Design and analysis of multi-level numerical experiments, with application to fire safety
Rémi Stroh, Julien Bect, Séverine Demeyer, Nicolas Fischer, Emmanuel Vazquez

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Abstract

To assess the conformity of a building in case of fire, fire engineers use numerical simulations. A popular software for fire simulations is Fire Dynamics Simulator (FDS). It is based on a finite difference method that takes into account the random behavior of the fire. Thus, the response of FDS is stochastic. The mesh size used in the numerical scheme can be chosen by the user. When the mesh size decreases, the accuracy and the computation time of simulations increase. At low accuracy, one simulation takes a few minutes to run, whereas it can be several weeks at high accuracy.

We consider the problem of estimating the behavior of fine-mesh simulations (high-fidelity) using a combination of coarse-mesh simulations (low-fidelity). This approach is called multi-fidelity, we propose to extend the Bayesian multi-fidelity models proposed by Picheny and Ginsbourger (2013) and Tuo et al. (2014) to the case of stochastic simulators.

Proposed model

Data:
• inputs: $(x_i, t_i) \in (\mathbb{X} \times \mathbb{T}) \subset (\mathbb{R}^d \times \mathbb{R}^e)$, where $t$ stands for the mesh size;
• outputs $(z_i) \in \mathbb{R}$.

Likelihood:
stochastic code + independent observations:

\[
(z_i)_{i \in \mathbb{N}} \sim \mathcal{N}(\xi(x_i), t_i); \quad \text{diag}(\Lambda(x_i, t_i)).
\]  

Prior:
1. $\xi$ is a Gaussian process:

\[
\xi(x, t) \sim \mathcal{GP}(m(x, t); k(x, t), (x_i, t_i^2));
\]

2. $\xi$ converges when $t$ tends to 0:

\[
\xi_0(x) = \lim_{t \to 0} \xi(x, t).
\]

Models are validated by comparing:
• predictions (posterior mean) with observations,
• distributions of normalized residual with the standard normal distribution.

Probability to exceed a threshold

Suppose $P_x$ a probability distribution on inputs.

Curves of posterior distributions: 1000 conditional simulations $\times$ 5000 points along $P_x$.

By comparison with H-F[100] posterior density:
• H-F[10] and M-F$_2$ have small variance, but their distributions do not agree with the posterior distribution of H-F[100];
• M-F$_1$ has a larger variance, but its posterior density maximum is in the posterior distribution of H-F[100];
• M-F$_2$ provides a better quantification of uncertainty.

Conclusion

• Contribution

A Bayesian model for multi-fidelity stochastic simulators has been proposed.
• Our model has been shown to provide, in a numerical experiment with FDS, a good quantification of uncertainty on predictions.
• Future work

• fully Bayesian inference for hyper-parameters,
• sequential design of experiments.

References