{k+N_p}_{\theta,k+1}\gamma\delta\theta_{max} \end{cases} \tag{27}$$

The regularized robust solution and the corresponding robustified weighting parameters that will be determined are based on the following theorem:

Theorem 1. Regularized Robust Design Criterion for Uncertain Data (Sayed et al., 2002) Consider the following optimization problem:
$$z^o = arg \min_z \max_{\delta A, \delta b} ||z||^2_V + ||(A + \delta A)z - (b + \delta b)||^2_W \tag{28}$$
where $V > 0$ and $W \ge 0$ are Hermitian weighting matrices.
δA denotes a perturbation matrix to the nominal matrix A and δb a perturbation vector to the nominal vector b which are assumed to satisfy the following model:
$$\begin{cases} \delta A = C\Delta E_a \\ \delta b = C\Delta E_b \end{cases} \tag{29}$$
where Δ denotes an arbitrary contraction with $||\Delta|| \le 1$.
The regularized robust least-squares problem (28) is a special case of a constrained two-player game problem, defined as follows:
$$z^o = arg \min_z \max_{||\kappa|| \le \pi(z)} ||z||^2_V + ||Az - b + C\kappa||^2_W \tag{30}$$

where $\pi(z)$ is a nonnegative function given by:
$$\pi(z) = ||E_a z - E_b|| \tag{31}$$
and κ is an unknown perturbation vector which is equal to $\Delta(E_a z - E_b)$.
The original optimization problem (28) is equivalent to:
$$\min_z [z^\top V z + (Az - b)^\top \hat{W}(\lambda)(Az - b) + \lambda\pi^2(z)] \tag{32}$$
This problem has a unique global minimum z^o given by:
$$z^o(\lambda^o) = [\hat{V}(\lambda^o) + A^\top\hat{W}(\lambda^o)A]^{-1}[A^\top\hat{W}(\lambda^o)b + \lambda^o E_a^\top E_b] \tag{33}$$
where the modified weighting matrices $\hat{W}(\lambda), \hat{V}(\lambda)$ are obtained from W, V via:
$$\begin{cases} \hat{W}(\lambda^o) = W + WC(\lambda^o I - C^\top WC)^\dagger C^\top W \\ \hat{V}(\lambda^o) = V + \lambda^o E_a^\top E_a \end{cases} \tag{34}$$
The notation $\alpha^\dagger$ denotes the pseudo inverse of α.
The invertibility of $\hat{V}(\lambda^o) + A^\top\hat{W}(\lambda^o)A$ is guaranteed by the positive definiteness of V. The nonnegative scalar parameter $\lambda^o \in \mathbb{R}$ representing the Lagrange multiplier (Sayed et al., 2002), is computed from the following minimization:
$$\begin{aligned}\lambda^o = arg \min_{\lambda \ge \lambda_l} &\ ||z(\lambda)||^2_V + \lambda||E_a z(\lambda) - E_b||^2 + \\ &\ ||Az(\lambda) - b||^2_{\hat{W}(\lambda)}\end{aligned} \tag{35}$$
with
$$\begin{cases} z(\lambda) = [\hat{V}(\lambda) + A^\top\hat{W}(\lambda)A]^{-1}[A^\top\hat{W}(\lambda)b + \lambda E_a^\top E_b] \\ \hat{W}(\lambda) = W + WC(\lambda I - C^\top WC)^\dagger C^\top W \\ \hat{V}(\lambda) = V + \lambda E_a^\top E_a \end{cases} \tag{36}$$
The lower bound on λ is denoted by λ_l, with:
$$\lambda_l = ||C^\top WC|| \tag{37}$$
Matrix norm, like e.g. $||A||$ is related to the maximum singular value of the corresponding matrix i.e. $||A|| = \sqrt{\bar{\sigma}(A^\top A)}$ with $\bar{\sigma}(A)$ the maximum eigenvalue of A.
For any value of λ in the semi-open interval $[\lambda_l, +\infty[$, the matrix $\hat{W}(\lambda)$ is nonnegative definite so that criterion (35) is nonnegative for $\lambda \ge \lambda_l$.

Proof: see (Sayed et al., 2002).

The robust nonlinear predictive problem which is defined by (26-27), is written in the form (28-29) with:
$$C = \bar{G}^{k+N_p}_{\theta,k+1}, \Delta = \gamma, E_a = 0, E_b = -\delta\theta_{max} \tag{38}$$
The application of the theorem 1 provides the following unique solution:
$$\begin{aligned}\tilde{u}^{k+N_p-1}_k &= \bar{u}^{k+N_p-1}_k + [R + \bar{G}^{k+N_p-1\top}_{u,k}\hat{P}(\lambda^o)\bar{G}^{k+N_p-1}_{u,k}]^{-1}\\ &\quad [\bar{G}^{k+N_p-1\top}_{u,k}\hat{P}(\lambda^o)(\bar{y}^{k+N_p}_{k+1} - \bar{G}^{k+N_p}_{nom,k+1})]\end{aligned} \tag{39}$$
with
$$\hat{P}(\lambda^o) = P + P\bar{G}^{k+N_p}_{\theta,k+1}(\lambda^o I - \bar{G}^{k+N_p\top}_{\theta,k+1}P\bar{G}^{k+N_p}_{\theta,k+1})^\dagger \bar{G}^{k+N_p\top}_{\theta,k+1}P \tag{40}$$
λ^o is computed from the following minimization problem:
$$\lambda^o = arg \min_{\lambda \ge ||\bar{G}^{k+N_p\top}_{\theta,k+1}P\bar{G}^{k+N_p}_{\theta,k+1}||} G(\lambda) \tag{41}$$
where the function $G(\lambda)$ is defined by:
$$\begin{aligned}G(\lambda) = &\ ||\bar{G}^{k+N_p-1}_{u,k}z(\lambda) - \bar{y}^{k+N_p}_{k+1} + \bar{G}^{k+N_p}_{nom,k+1}||^2_{\hat{P}(\lambda)} + \\ &\ ||z(\lambda)||^2_R + \lambda||\delta\theta_{max}||^2\end{aligned} \tag{42}$$

with

$$z(\lambda) = [R + \bar{G}_{u,k}^{k+N_p-1^\top} \hat{P}(\lambda) \bar{G}_{u,k}^{k+N_p-1}]^{-1}$$
$$[\bar{G}_{u,k}^{k+N_p-1^\top} \hat{P}(\lambda)(\bar{y}_{k+1}^{k+N_p} - \bar{G}_{nom,k+1}^{k+N_p})] \quad (43)$$

and $\hat{P}(\lambda)$ as in (40).

The minimum λ^o of the unidimensional function $G(\lambda)$ is found using the golden section search algorithm.

As a conclusion, the predictive controller consists in solving online a unidimensional optimization problem (41) at each sampling time, instead of solving min-max problem (15). In the sequel, this predictive control law will be called as linearized robust model predictive control (LRMPC).

6. RESULTS AND DISCUSSION

In this section, the efficiency of the proposed control strategy is validated in simulation. The performances of the above mentioned algorithms are compared for a worst uncertain parameters case. The worst case biomass prediction can be approximated using parameter bounds $\{\theta^-, \theta^+\}$ only, rather than by exploring the full parameter space (Goffaux and Vande Wouwer, 2008). Then to reduce the search in the whole parameter subspace to a search on its boundary, $2^{dim(\theta)}$ tests have been realized using NMPC law. The parameters values of the system are chosen on the parameter subspace border ($\theta_{real} = [\rho_m^+, K_s^-, \bar{\mu}^+, K_Q^-, K_{sI}^-, K_{iI}^+]$) and correspond to one of the 4 worst-case model mismatches (Benattia et al., 2014b), where the uncertain parameters subspace $[\theta^-, \theta^+]$ is given by $[0.8\theta, 1.2\theta]$. The initial biomass concentration value is set close to the setpoint in order to cancel the transient effect and to focus only on the behavior during setpoint changes (rising and falling edge respectively), with a maximal admissible dilution rate D_{max} equal to 1.6 d^{-1}. The simulation time is set to $T_f = 2$ days but the optimal control law is applied between 0 and $T_f - N_p T_s$. The inlet substrate concentration S_{in} is assumed to be perfectly known. The light intensity is assumed to be measured online, non-corrupted with noise. The weighting matrices are chosen as $P = I_{N_p}$ and $R = I_{N_p}$ (the same for all controllers).

First, simulations have been carried out considering the uncertain parameter worst case cited previously. The results as depicted in Fig. 1 show the influence of the sampling time T_s on the biomass concentration tracking error and dilution rate evolution for NMPC and LRMPC ($N_p = 5$) without considering measurements affected by noise. It can be observed that the model should be sampled sufficiently fast in order to guarantee that the first order Taylor series expansion is accurate as much as possible. It should be noted that a compromise is required to properly select an appropriate sampling time taking into account the computation burden due to potential state estimator and/or online determination of the optimal trajectory. It appears clearly that $T_s = 10$ min allows satisfying a good trade off between linearization accuracy and computational burden.

Secondly, the choice of the prediction horizon N_p is studied for the same conditions cited previously with $T_s = 10$ min. The prediction horizon N_p is chosen to satisfy a compromise between the computation time and a sufficient vision of the system behaviour. Figure 2 shows that increasing the prediction horizon leads to a loss of accuracy which is due to the prediction using an approximated model (in the linearization procedure).

Finally, three predictive control laws will be tested (Fig. 3): a classical Nonlinear Model Predictive Control (denoted as
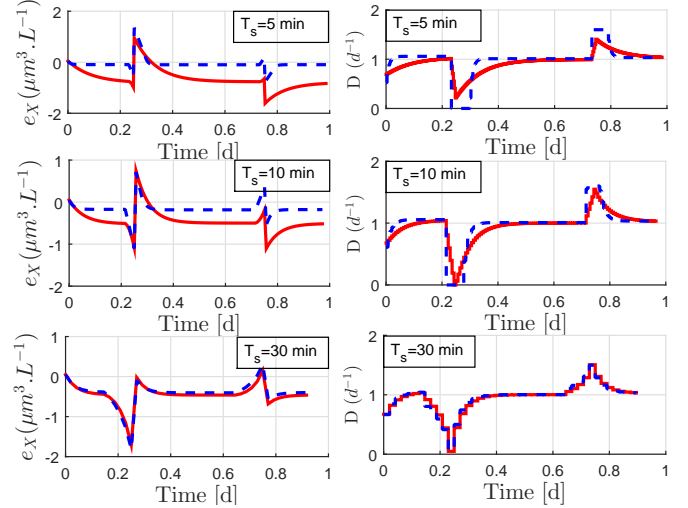

Fig. 1. Biomass concentration tracking error and dilution rate evolution with time for NMPC ($-$ red) and LRMPC ($--$ blue) strategies.
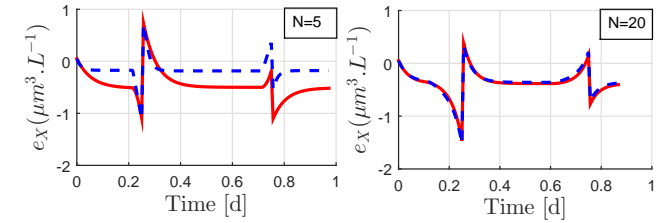

Fig. 2. Biomass concentration tracking error evolution with time for NMPC ($-$ red) and LRMPC ($--$ blue) strategies.

NMPC), a robust one using criterion (15) (denoted as RNMPC) and the proposed one (LRMPC). The tuning parameters are the same for all strategies (T_s=10 min, $N_p = 5$ and $P = R = I_{N_p}$). Biomass concentration measurements, y_k, are assumed to be corrupted by a centred Gaussian white noise with 0.1 standard deviation.
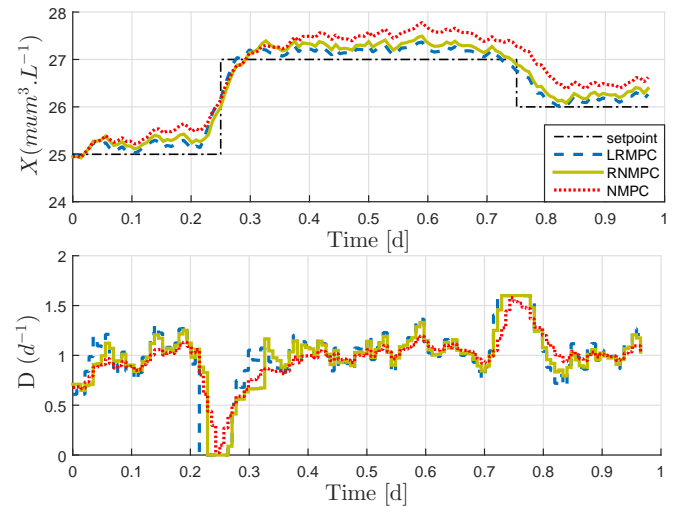

Fig. 3. Biomass concentration and dilution rate evolution with time for NMPC, RNMPC and LRMPC strategies.

It can be noticed the anticipation behavior to a setpoint change (Fig. 3), due to the prediction of the setpoint trajectory future

evolution over the moving horizon. The dilution rate decrease leads to an increase of the cell concentration and vice versa which agrees with the biological aspect. The obtained results show that both RNMPC and LRMPC have better performances than the classical NMPC under parameter uncertainties and measurement noise. In the NMPC law, the biomass concentration is not able to track the specified setpoint in the presence of parameters uncertainties, due to the fact that the mismatch between the system and the model is not considered during the prediction step inside the minimization procedure. Furthermore, both RNMPC and LRMPC in term of accuracy are slightly the same. The LRMPC algorithm performs well and offers a very significant computational load reduction comparing with RNMPC as shown in Table 2 (in purpose of comparing the computation time, upper bound constraint on D is omitted in the NMPC law).

Table 2. Comparison of the proposed algorithms in terms of computation time at each sampling time (worst case).

Algo. \ Perf. indices	Computation time (s)		
	min	mean	max
NMPC	$< 10^{-5}$	0.024	0.31
RNMPC	0.43	3.32	47.5
LRMPC	$< 10^{-5}$	0.016	0.1

In the case of LRMPC law, the static error is due to the superposition of two phenomena: the approximation of the model through linearization and the model mismatch. In the latter case, a possible improvement could be to use the difference between the system and the model outputs at each sampling time during the prediction step (Benattia et al., 2014a).

7. CONCLUSION

In this paper, a new robust NMPC law is proposed. The min-max problem is solved in two ways: first, the optimal control sequence is determined so that the maximum deviation for all trajectories over all possible data scenarii is minimized. Secondly, a linearization of the predicted trajectory is performed to turn the original min-max problem into a simple scalar minimization problem. Tests in simulation show good performance of the proposed strategy with respect to worst case model uncertainties. Moreover, it allows to significantly reduce the computational load with a good tracking trajectory accuracy.

In order to increase the quality of linearized model, there are several issues that deserve further investigation. An interesting perspective may be considering a second order expansion rather than the first order approximation to improve the robustness and accuracy of the proposed control strategy. Future works will be directed on the determination of sufficient conditions ensuring robust stability of LRMPC with bounded uncertainties including inequality constraints. Furthermore, an estimation algorithm to reconstruct biomass concentration will be developed and coupled to the proposed control strategy.

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