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Nonlinear imaging of 3D defect in anisotropic laminate using joint sparsity constraints

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Abstract

In the following the reconstruction of 3D defect affecting an anisotropic laminate is delt with. The nonlinear problem is solved using a two step procedures in which the equivalent current are first search for by enforcing a joint sparsity prior. Once the equivalent currents known, the total field is computed via the resolution of the state integral equation and the contrast is estimated. Preliminary results will be presented in a microwave regime.

1 Introduction

From eddy currents to test graphite-based materials to microwaves and beyond to test glass-based composite structures, one aims to obtain images of the possibly damaged parts with robust, fast inversion algorithms. In this contribution, such algorithms are tailored to detect small inclusions affecting the structures mentioned above. These inclusions may be voids, fluid-filled cavities (isotropic) or uniaxial ones. The inverse problem being known to be nonlinear and ill-posed sparsity promoting regularization has become an interesting alternative to overcome the non-uniqueness and/or numerical instability of the inversion process [1, 2]. In [3] a Bayesian Compressive Sensing solver is used whereas, in this work, an algorithm exploiting a joint sparsity regularization is proposed.

2 Formulation of the direct problem

The time-harmonic dependence $\mathrm{e}^{-\mathrm{i}\omega t}$ is assumed and dropped, all the calculations being carried at a single angular frequency ω. Let us consider a multi-layer structure, each n layer is a non-magnetic ($\mu = \mu_0$) homogeneous uniaxial medium and characterized by its diagonal complex permittivity tensor in the local coordinate system (material frame)

$$\bar{\bar{\epsilon}}_e^{(n)} = \begin{bmatrix} \epsilon_{11}^{(n)} & 0 & 0 \\ 0 & \epsilon_{22}^{(n)} & 0 \\ 0 & 0 & \epsilon_{22}^{(n)} \end{bmatrix} \qquad \bar{\bar{\Xi}}(\theta_n) = \begin{bmatrix} \cos\theta_n & \sin\theta_n & 0 \\ -\sin\theta_n & \cos\theta_n & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{1}$$

one is able to carry the local coordinate system to the global one via the rotation matrix $\bar{\bar{\Xi}}(\theta_n)$ where θ_n is the Euler rotation angle. The electromagnetic response of such a structure is computed as (theoretical and numerical details can be found in [4])

$$\mathbf{E}_j^{\mathrm{sca}}(\mathbf{r}) = \mathrm{i}\omega\mu_0 \int_V \bar{\bar{G}}^{\mathrm{ee}}(\mathbf{r},\mathbf{r}') \cdot \bar{\bar{\chi}}(\mathbf{r}') \cdot \mathbf{E}_j^{\mathrm{tot}}(\mathbf{r}')\,\mathrm{d}\mathbf{r}', \text{ with } \bar{\bar{\chi}}(\mathbf{r}) = -\mathrm{i}\omega\epsilon_0 \bar{\bar{\Xi}}^{-1}(\theta_n) \cdot \left(\bar{\bar{\epsilon}}_i - \bar{\bar{\epsilon}}_e^{(n)}\right) \cdot \bar{\bar{\Xi}}(\theta_n) \tag{2}$$

$$\bar{\bar{\chi}}(\mathbf{r}) \cdot \mathbf{E}_j^{\text{inc}}(\mathbf{r}) = \frac{\mathbf{J}_j(\mathbf{r})}{-i\omega\epsilon_0} - \bar{\bar{\chi}}(\mathbf{r}) \cdot i\omega\mu_0 \int_V \bar{\bar{G}}^{\text{ee}}(\mathbf{r}, \mathbf{r}') \cdot \mathbf{J}_j(\mathbf{r}')\, \mathrm{d}\,\mathbf{r}' \tag{3}$$

where $\bar{\bar{G}}^{\text{ee}}(\mathbf{r}, \mathbf{r}')$ is the electric-electric dyadic Green's function, $\bar{\bar{\epsilon}}_i$ and volume V the permittivity tensor affecting, the background medium, $\mathbf{E}_j^{\text{sca}}(\mathbf{r})$, $\mathbf{E}_j^{\text{tot}}(\mathbf{r})$ and $\mathbf{E}_j^{\text{inc}}(\mathbf{r})$ the scattered, total and incident field a due to the jth source.

3 Two-Step Inversion Procedure

Instead of solving directly the nonlinear inverse problem a two step method which leads to deal with two linear minimization problems solved by exploiting the jointly-sparse aspect of the sought equivalent currents solution of the first step and a classical l^2-minimization of a linear problem for the second step is proposed.

First step: reconstruction of the equivalent currents The following optimization problem is solved

$$\mathbf{J}_i^\star = \underset{\mathbf{J}_i}{\text{argmin}} \left[\frac{1}{2} |\zeta_i - \mathbf{G}\mathbf{J}_i|_2 \right] \quad i = 1, \cdots, N_s. \tag{4}$$

where ζ_i is a vector of size N_r which gathered the signal due to the source #i measured by the N_r receivers. The main idea is that $\mathbf{J}_i$ and χ share the same support, (4) can then be recast as a joint sparsity minimization problem via a (weighted) $l_{2,1}$-regularization as

$$\min_{\mathbf{J}} \|\mathbf{J}\|_{2,1} := \sum_{i=1}^{N} \|\mathbf{J}^i\|_2 \quad \text{s. t.} \quad \mathbf{G}\mathbf{J}_i = \zeta_i, \qquad i = 1, \cdots, N_s \tag{5}$$

where $\mathbf{J}^i$ denotes the i-th row of $\mathbf{J}$ [5].

Second step: reconstruction of the contrast function Once $\mathbf{J}_i$ is known, the contrast function is obtained by solving the following minimization problem [6]

$$\chi^\star = \text{argmin} \sum_{i=1}^{N_s} \|\mathbf{J}_i - \bar{\bar{\chi}} \cdot \mathbf{E}_i\|_2 \tag{6}$$

4 Conclusion

The preliminary results obtained using this two steps joint sparsity algorithm already shows some good results when the obstacle is not too large and/or its contrast not too high. If one of the two conditions is not fulfilled the approach failed due to the hight nonlinearity of the problem.

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