New insights into the statistical properties of M-estimators and application to signal detection

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Keywords. *M-estimators; Complex Elliptical Symmetric distributions; Robust estimation; Wishart distribution; Signal Detection.*

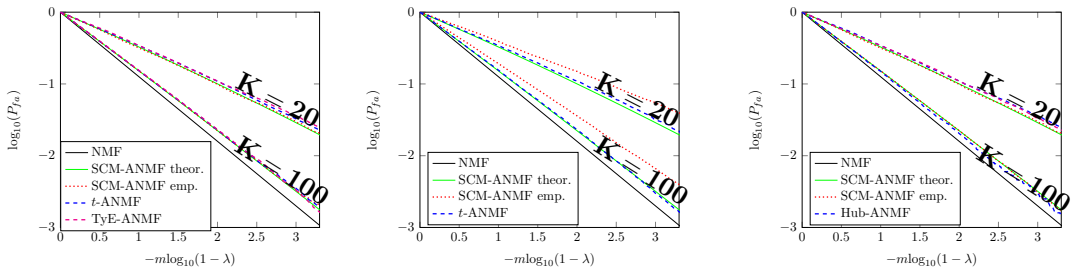
1 General context

In signal processing applications, the knowledge of scatter matrix is of crucial importance. It arises in diverse applications such as filtering, detection, estimation or classification. Generally, data can be locally modelled by a multivariate zero-mean circular Gaussian stochastic process, which is completely determined by its covariance matrix. In that case, the classical covariance matrix estimator is the sample covariance matrix (SCM), that is Wishart-distributed. Nevertheless, the complex normality sometimes presents a poor approximation of underlying physics. Noise and interference can be spiky and impulsive. An alternative is to use elliptical distributions, namely the Complex Elliptically Symmetric (CES) distributions [Ollila et al., 2012]. In this context, SCM performance can be very degraded and robust estimators, namely M-estimators [D. E. .Tyler , 1982, R. A. Maronna , 1976] are generally used. These estimators are defined by fixed-point equations making their statistical analysis very difficult. Thus, so far, they have been analyzed only in a standard asymptotic regime. In this work, we present a new approach to analyze the statistical properties of these estimators.

2 Main results and application to detection

In this paper, we propose to study a sort of distance between M-estimators and the SCM in order to propagate SCM non-asymptotic properties towards M-estimators. A new "Gaussian-cores" representation is introduced to better explained the relation between observations used to build an M-estimators and "fictive" data used

for theoretical construction of the SCM. The correlation between these two estimators is derived and the obtained results give new information on the "proximity" between M-estimators and the Gaussian-based SCM. More precisely, it is shown that the second order statistics of M-estimators when centering around a Wishart distributed matrix are much smaller than the ones when centering around the true scatter matrix. It is also revealed that this difference is even more meaningful for high-dimensional data, in agreement with [R. Couillet et al. , 2015] where large random matrix theory is used. Such results are of great interest for signal detection problems [S. Kraut et al. , 2001]. We analyzed the properties of the adaptive normalized matched filter (ANMF). The distribution of this detector and the theoretical relationship between the detection threshold and the Probability of False Alarm are known only for the test built with true scatter matrix (NMF) and for the one built with the SCM in the Gaussian framework (SCM-ANMF). Our results justify the use of M-estimators in practice as well as the approximation of the properties with ones based on the SCM-ANMF. Figure 2 shows the PFA-threshold relationships for the empirical M-ANMF built with Tyler's M-estimator, Student M-estimator and Huber's M-estimator compared to the theoretical SCM-ANMF and theoretical NMF. One can notice the good match of the empirical distributions of M-ANMFs with the theoretical distribution of the SCM-ANMF. Moreover, this shows that even for a small K the behavior of M-estimators is better approximated with the one of the SCM (green line) than with the true scatter (covariance) matrix (dark line).



(a) Gaussian distribution (b) Student t-distribution (c) Gaussian distribution

References

Ollila, E. , Tyler, D. E., Koivunen, V. & Poor, H. V. (2012).Complex Elliptically Symmetric Distributions: Survey, New Results and Applications. *IEEE Transactions on Signal Processing*, **60**, 5597-5625.

Tyler, D. E. (1982). Radial estimates and the test for sphericity. *Biometrika*.

Maronna, R. A.(1976). Robust M-Estimators of Multivariate Location and Scatter. *Annals of Statistics*, **17**, 51-67.

Kraut, S. and Scharf, L. L and McWhorter, L. T. (2001). Adaptive subspace detectors. *IEEE Transactions on Signal Processing*, **16**, 1-16.

Couillet, R. and Pascal, F. and Silverstein, J. W. (2015). The Random Matrix Regime of Maronna's M-estimator with elliptically distributed samples. *Journal of Multivariate Analysis*, **139**, 56-78.