



**HAL**  
open science

# Algorithmic scheme for hybrid computing with CPU, Xeon-Phi/MIC and GPU devices on a single machine

Sylvain Contassot-Vivier, Stéphane Vialle

► **To cite this version:**

Sylvain Contassot-Vivier, Stéphane Vialle. Algorithmic scheme for hybrid computing with CPU, Xeon-Phi/MIC and GPU devices on a single machine. International Conference on Parallel Computing 2015 (ParCo 2015), Sep 2015, Edinburgh, United Kingdom. pp.1-10, 10.3233/978-1-61499-621-7-25 . hal-01258499

**HAL Id: hal-01258499**

**<https://centralesupelec.hal.science/hal-01258499v1>**

Submitted on 17 Feb 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons Attribution - NoDerivatives 4.0 International License

# Algorithmic scheme for hybrid computing with CPU, Xeon-Phi/MIC and GPU devices on a single machine

Sylvain CONTASSOT-VIVIER<sup>a</sup> and Stephane VIALLE<sup>b</sup>

<sup>a</sup> *Loria - UMR 7503, Université de Lorraine, Nancy, France*

<sup>b</sup> *CentraleSupélec & UMI GT-CNRS 2958, Metz, France*

**Abstract.** In this paper, we address the problem of the efficient parallel exploitation of different types of computing devices inside a single machine, to solve a scientific problem. As a first step, we apply our scheme to the Jacobi relaxation. Despite its simplicity, it is a good example of iterative process for scientific simulation. Then, we evaluate and analyze the performance of our parallel implementation on two configurations of hybrid machine.

**Keywords.** Accelerator, Xeon-Phi/MIC, GPU, hybrid computing, heterogeneous computing, offload computing.

## Introduction

According to the hardware evolution in the last decades, the architecture of parallel systems becomes more and more complex. In particular, the development of many-core devices such as GPU (Graphical Processing Unit) and MIC (Many Integrated Cores) have induced an additional hierarchical level of parallelism in supercomputers. Indeed, current parallel systems are typically organized as big clusters of nodes and the many-core devices provide much larger computational power at the node level. However, the efficient programming of all that gathered power is still a major difficulty in the domain of High Performance Computing, partly due to the hierarchy in the system and to the communications between the different levels. The main issue is to design and implement efficient parallel schemes, as general as possible, that allows an efficient cooperation between all the computing units in a parallel system. The study presented in this paper is, to the best of our knowledge, one of the first attempt to solve a scientific application by using three different types of computing units inside a single node: the CPU cores, a GPU and a MIC.

After a brief review of the previous works over the programming of *hybrid* machines (containing different kinds of computing devices) in Section 1, our application and hardware testbeds are described in Section 2. Then, Section 3 details the different algorithms designed and implemented for each kind of device (CPU, GPU, MIC). Finally, a multiple devices solution is proposed in Section 4. An experimental performance comparison and analysis, proposed in Section 5, allows us to evaluate the efficiency of our scheme and to point out the major difficulties in such cooperation.

## 1. Related works

In the past, we have investigated some ways to efficiently design algorithms and codes for hybrid nodes (one PC with a GPU) and clusters of hybrid nodes (cluster of multi-core nodes with GPUs) [1,2]. Overlapping of computations with communications was a key point to achieve high performance on hybrid architectures. The processing speed of each node increases when using accelerators, while interconnection networks remains unchanged, and data transfer times between CPU main memory and accelerator memory introduce new overheads.

Today, scientific computing on GPU accelerators is common, but using Xeon Phi accelerators has still to be explored, although some comparison works have been achieved. In [3], authors point out the need to optimize data storage and data accesses in different ways on GPU and Xeon Phi, but no dot attempt to use both accelerators in the same program. Another strategy is to use a generic programming model and tool to program heterogeneous architectures, like OpenCL [4]. But usually it does not hide the different architectures requirements to achieve optimal performance, and it still requires an (important) algorithmic effort to design high performance codes running concurrently on different accelerators.

## 2. Benchmark application and testbeds

### 2.1. Jacobi relaxation application

According to the scope of this paper (hybrid computing with CPU, GPU and MIC), we have chosen an application with a regular domain, that is quite representative of the scientific problems adapted to the constraints of the studied devices (especially the GPU). Indeed, the objective of this work is not to propose parallel schemes for general numerical methods but to study the best ways to make the different internal devices of a hybrid computer work together.

The Jacobi relaxation is a classical iterative process providing a simple modeling of heat transfer or electrical potential diffusion in 2D or 3D discrete domain (regular grid). The objective of this application is to compute the stable state over the entire domain for some given fixed boundary conditions. An explicit iterative 2D scheme is performed as:

$$crt[l][c] = \frac{pre[l-1][c] + pre[l][c-1] + pre[l][c] + pre[l][c+1] + pre[l+1][c]}{5} \quad (1)$$

where  $crt[l][c]$  is the value of the grid point at line  $l$  and column  $c$  at the current iteration, while  $pre[l][c]$  gives the value of the same grid point at the previous iteration. The other four grid points involved are the direct neighbors (in 4-connexity) of the current point.

This iterative process is performed until the termination condition is reached. As the quantities in the grid are generally coded by real numbers, the strict stabilization may not be reachable in reasonable time. Among the different solutions to get around this problem, we have chosen to fix the number of iterations. This presents the advantage of providing a complete and accurate control over the amount of computation. In fact, this parameter is essential to study some aspects of parallel algorithms, such as the scalability.

### 2.2. Testbeds

The machine used at CentraleSupélec is a Dell R720 server containing two 6-cores Intel(R) Xeon(R) CPU E5-2620 at 2.10GHz, with two accelerators on separate PCIe buses.

One accelerator is an Intel MIC *Xeon-Phi 3120* with 57 physical cores at 1.10 GHz, supporting 4 threads each. The second accelerator is a Nvidia GPU *GeForce GTX Titan Black* (Kepler architecture) with 2880 CUDA cores. The machine used at Loria is a Dell R720 server containing two 8-cores Intel(R) Xeon(R) CPU E5-2640 at 2.00GHz, with two accelerators on separate PCIe buses. One accelerator is an Intel MIC *Xeon-Phi 5100* with 60 physical cores at 1.05 GHz supporting 4 threads each. The second accelerator is a Nvidia GPU *Tesla K40m* (Kepler architecture) with 2880 CUDA cores.

In this paper, we study the behavior of our parallel scheme on these two different machines, taking into account the relative computation powers of CPU, GPU and MIC.

### 3. Optimized kernels for single architecture and device

#### 3.1. Multi-core CPU with OpenMP

A first version of the multi-core CPU kernel to perform the Jacobi relaxation consists in a classical domain decomposition in horizontal strips through the cores. This is achieved by inserting the parallelism at the level of the loop over the lines of the domain inside the main iterative loop. That main loop updates the current version of the grid according to the previous one. The corresponding parallel scheme is given in Listing 1.

Listing 1: Simple OpenMP scheme for the Jacobi relaxation

```

1 #pragma omp parallel num_threads(nbT) // Threads creation
2 {
3   ... // Local variables and array initializations
4   for(iter=0; iter<nbIters; ++iter){ // Main iterative loop
5     // Parallel parsing of horizontal strips of the domain
6     #pragma omp for
7     for(lig=1; lig<nLig-1; ++lig){ // Lines in each strip
8       for(col=1; col<nCol-1; ++col){ // Columns in each line
9         ind = lig * nCol + col;
10        crt[ind] = 0.2 * (prec[ind - nCol] + prec[ind-1] + prec[ind]↔
11          + prec[ind+1] + prec[ind+nCol]);
12      }
13    }
14    #pragma omp single
15    { ... // Arrays exchange for next iteration (avoids copy) }
16  }

```

Although this simple version works quite well for small and medium sizes of grids, it is not fully scalable for grids with large lines, due to the L2 cache use that is not optimized. We remind the reader that one L2 cache is present in each core of a CPU. So, a second version has been implemented explicitly taking into account the cache management in each core. Due to the data dependencies in our application and to the cache mechanism, the modifications mainly consist in changing the

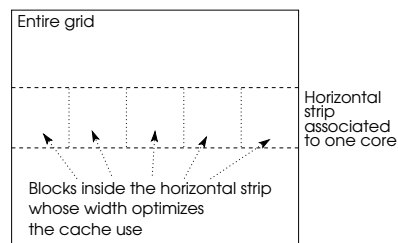


Figure 1.: Blocks in horizontal strips to optimize the cache use

update order of the elements in each horizontal strip. In the first version, the updates are performed by following the order of the entire lines of the grid. In the second version, the horizontal strips are divided in blocks along their width and their updates are performed block by block. The height of the blocks in a given strip is the same as the height of the strip, but their width may be smaller as it is directly deduced from the cache size and the line width ( $lw$ ), as illustrated in Figure 1. In fact, the optimal block width ( $obw$ ) is deduced from the cache size. Then, the number of blocks ( $nbb$ ) per strip is computed. Finally, the actual block width ( $abw$ ) is computed in order to obtain blocks of the same width in the horizontal strip.

$$nbb = \left\lceil \frac{lw}{obw} \right\rceil, \quad abw = \frac{lw}{nbb} \quad (2)$$

### 3.2. Many-core MIC with offloaded OpenMP

In order to use the MIC Xeon-Phi, Intel proposes an extension of the OpenMP library in its C/C++ compiler. It mainly consists in additional directives that allows the programmer to control the MIC directly from the CPU. It must be noticed that any classical OpenMP program can be run directly on a MIC. However, in this context, the MIC acts as an autonomous multi-core machine but it cannot cooperate (via OpenMP) with the central CPU cores. So, in the perspective of making the MIC cooperate with other devices (CPU, GPU,...), it is required to use the MIC as a co-processor of the central CPU (see [5] for an introduction to *offload* programming paradigm). One great advantage of the MIC, compared to other devices such as GPU, is that the same OpenMP code that runs on the CPU can be executed on the MIC without modification. Hence, Listing 1 can be directly executed on a MIC. However, the MIC has its own memory and can only process data in its memory. This implies the need of explicit data transfers between the central memory of the node and the memory on the MIC board.

The execution and data transfers can be expressed with the same directive, called `offload`, as depicted in Listing 2.

Listing 2: Offloading of the Jacobi relaxation on a MIC with synchronous data transfers

```
1 #pragma offload target(mic:0) inout(tabM:length(nLig*nCol) align(64))
2 { // Computes nbIters iterations of Jacobi over array tabM
3   // with nbTMic cores on the MIC
4   jacobi(tabM, nLig, nCol, nbIters, nbTMic);
5 }
```

In this example, `target` gives the identifier of the MIC device to use, and the `inout` parameter specifies that the array `tabM` (whose size must be given in number of elements) is an input as well as an output of the offload. That means that before the start of the computation on the MIC, the array is copied from central RAM to the MIC RAM. And once the computation is over, the array is copied back from the MIC RAM to the central RAM (at the same location). The scalar variables passed as parameters of the `jacobi` function are implicitly copied from the central RAM to the MIC RAM. Finally, the `align` parameter is optional and forces the memory allocations for the data on the MIC to be aligned at boundaries greater or equal to the specified number of bytes. Such memory alignments improve the performance of data transfers.

It is worth noticing that the offload presented in Listing 2 is blocking. So, the core CPU that executes this offload will wait for the end of the execution of `jacobi` on the

MIC and for the completion of the output data transfer from the MIC memory to the central one, before resuming its execution. When the MIC is used alone, without cooperating with the CPU, this synchronous scheme is pertinent. Nevertheless, it prevents any computation by the CPU while the MIC is running. We will see in Section 4 how to perform asynchronous (non-blocking) offloads, in order to allow the CPU to work during the MIC execution. Also, we will point out the need to replace blocking data transfers by asynchronous ones, in order to overlap communication with computation.

### 3.3. Many-core GPU with CUDA

We designed a single CUDA kernel to process the GPU part of the Jacobi relaxation. It is called two times per iteration: to quickly and early compute the boundary of the GPU part of the Jacobi grid, and to compute the (large) rest of this grid part. We optimized our algorithm and code to make fast *coalescent* memory accesses, to use the *shared memory* of each vectorial multiprocessor of the GPU, and to *limit the divergence* of the thread of a same block (when not executing exactly the same instructions). See [6] for efficient CUDA programming rules.

Each thread of this kernel updates one point of the Jacobi Grid during one cycle, and threads are grouped in 2-dimensional blocks of a 2-dimensional grid. This kernel has been optimized using the *shared memory* of each multiprocessor of the GPU, allowing each thread to read only one data from the GPU global memory, to share this data with others threads of its block, and efficiently access the 5 input data it requires to update its Jacobi grid point. Global memory read and write are achieved in a coalescent way. Considering a block of size  $BSY \times BSX$ , all the threads (in the range  $[0; BSY - 1] \times [0; BSX - 1]$ ) load data from the global memory into the shared memory, and  $(BSY - 2) \times (BSX - 2)$  threads in the range  $[0; BSY - 3] \times [0; BSX - 3]$  achieve computations, limiting the divergence of the threads inside a block.

Listing 3: Optimized CUDA kernel

```

1 __global__ void update(double *gpuPrec, double *gpuCrt, int gpuLigs, ←
    int cols)
2 {
3     int idx, lig, col;
4     __shared__ double buf[BLOCKSIZEY][BLOCKSIZEX];

6     // Coordinates of the Jacobi grid to load in shared memory
7     col = blockIdx.x * (BLOCKSIZEX - 2) + threadIdx.x;
8     lig = blockIdx.y * (BLOCKSIZEY - 2) + threadIdx.y;
9     // If valid coordinates: load data and compute
10    if(col < cols + 2 && lig < gpuLigs + 2){
11        idx = lig * (cols + 2) + col;
12        buf[threadIdx.y][threadIdx.x] = gpuPrec[idx];
13        __syncthreads();
14        lig++; col++; // shift coordinates to point out element to compute
15        // if new coordinates are valid: achieve computation
16        if(col <= cols && lig <= gpuLigs && threadIdx.x < BLOCKSIZEX-2 ←
            && threadIdx.y < BLOCKSIZEY-2){
17            idx = lig * (cols + 2) + col;
18            gpuCrt[idx] = 0.2 * (buf[threadIdx.y][threadIdx.x+1] +
19                buf[threadIdx.y+1][threadIdx.x] +
20                buf[threadIdx.y+1][threadIdx.x+1] +

```

```

21         buf[threadIdx.y+1][threadIdx.x+2] +
22         buf[threadIdx.y+2][threadIdx.x+1]);
23     }
24 }
25 }

```

Moreover, CPU memory arrays involved in the CPU-GPU data transfers have been locked in memory, using `cudaHostAlloc(...)` routine, in order to support asynchronous and faster data transfers. Finally, we used some CUDA *streams* to efficiently manage and run concurrent data transfers and kernel computations, so that we obtain a maximal overlapping.

#### 4. Multiple architectures and devices solution

##### 4.1. General asynchronous scheme and data distribution

The GPU remains a scientific co-processor, and we use the MIC in *offload* mode. So, our hybrid CPU+MIC+GPU solution still uses the CPU to run the `main` function, to launch all computation steps on the GPU, on the MIC and on its own cores, and to launch the data transfers between the CPU and the accelerators. The CPU memory hosts the entire current (`crt`) and previous (`prev`) Jacobi grids, but the top part is transferred on the GPU and the bottom part on the MIC, see Figure 2. We name CPU boundaries the first and last lines computed by the CPU, GPU boundary the last line computed by the GPU, and MIC boundary the first line computed by the MIC. We name *corpus* the other lines computed by a computing device. So, each *computing device* (CPU, MIC and GPU) stores its parts of the Jacobi grids and the adjacent boundary(ies) of other device(s). In order to save memory and optimize the transfers, our parallel algorithm

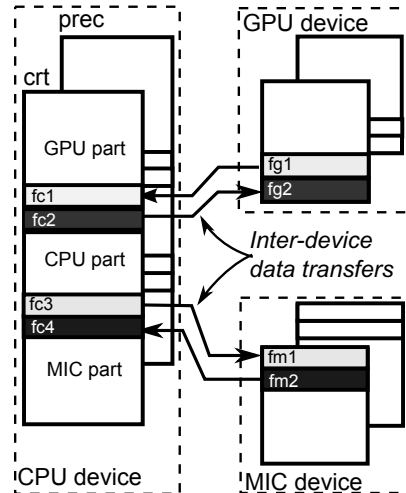


Figure 2.: Data structures implemented on the three devices (CPU, MIC and GPU)

has been designed to allow the direct transfer of each frontier to its right place in the local array on the destination device. So, no intermediate array is required to store the received frontiers coming from another device. Also, some efforts have been made to obtain a CPU algorithm that uses symmetric data structures and interactions for both accelerators. Figure 3 introduces our multi-device algorithm, based on the following principles:

- Before to enter a new computation step, a processor has its previous Jacobi grid entirely updated, including the boundary of the adjacent processor. So it can compute all its part of its current grid.
- A processor sends its newly updated boundary to the adjacent processor while it receives the updated boundary of this adjacent processor.
- Boundary(ies) computation and transfer of a processor are sequentially linked, but achieved in parallel of its corpus computation. The objective is to overlap as

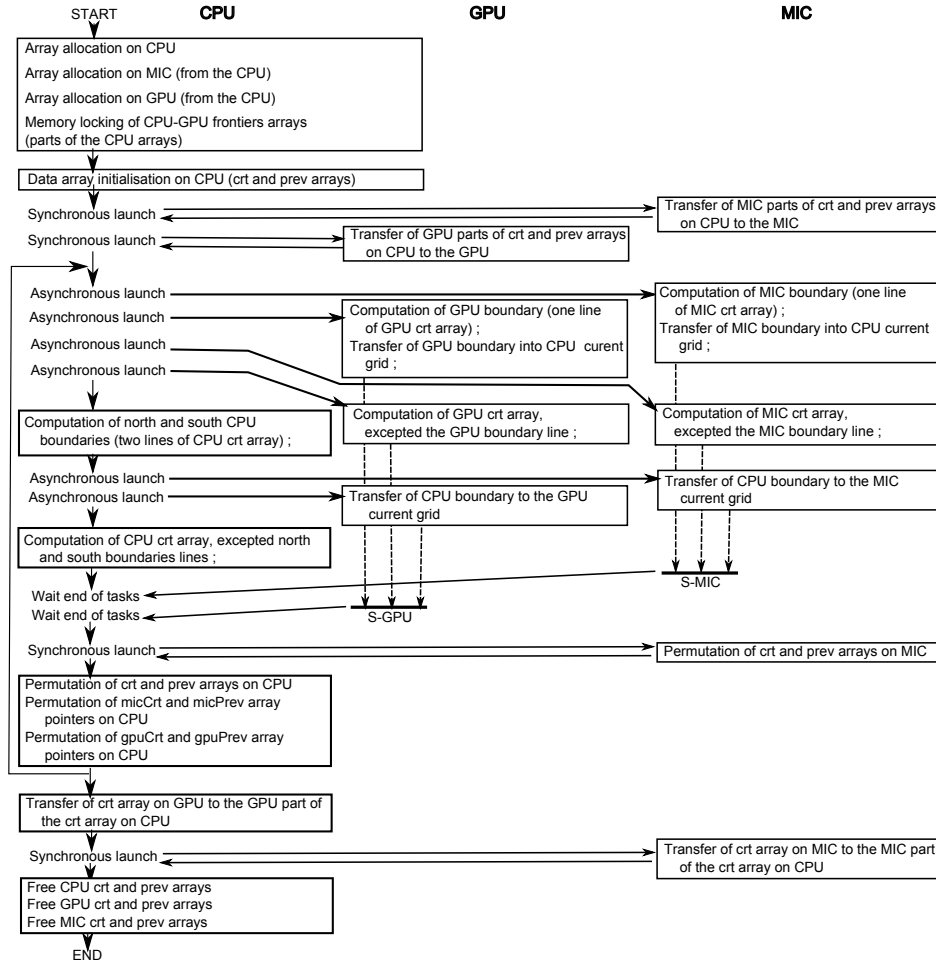


Figure 3. CPU-MIC-GPU algorithm

much as possible the data transfers with large computations, as well as to avoid that a processor is underused by processing only its boundary(ies).

- The CPU launch asynchronous computations on accelerators and asynchronous data transfers from and to the accelerators. So, the two accelerators and the CPU can compute in parallel, and the different data transfers can exploit the two PCI express buses in parallel.

Obviously, two synchronization points, S-MIC and S-GPU, are mandatory to ensure that data transfers and computations are finished respectively on MIC and GPU, before to switch the arrays (current and previous grids) and to enter the next iteration.

A slight asymmetry appears between the MIC and GPU concerning the arrays switching management (pointers switching). In fact, pointers on GPU arrays are stored in the CPU memory and sent to the GPU computing kernel as parameters when launching the kernel. So, these pointers can be switched directly in the CPU memory by the CPU process. On the contrary, array pointers on MIC are stored in the MIC memory and managed by the MIC. So, the CPU needs to launch a short task on the MIC to make it switch its local array pointers.



| 4000 × 4000 pts, 2000 iterations |               |            |            |            |
|----------------------------------|---------------|------------|------------|------------|
| Testbed                          | Measure       | CPU        | MIC        | GPU        |
| CentraleSupelec<br>machine       | Pts Updates/s | 1.28E + 09 | 2.81E + 09 | 9.16E + 09 |
|                                  | Speedup       | 1.0        | 2.20       | 7.15       |
| Loria<br>machine                 | Pts Updates/s | 1.62E + 09 | 4.23E + 09 | 7.84E + 09 |
|                                  | Speedup       | 1.0        | 2.61       | 4.84       |

**Table 1.** Absolute and relatives performances of the three devices

Finally, we obtain an efficient and rather generic and symmetric parallel scheme that make cooperate CPU, MIC and GPU devices to solve the same problem.

#### 4.2. Implementation details

To achieve asynchronous transfers between CPU and GPU, three CUDA streams are used together with two CUDA registrations of the memory banks concerned by the transfers to lock them and avoid their swapping. One stream is used to compute and send the *FG1* line (cf. Fig.2) to the CPU (*FC1*), another one is used to receive the *FG2* line from the CPU (*FC2*), and the last one is used to control the asynchronous computation of the GPU part. The two registrations concern the two frontier lines (*FG1* and *FG2*). The `cudaMemcpyAsync` and `cudaStreamSynchronize` functions are used to perform the asynchronous transfers and to ensure their completion before to proceed to the following computations.

Concerning the asynchronous data transfers between CPU and MIC, the `signal` clause is used in the `offload` directive computing and sending (with a `out` clause) the *FM2* line to the CPU (*FC4*). It is also used in the `offload_transfer` directive related to the reception of *FM1* from the CPU (*FC3*). There is also a signaled `offload` to asynchronously perform the computation of the MIC part. Then, the `offload_wait` directive is used to ensure the transfer completions before performing the following computations.

## 5. Experiments

### 5.1. Individual performances of the devices

Table 1 shows the absolute and relative performances of the three devices (computing units) of each testbed machine, during 2000 iterations on a grid of 4000 × 4000 points. In both cases the CPU cores of the motherboard is the less powerful, the MIC device is medium, and the GPU is the most powerful on this problem. But the MIC achieves higher absolute and relative performance (compared to the motherboard CPU) on the Loria testbed, while the GPU is faster on the CentraleSupelec (CS) testbed. So, our two testbed machines could have different behaviors when running multi-device codes.

### 5.2. Performance of heterogeneous computing on the three devices

Table 2 shows absolute performance (in point update/s) of an heterogeneous computing concurrently on the three devices of each machine (again during 2000 iterations on a grid of 4000 × 4000 points). The cutting lines specify the load balancing of the grid on the three devices: first result line of table 2 summarizes some experiments with lines 1 – 999

| 4000 × 4000 pts, 2000 iterations |                         |                         |                  |                  |                  |                    |
|----------------------------------|-------------------------|-------------------------|------------------|------------------|------------------|--------------------|
| Testbed                          | GPU/CPU<br>cutting line | CPU/MIC<br>cutting line | CPU<br>Pts Ups/s | MIC<br>Pts Ups/s | GPU<br>Pts Ups/s | CPU + MIC<br>+ GPU |
| CS<br>machine                    | 1000                    | 3000                    | 1.28E + 09       | 2.81E + 09       | 9.16E + 09       | 2.11E + 09         |
|                                  | 2000                    | 2500                    | 1.27E + 09       | 2.81E + 09       | 9.22E + 09       | 5.47E + 09         |
| Loria<br>machine                 | 1000                    | 3000                    | 1.62E + 09       | 4.23E + 09       | 7.84E + 09       | 2.68E + 09         |
|                                  | 2000                    | 2500                    | 1.71E + 09       | 4.12E + 09       | 7.81E + 09       | 5.32E + 09         |

**Table 2.** Absolute performance (points updates/s) of heterogeneous computing on CPU, MIC and GPU

| 4000 × 4000 pts, 2000 iterations |                         |                  |                  |                        |                                    |
|----------------------------------|-------------------------|------------------|------------------|------------------------|------------------------------------|
| Testbed                          | CPU/MIC<br>cutting line | CPU<br>Pts Ups/s | MIC<br>Pts Ups/s | CPU + MIC<br>Pts Ups/s | Speedup CPU+MIC<br>vs max(CPU,MIC) |
| CS machine                       | 700                     | 1.26E + 09       | 2.77E + 09       | 2.95E + 09             | 1.06                               |
|                                  | 750                     | 1.25E + 09       | 2.78E + 09       | 2.96E + 09             | 1.07                               |
|                                  | 850                     | 1.26E + 09       | 2.77E + 09       | 2.88E + 09             | 1.04                               |
|                                  | 950                     | 1.27E + 09       | 2.75E + 09       | 2.82E + 09             | 1.03                               |

**Table 3.** Absolute performance (points updates/s) of heterogeneous computing on CPU and MIC

processed on the GPU, lines 1000 – 2999 processed on the CPU, and lines 3000 – 3999 processed on the MIC.

As expected, we observe the sensitivity of the heterogeneous computing performance to the load balancing on the three devices. The last column introduces this performance that varies from 2.11 to 5.47 Giga Point Updates/s on the CS machine, and from 2.68 to 5.32 Giga Point Updates/s on the Loria machine, for two different load balancing. First load balancing is very bad: global performance is greater than pure CPU performance, but less than GPU and MIC ones on the two testbeds. Second load balancing is better: global performance are greater than CPU and MIC performances on the two testbeds. But they remain less than the GPU performance.

In fact, we experimented other load balancing, and we never succeeded to obtain greater performances than pure GPU executions. When using simultaneously several devices we add data transfers between CPU and GPU, and between CPU and MIC, at each time step. Despite our efforts to overlap communication with computation on the different devices (see section 4), the GPU are so powerful on this application, compared to others devices, that:

- when processing significant parts of the problem on the CPU or the MIC, we slow down the execution (compared to a pure GPU execution),
- when processing only small parts of the problem on the CPU or the MIC, to try to load balance the computation with the GPU, the computations times are low, and the data transfers can not be hidden and slow down the application.

According to these preliminary results, the simultaneous use of the three devices in each machine does not bring significant improvement in performance. However, further study should be done to look for contexts where the three-devices version is faster than the GPU alone.

### 5.3. Performance of heterogeneous computing on CPU and MIC

Table 3 shows that it is possible to run faster on both CPU and MIC, than on CPU or MIC alone, on the CS machine. We achieved a small speedup close to 1.07 with a cutting line at 750 (lines 1 – 749 processed on the CPU, and lines 750 – 3999 processed on the MIC). However, we never succeeded to achieve a speedup on the Loria machine. The MIC/CPU speed ratio was close to 2.15 on the CS machine, while it was close to 2.58 on the Loria machine. It seems possible to speedup using simultaneously CPU and MIC when their computing power are not so different, but it seems more difficult when their computing power ratio increases.

## 6. Conclusion

A parallel scheme has been described that allows the cooperation of different computing devices inside a single hybrid machine. The major difficulties in exploiting those devices together come from the data transfers between the central memory of the system and the local memory on each device. To obtain good efficiency, it is required to make extensive use of asynchronism between the different computing devices, especially by overlapping computations with communications thanks to asynchronous data transfers.

Our experiments have validated our multi-devices parallel scheme: results were qualitatively identical using one, two or three devices. Two series of asynchronous data transfers (CPU  $\leftrightarrow$  GPU and CPU  $\leftrightarrow$  MIC) have been implemented with different mechanisms. However, the global performance obtained when using simultaneously the CPU, the MIC and the GPU, are lower than when using only the most powerful device and computing kernel for this problem (the GPU alone and our optimized CUDA kernel). On some testbeds, we succeeded to achieve significant speedups using CPU and MIC devices compared to MIC or CPU alone, with a finely tuned load balancing. This points out the difficulty to efficiently balance load on hybrid heterogeneous systems.

The next steps in this work should be to adapt our parallel scheme to more complex scientific applications, to the coupling of different solvers running on different devices, and to larger problems that could not fit in the memory of a single device.

## References

- [1] S. Vialle and S. Contassot-Vivier. *Patterns for parallel programming on GPUs*, chapter Optimization methodology for Parallel Programming of Homogeneous or Hybrid Clusters. Saxe-Coburg Publications, 2014. ISBN: 978-1-874672-57-9.
- [2] S. Contassot-Vivier, S. Vialle, and J. Gustedt. *Designing Scientific Applications on GPUs*, chapter Development Methodologies for GPU and Cluster of GPUs. Chapman & Hall/CRC Numerical Analysis and Scientific Computing series. Chapman & Hall/CRC, 2013. ISBN 978-1-466571-64-8.
- [3] J. Fang, A. L. Varbanescu, B. Imbernon, J. M. Cecilia, and H. Perez-Sanchez. Parallel computation of non-bonded interactions in drug discovery: Nvidia GPUs vs. Intel Xeon Phi. In *2nd International Work-Conference on Bioinformatics and Biomedical Engineering (IWBBIO 2014)*, Granada, Spain, 2014.
- [4] B. Gaster, L. Howes, D. Kaeli, P. Mistry, and D. Schaa. *Heterogeneous Computing with OpenCL*. Morgan Kaufmann, 2nd edition, 2012. ISBN 9780124058941.
- [5] J. Jeffers and J. Reinders. *Intel Xeon Phi coprocessor high-performance programming*. Elsevier Waltham (Mass.), 2013. ISBN 978-0-12-410414-3.
- [6] J. Sanders and E. Kandrot. *CUDA by Example: An Introduction to General-Purpose GPU Programming*. Addison-Wesley Professional, 1st edition, 2010. ISBN-10 0131387685, ISBN-13 9780131387683.