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To cite this version:
Rémi Stroh, Julien Bect, Séverine Demeyer, Nicolas Fischer, Emmanuel Vazquez. Design and analysis of multi-level numerical experiments, with application to fire safety. Journées annuelles du GdR MASCOT NUM (MASCOT NUM 2016), Mar 2016, Toulouse, France. hal-01331171

HAL Id: hal-01331171
https://centralesupelec.hal.science/hal-01331171
Submitted on 13 Jun 2016

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Design and analysis of multi-level numerical experiments, with application to fire safety

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Abstract
To assess the conformity of a building in case of fire, fire engineers use numerical simulations. A popular software for fire simulations is Fire Dynamics Simulator (FDS). It is based on a finite difference method that takes into account the random behavior of the fire. Thus, the response of FDS is stochastic. The mesh size used in the numerical scheme can be chosen by the user. When the mesh size decreases, the accuracy and the computation time of simulations increase. At low accuracy, one simulation takes a few minutes to run, whereas it can be several weeks at high accuracy. We consider the problem of estimating the behavior of fine-mesh simulations (high-fidelity), using a combination of low- and coarse-mesh simulations (low-fidelity). This approach is called multi-fidelity. We propose to extend the Bayesian multi-fidelity models proposed by Picheny and Ginsbourger [2013] and Tuo et al. [2014] to the case of stochastic simulators.

Fire Dynamics Simulator

FDS has two main characteristics:
• finite difference methods ⇒ mesh size can be tuned;
• random behavior of fire ⇒ stochastic simulator.

Proposed model

Data:
• inputs: (x, t) ∈ (X × T) ↷ (R^d × R^t), where t stands for the mesh size;
• outputs: (z) ∈ R.

Likelihood:
stochastic code + independent observations:

\( (z) \sim \mathcal{N}(\xi(x, t), \delta(x)) \)\n
Prior:
1. ξ is a Gaussian process:

\( \xi(x, t) \sim \mathcal{GP}(m(x, t); k(x, t), \mathcal{M}(x, t)) \)\n
2. ξ converges when t tends to 0:

\( \xi(x) = \lim_{t \to 0} \xi(x, t) \)\n
Models are validated by comparing:
• predictions (posterior mean) with observations,
• distributions of normalized residual with the standard normal distribution.

Quality of prediction:
• H-F[10] has bad predictions;
• M-F1 and M-F2 give similar quality of predictions;
• H-F[100] is the best, but its design is 11 times more costly.

Probability to exceed a threshold
Suppose Pz a probability distribution on inputs.

Numerical experiments

One numerical experiment on FDS:
• d = 8 inputs + the tuning parameter;
• 1 output: maximal temperature at 1.8 m.

To check efficiency of our model, 4 models are compared:
• M-F1: our model (see above);
• M-F2: same as M-F1, but instead of assumptions 3, 4, and 5, covariance k is a stationary Matérn covariance on X × T;
• H-F[10]: a high-fidelity model. Constant mean, Matérn covariance on X, homogeneous noise;
• H-F[100]: same as H-F[10], but with more points. This model serves us as reference.

The following designs are used:

<table>
<thead>
<tr>
<th>Model</th>
<th>Cost</th>
<th>Design</th>
<th>Observations</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-F1</td>
<td>1 2 5</td>
<td>10000</td>
<td>100</td>
</tr>
<tr>
<td>H-F[10]</td>
<td>1 2 5</td>
<td>1000</td>
<td>40</td>
</tr>
<tr>
<td>H-F[100]</td>
<td>1 2 5</td>
<td>10000</td>
<td>300</td>
</tr>
</tbody>
</table>

Conclusion

• Contribution
  A Bayesian model for multi-fidelity stochastic simulators has been proposed.
• Our model has been shown to provide, in a numerical experiment with FDS, a good quantification of uncertainty on predictions.
• Future work
  • fully Bayesian inference for hyper-parameters;
  • sequential design of experiments.

References