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A. Pyrkin[1], R. Ortega[2], A. Bobtsov[1,3], S. Aranovskiy[1] and D. Gerasimov[1]

***Abstract*— A novel approach to the problem of partial state estimation of a class of nonlinear systems is proposed. In this paper we apply the new adaptive observer design technique to power converters. For the sake of ease of exposition we preferred to concentrate on the specific example of the Ćuk power converter. The main idea is to translate the state estimation problem into one of estimation of *constant, unknown parameters* related to the systems initial conditions. The proposed observer is shown to be applicable for the reconstruction of the state of power converters for further using in the full-state feedback controller. Comparison with Immersion and Invariance technique for the observer design is made via numerical examples.**

I. Introduction

The problem of regulating the output voltage of switched power converters has attracted the attention of many control researchers for several years now. Besides its practical relevance in modern power systems for satellites, non-civilian, industrial and consumer electronic applications, these systems are an interesting theoretical case study because they are switched devices whose averaged dynamics are described by a bilinear second order non-minimum phase system with saturated input and a highly uncertain parameters — the load resistance and the input voltage.

In this paper we apply the new adaptive observer design technique to power converters. For the sake of ease of exposition, instead of developing a—notationally cumbersome—general theory for a broader class of power converters, we preferred to concentrate on the specific example of the Ćuk power converter, depicted in Fig. 1.

We use a nonlinear controller proposed in [1]. In contrast to other schemes reported in the literature [2]–[5] this type of controllers do not rely on the systems inversion, hence they can directly regulate the output signal, which as is well-known is a non-minimum phase output. Furthermore, the control laws are simple static state feedback. See [6] for a list of references and [7] for a comparative experimental study for the Boost converter case.

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This article is supported by Government of Russian Federation (GOSZADANIE 2014/190 (project 2118), grant 074-U01) and the Ministry of Education and Science of Russian Federation (project 14.Z50.31.0031).

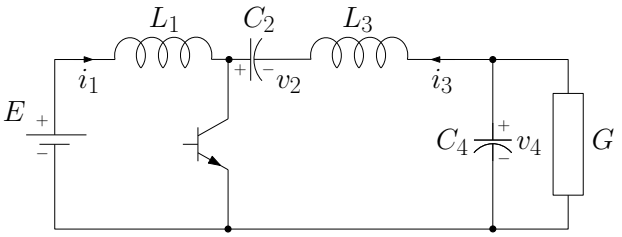

Fig. 1: DC–DC Ćuk converter circuit

The main problem is the adaptive observer design for Ćuk converter states that are currents and voltages. It is assumed that a part of states are measurable. The rest of state vector is to be observed and estimates should be substituted to the full-state controller. A new framework for constructing globally convergent (reduced-order) observers for a well–defined class of nonlinear systems is presented (see also [8], [9]). Instrumental to this development is to formulate the observer design problem as a problem of *parameter estimation*, which represents the *initial conditions* of the unknown part of the state. The class of averaged Ćuk converter model is suitable for the proposed observer design technique.

The remaining of the paper is organized as follows. Section II presents the problem formulation. Section III is devoted to a discussion of the main theoretical results. Section IV illustrates the application of the technique to Ćuk converter with two scenarios of work. The paper is wrapped–up with simulation results and concluding remarks in Section V.

II. Problem Statement

The Observer Design procedure starts by writing the averaged dynamic of the power converters in the following PCH form [10]

$$\dot{X} = [\mathcal{J}(u) - R]\frac{\partial H}{\partial X}(X) + gE, \tag{1}$$

where X is the state vector, consisting of charges and fluxes, E is the input voltage, u represents the duty ratio, $H(x)$ is the total energy and $\mathcal{J}(u)$, R, g represent the internal interconnection, damping and external interconnection matrices, respectively. PCH models result from the network modeling of energy conserving lumped-parameter physical systems with independent storage elements [11].

The Cuk DC-to-DC power converter shown in Fig. 1 can

be written in PCH form with

$$\mathcal{J} = \begin{bmatrix} 0 & -(1-u) & 0 & 0 \\ (1-u) & 0 & u & 0 \\ 0 & -u & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \tag{2}$$

$$\mathcal{R} = \text{diag}\{0,0,0,\frac{1}{R}\}, \quad g = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^\top, \tag{3}$$

and then the average model of this device is given by

$$L_1 \frac{di_1}{dt} = -(1-u)v_2 + E \tag{4}$$

$$C_2 \dot{v}_2 = (1-u)i_1 + ui_3 \tag{5}$$

$$L_3 \frac{di_3}{dt} = -uv_2 - v_4 \tag{6}$$

$$C_4 \dot{v}_4 = i_3 - Gv_4, \tag{7}$$

where L_1, C_2, L_3, C_4 are inductances and capacities, $G = \frac{1}{R}$ are positive constants. We refer the reader to [1] for further details on the model.

The destination of the Ćuk converter is to regulate the voltage across the load (i.e., the capacitor voltage v_4) to a constant value V_d. One can apply the control law proposed in [1], [10].

III. Observer Design

In this paper we consider the problem when a part of state variables are unmeasurable and are to be observed. Further a theoretical discussion about this issue follows.

Consider the dynamical system

$$\begin{aligned} \dot{x} &= f_x(x,y,u) \\ \dot{y} &= f_y(x,y,u), \end{aligned} \tag{8}$$

where x is an unmeasurable part of X, y are measurable states, $f_x : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \times \mathbb{R}^m \to \mathbb{R}^{n_x}$ and $f_y : \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \times \mathbb{R}^m \to \mathbb{R}^{n_y}$ are smooth mappings. Assume that the input signal vector $u : \mathbb{R}_+ \to \mathbb{R}^m$ is such that all trajectories of the system are bounded.

Find, if possible, mappings $F : \mathbb{R}^{n_\xi} \times \mathbb{R}^{n_y} \times \mathbb{R}^m \to \mathbb{R}^{n_\xi}$ and $G : \mathbb{R}^{n_\xi} \times \mathbb{R}^{n_y} \times \mathbb{R}^m \to \mathbb{R}^{n_x}$, for some positive integer n_ξ, such that the (partial state) observer

$$\begin{aligned} \dot{\xi} &= F(\xi,y,u) \\ \hat{x} &= G(\xi,y,u), \end{aligned} \tag{9}$$

ensures that ξ is bounded and

$$\lim_{t\to\infty} |\hat{x}(t) - x(t)| = 0, \tag{10}$$

for all initial conditions $(x(0), y(0), \xi(0)) \in \mathbb{R}^{n_x+n_y+n_\xi}$ and a well defined class of input signals $u \in \mathcal{U}$.

Assumption 1: There exists three mappings

$$\begin{aligned} \phi &: \mathbb{R}^{n_x} \times \mathbb{R}^{n_y} \to \mathbb{R}^{n_z} \\ \phi^{\text{L}} &: \mathbb{R}^{n_z} \times \mathbb{R}^{n_y} \to \mathbb{R}^{n_x} \\ h &: \mathbb{R}^{n_y} \times \mathbb{R}^m \to \mathbb{R}^{n_z}, \end{aligned}$$

with $n_z \geq n_x$, verifying the following conditions.

(i) (Left invertibility of $\phi(\cdot,\cdot)$ with respect to its first argument)

$$\phi^{\text{L}}(\phi(x,y),y) = x, \quad \forall x \in \mathbb{R}^{n_z},\ \forall y \in \mathbb{R}^{n_y}.$$

(ii) (Transformability into cascade form)

$$\frac{\partial \phi}{\partial x} f_x(x,y,u) + \frac{\partial \phi}{\partial y} f_y(x,y,u) = h(y,u). \tag{11}$$

An immediate corollary of (ii) in Assumption 1 is that the partial change of coordinates

$$z = \phi(x,y), \tag{12}$$

ensures

$$\dot{z} = h(y,u). \tag{13}$$

Moreover, the left invertibility condition (i) ensures that the partial state x can be recovered from z and y, that is,

$$x = \phi^{\text{L}}(z,y). \tag{14}$$

Define the dynamic extension

$$\dot{\chi} = h(y,u), \tag{15}$$

with $\chi(0) \in \mathbb{R}^{n_z}$.

From (13) and (15) we get $\dot{z} = \dot{\chi}$. Hence, integrating this equation yields

$$z(t) = \chi(t) + \theta, \tag{16}$$

where

$$\theta := z(0) - \chi(0). \tag{17}$$

Based on (14) and (16) we get the state observer

$$\hat{x} = \phi^{\text{L}}(\chi + \hat{\theta}, y), \tag{18}$$

where $\hat{\theta} : \mathbb{R}_+ \to \mathbb{R}^{n_z}$ is an *on–line estimate* of the vector θ.

Therefore, the problem of observation of the unmeasurable state x is translated into a *parameter estimation* problem for the regression model that is to be found later. To complete the observer design it is necessary to ensure the existence of a consistent estimator for the unknown parameter θ.

Proposition 1: Consider the system (8) *verifying Assumption 1 and the dynamic extension* (15)*. We can compute a mapping $\Phi : \mathbb{R}^{n_z} \times \mathbb{R}^{n_y} \times \mathbb{R}^m \times \mathbb{R}^{n_z} \to \mathbb{R}^{n_y}$ such that*

$$\dot{y} = \Phi(\chi,y,u,\theta) \tag{19}$$

$$x = \phi^{\text{L}}(\chi + \theta, y), \tag{20}$$

where $\theta \in \mathbb{R}^{n_z}$ is a vector of constant*, unknown parameters.*

The proof is given by replacing (16) in (14) which yields (20), and the regression model (19) is obtained replacing (20) in (8) to get $f_y(\phi^{\text{L}}(\chi+\theta,y),y,u) =: \Phi(\chi,y,u,\theta)$.

IV. Ćuk Converter Observer Design

To illustrate the generality of the approach we consider two different measurement scenarios. In the first one we assume that (v_2, i_3) are measurable, while in the second one (v_2, v_4) are measurable. Although from the practical viewpoint it is "easier" to measure voltages, we also consider the first one since, as shown in [1], is the one that can be solved with immersion and invariance (I&I) observers, with which we compare our observer in simulations below.

Case I Denoting $y := \text{col}(v_2, i_3),\ x := \text{col}(i_1, v_4)$ we get from (4) and (7)

$$\begin{aligned}\dot{x}_1 &= -\frac{1}{L_1}(1-u)y_1 + \frac{E}{L_1}\\ \dot{x}_2 &= \frac{1}{C_4}y_2 - \frac{G}{C_4}x_2.\end{aligned}$$

The right hand side of the second equation depends on x_2, therefore we propose the mapping

$$\phi(x,y) = x - \begin{bmatrix} 0 \\ \frac{GL_3}{C_4}y_2 \end{bmatrix},$$

that, introducing the partial change of coordinates $z = \phi(x,y)$, yields the required form

$$\dot{z} = \begin{bmatrix} -\frac{1}{L_1}(1-u)y_1 + \frac{E}{L_1} \\ \frac{1}{C_4}y_2 + \frac{G}{C_4}uy_1 \end{bmatrix} =: h(y,u).$$

The dynamic extension is then given by $\dot{\chi} = h(y,u)$, and the regression model is of the form

$$\begin{aligned}\dot{y} &= \Phi_0(\chi, y, u) + \Phi_1(u)\theta &(21)\\ \theta &:= x(0) - \chi(0) - \begin{bmatrix} 0 \\ \frac{GL_3}{C_4}y_2(0) \end{bmatrix}, &(22)\end{aligned}$$

where

$$\Phi_0(\chi,y,u) := \begin{bmatrix} \frac{1}{C_2}(1-u)\chi_1 + \frac{1}{C_2}uy_2 \\ -\frac{1}{L_3}uy_1 - \frac{1}{L_3}\chi_2 - \frac{GL_3}{C_4}y_2 \end{bmatrix}, \quad (23)$$

$$\Phi_1(u) := \begin{bmatrix} \frac{1}{C_2}(1-u) & 0 \\ 0 & -\frac{1}{L_3} \end{bmatrix}. \quad (24)$$

The model (21) contains the time derivative of the output y. To get a classical (static) regression model we use the standard filtering technique [12] and define the filtered signals

$$\overline{(\cdot)} := \frac{\alpha}{p+\alpha}(\cdot),$$

where $p := \frac{d}{dt}$ and $\alpha > 0$ is a design parameter. Applying the filter to (21) we obtain the standard linear, static regression model

$$\vartheta = \overline{\Phi}_1\theta + \epsilon \quad (25)$$

where

$$\vartheta := \frac{\alpha p}{p+\alpha}y - \frac{\alpha}{p+\alpha}\Phi_0$$

is clearly measurable (without differentiation) and ϵ is an exponentially decaying signal that depends on the filter initial conditions and the filter time constant $\frac{1}{\alpha}$.

The regression model (25) is used for the parameter estimator, which is the classical gradient estimator

$$\dot{\hat{\theta}} = \Gamma\overline{\Phi}_1^\top(\vartheta - \overline{\Phi}_1\hat{\theta}),$$

where the adaptation gain $\Gamma = \Gamma^\top > 0$ is a design parameter.

The state observer takes the form

$$\hat{x} = \hat{\theta} + \chi + \begin{bmatrix} 0 \\ \frac{GL_3}{C_4}y_2 \end{bmatrix}.$$

The regressor matrix $\Phi_1(u)$ given in (23) has a very simple form. Indeed, the matrix is diagonal with the second term in the regression simply the constant $\frac{-1}{L_3}$. Clearly, for this case we also have $\mathcal{U} = \{u : \mathbb{R}_+ \to (0,1)\}$ and consistent estimation is always guaranteed.

In Proposition 8.3 of [1] the following I&I observer is proposed

$$\begin{aligned}\hat{x}_{I\&I} &= \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix} + \begin{bmatrix} C_2\gamma_1 y_1 \\ L_3\gamma_2 y_2 \end{bmatrix}\\ \dot{\hat{\zeta}}_1 &= \frac{1}{L_1}[-(1-u)y_1 + E]\\ &\quad -\gamma_1[(1-u)(\hat{\zeta}_1 + C_2\gamma_1 y_1) + uy_2] &(26)\\ \dot{\hat{\zeta}}_2 &= \frac{1}{C_4}[y_2 - G(\hat{\zeta}_2 - L_3\gamma_2 y_2)]\\ &\quad -\gamma_2[uy_1 + \hat{\zeta}_2 - L_3\gamma_2 y_2], &(27)\end{aligned}$$

where $\gamma_1, \gamma_2 > 0$ are design parameters. It should be noted that in the latter reference the parameters E and G are treated as unknown and are also estimated. If they are assumed known the I&I observer takes the form given above.

Simulations were carried out to evaluate the performance of the proposed observer. The simulations were done for the model (4)–(7) in closed–loop with the *certainty equivalent* version of the full–state feedback I&I controller given in Proposition 8.2 of [1]. That is, the control law was defined by

$$u = \frac{|V_d|}{|V_d| + E} + \lambda\frac{G|V_d|v_2 + E(i_3 - \hat{x}_1)}{1 + (G|V_d|v_2 + E(i_3 - \hat{x}_1))^2} \quad (28)$$

where $V_d < 0$ is the *reference* imposed to the output voltage v_4 and λ is chosen as

$$\lambda = \lambda_0\ \min\left(\frac{|V_d|}{|V_d| + E}, \frac{E}{|V_d| + E}\right),$$

with $0 < \lambda_0 < 2$. The full-state version of this controller, *i.e.*, replacing $\hat{x}_1$ by i_1, respectively, ensures global asymptotic stability of the desired equilibrium as well as verification of the saturation constraints in the input signal.

The performance of our observer was compared with the I&I observer (27) via numerical simulations which were performed with the following values of the converter parameters $L_1 = 10$ mH, $C_2 = 22.0$ μF, $L_3 = 10$ mH and $C_4 = 22.9$ μF, $G = 0.0447$ S and $E = 12$ V. The initial conditions for all simulations are set to $x(0) = (1, -2)$, $y(0) = (4, -2)$.

The initial set point for the output voltage is $V_d = -5$ V, and then this is changed at $t = 0.2$ s to $V_d = -40$ V, at

$t = 0.4$ s to $V_d = -10$ V, at $t = 0.6$ s to $V_d = -25$ V, at $t = 0.8$ s to $V_d = -15$ V. The simulation results are presented in Figs. 2, 3, 4 with different observer gains and the control law (28) with $\lambda = 0.5$.

Case II Denoting now $y := \text{col}(v_2, v_4),\ x := \text{col}(i_1, i_3)$ we get from (4), (6)

$$\dot{x}_1 = -\frac{1}{L_1}(1-u)y_1 + \frac{E}{L_1}$$
$$\dot{x}_2 = -\frac{1}{L_3}uy_1 - \frac{1}{L_3}y_2.$$

Since the right hand side of these equations is independent of x we can directly select

$$\phi(x, y) = x.$$

The dynamic extension is given by

$$\dot{\chi} = \begin{bmatrix} -\frac{1}{L_1}(1-u)y_1 + \frac{E}{L_1} \\ -\frac{1}{L_3}uy_1 - \frac{1}{L_3}y_2 \end{bmatrix} =: h(y, u),$$

and the regression form is

$$\begin{aligned} \dot{y} &= \Phi_0(\chi, y, u) + \Phi_1(u)\theta \\ \theta &:= x(0) - \chi(0) \end{aligned} \tag{29}$$

where

$$\Phi_0(\chi, y, u) := \begin{bmatrix} \frac{1}{C_2}(1-u)\chi_1 + \frac{1}{C_2}u\chi_2 \\ \frac{1}{C_4}\chi_2 - \frac{G}{C_4}y_2 \end{bmatrix}, \tag{30}$$

$$\Phi_1(u) := \begin{bmatrix} \frac{1}{C_2}(1-u) & \frac{1}{C_2}u \\ 0 & \frac{1}{C_4} \end{bmatrix}. \tag{31}$$

The state observer is defined as $\hat{x} = \hat{\theta} + \chi$, where the parameter estimator has the same form as in Case I.

It is important to underscore that the regressor matrix $\Phi_1(u)$ given in (30) has an extremely simple form. Indeed, due to its upper triangular form, the estimation of the second parameter is decoupled from the first one and, moreover, the corresponding term in the regression is simply the constant $\frac{1}{C_4}$. Also, since the matrix depends only on the input signal u the set $\mathcal{U}$ is defined as

$$\mathcal{U} := \{u : \mathbb{R}_+ \to (0,1) \mid \int_t^{t+T} \begin{bmatrix} 1-u(s) & (1-u(s))u(s) \\ (1-u(s))u(s) & u^2(s) + \frac{C_2^2}{C_4^2} \end{bmatrix} ds \geq \delta > 0\}.$$

Some simple calculations show that the matrix inside the integral is *positive definite* for any $u \in (0,1)$. Hence, $\mathcal{U} = \{u : \mathbb{R}_+ \to (0,1)\}$ and consistent estimation is always guaranteed.

The performance of our observer was illustrated via numerical simulations. They were done under the same scenario as the ones done for Case I, but now with the certainty equivalent observer that results replacing i_1 and i_3 by $\hat{x}_1$ and $\hat{x}_2$, respectively. The controller is given by

$$u = \frac{|V_d|}{|V_d| + E} + \lambda \frac{G|V_d|v_2 + E(\hat{x}_2 - \hat{x}_1)}{1 + (G|V_d|v_2 + E(\hat{x}_2 - \hat{x}_1))^2} \tag{32}$$

The simulation results are presented in Figs. 5.

V. Conclusions

A radically new approach to design state observers for nonlinear systems has been proposed. The key idea is to translate the state observation problem into one of parameter estimation.

The proposed technique has been shown to be applicable to position estimation of a class of power converters and application for Ćuk converter is presented. We expect to identify other classes of physical systems to which the proposed method is applicable in future.

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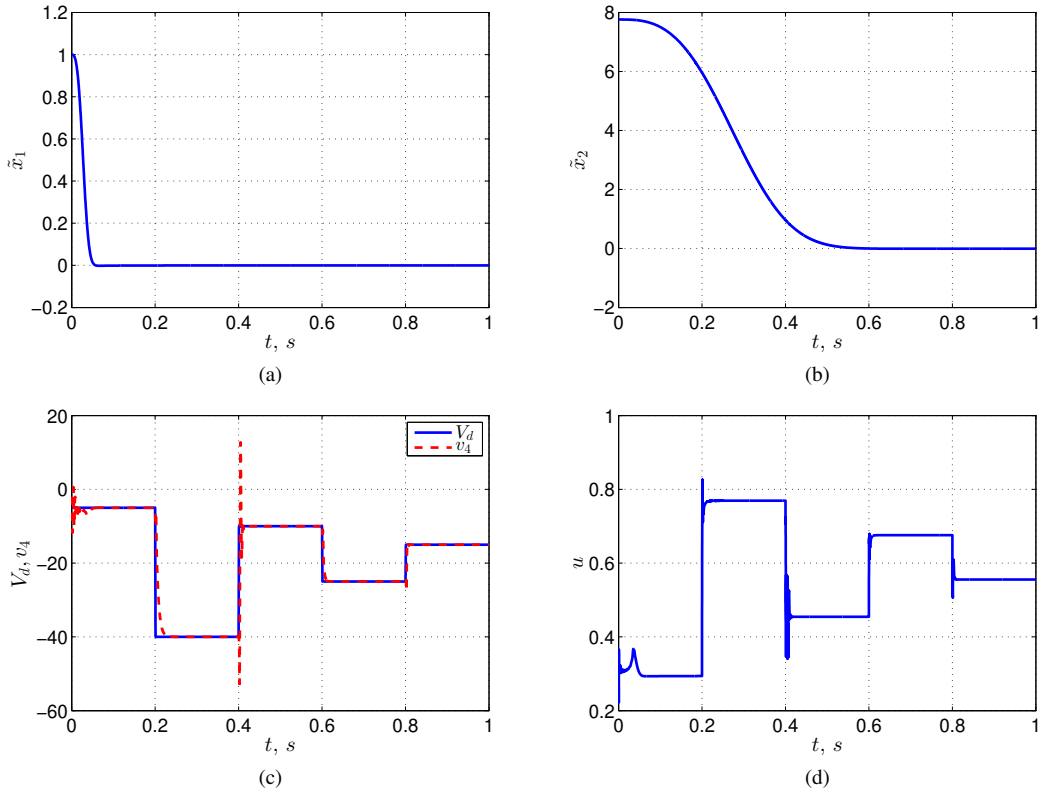

Fig. 2: Transients of the observation errors (a) $\tilde{x}_1 := \hat{x}_1 - i_1$, (b) $\tilde{x}_2 := \hat{x}_2 - v_4$, (c) the voltage reference V_d and voltage output v_4 and (d) the control input u for the proposed observer with tuning gains $\alpha = 0.1$, $\Gamma = \text{diag}\{0.01, 1\}$.

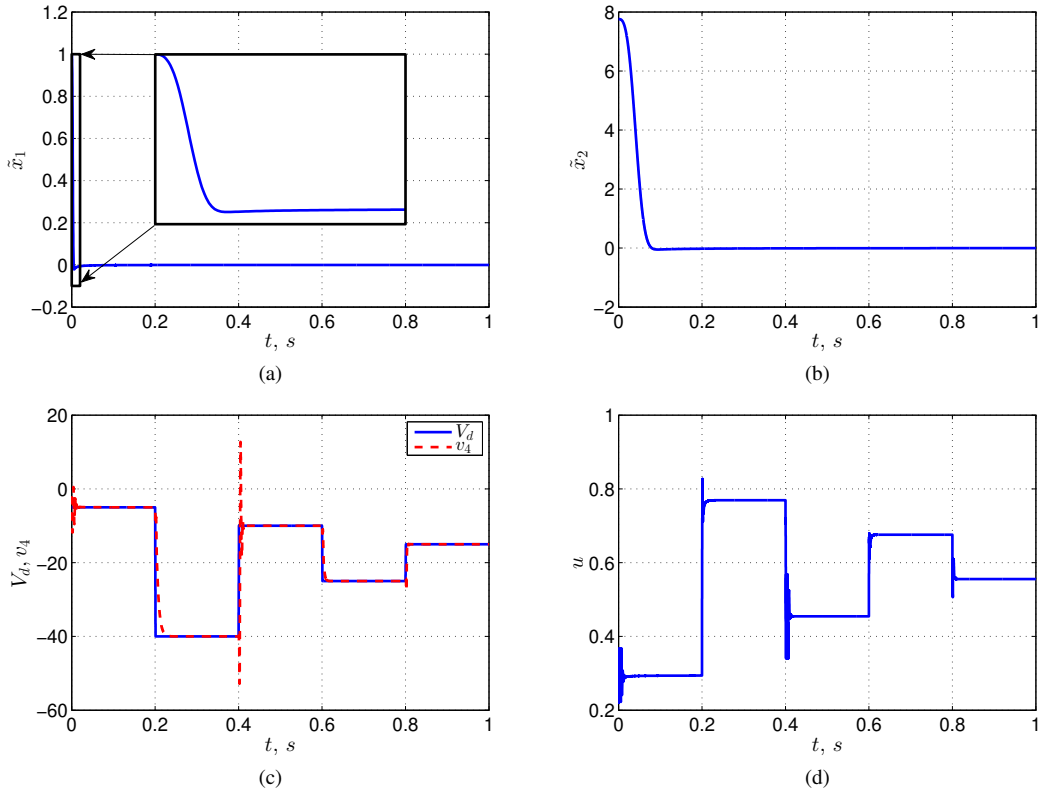

Fig. 3: Transients of the observation errors (a) $\tilde{x}_1 := \hat{x}_1 - i_1$, (b) $\tilde{x}_2 := \hat{x}_2 - v_4$, (c) the voltage reference V_d and voltage output v_4 and (d) the control input u for the proposed observer with tuning gains $\alpha = 1$, $\Gamma = \text{diag}\{0.1, 3\}$.

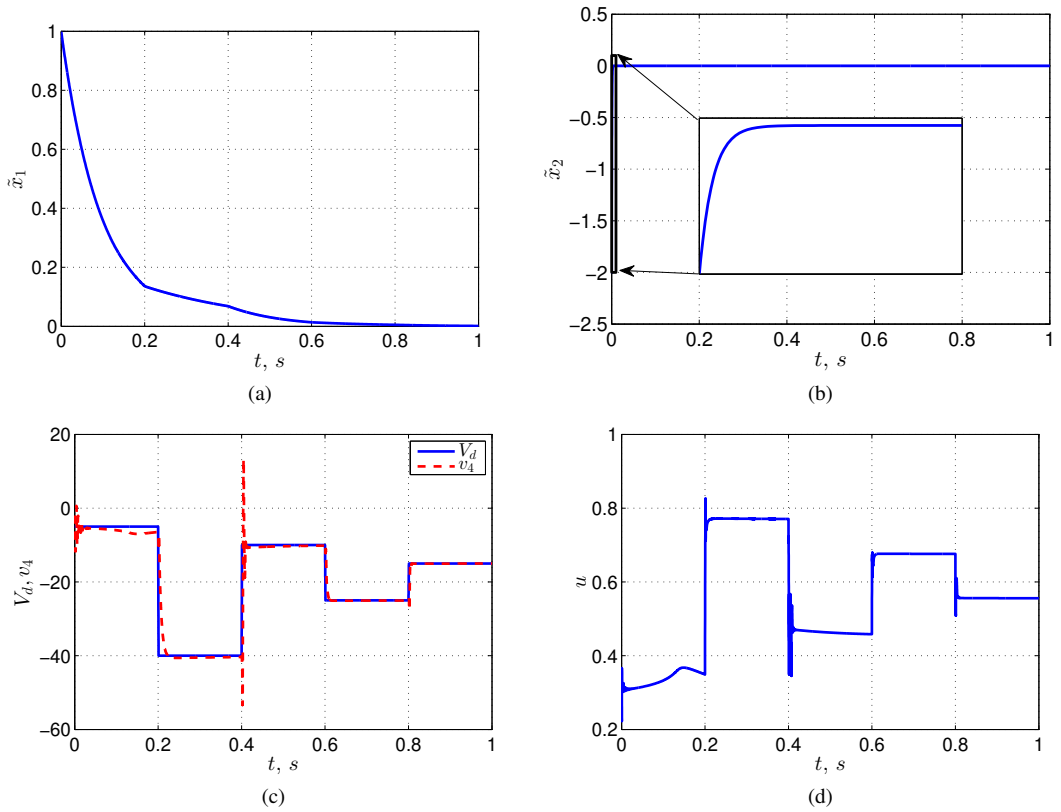

Fig. 4: Transients of the observation errors (a) $\tilde{x}_1$, (b) $\tilde{x}_2$, (c) the voltage reference V_d and voltage output v_4 and (d) the control input u for the I&I observer (27) with tuning gains $\gamma_1 = 15$, $\gamma_2 = 2$.

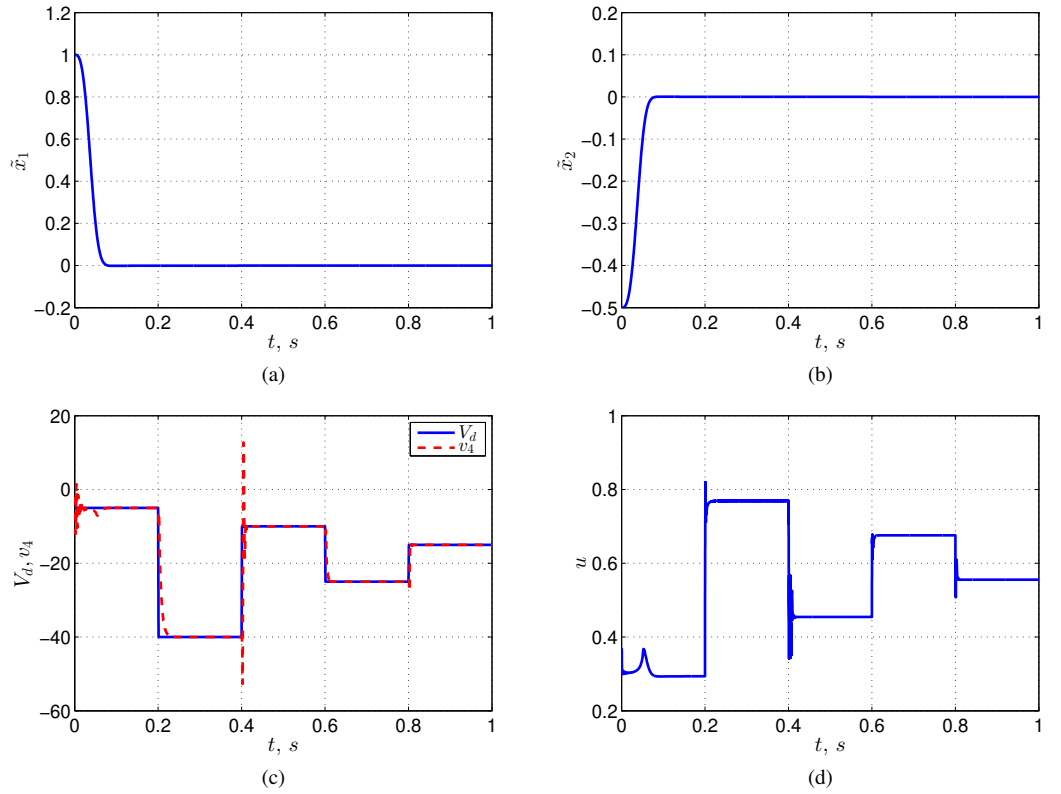

Fig. 5: Transients of the observation errors (a) $\tilde{x}_1 := \hat{x}_1 - i_1$, (b) $\tilde{x}_2 := \hat{x}_2 - i_3$, (c) the voltage reference V_d and voltage output v_4 and (d) the control input u for the tuning gains $\alpha = 0.5$, $\Gamma = 0.001 I_2$.