Observer Design for Triangular Nonlinear Systems Using Delayed Sampled-Output Measurements

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Abstract— **The problem of observer design is addressed for a class of triangular nonlinear systems in presence of output measurement sampling and time-delay. A major difficulty with the considered nonlinear systems is that the state matrix is dependent on the "undelayed output signal" which is not accessible to measurement, making existing observers inapplicable. A new observer is designed where the effects of time-delay and sampling are compensated for using an output predictor. Defined by a couple of first-order ordinary differential equations (ODEs), the present predictor turns out to be much simpler compared to previous predictors involving output and state predictors. Using the small gain technique, sufficient conditions for the observer to be exponentially convergent are established in terms of the maximum time-delay and sampling interval.**

I. Introduction

Time-delay (or dead-time) is a natural phenomenon in most physical systems, see several examples in the books [9], [16], [17]. But, control components such as sensors and actuators, may also introduce additional delay in control systems. In this respect, the penetration of digital technology during the three last decades has led to networked control systems where time-delay is inevitable due to communication constraints [11]. It is well established that the negligence of time-delays in control design may cause the instability of control systems. For this reason an intensive research activity has been devoted to control design of delayed systems, leading to dozens of publications especially over the last fifteen years, see the recent book [13] and references therein. In output feedback control systems, involving online state estimation, the system time-delay must be accounted for in the underlying observer design. Although time-delays are fundamentally distributed parameter nature, observer design in presence of these elements has often been dealt with using finite-dimensional design tools. Accordingly, one starts with exponentially convergent state observers of ODEs (without time-delay) and modify them so that exponential convergence is preserved in presence of time-delay. The main modification amounts to introduce one or several predictors of the output and/or the state. In the more challenging case of nonlinear systems, the approach has been illustrated with observers based on drift-observability property ([10]) and with high-gain observers [1], [7]. The involved predictors prove to be useful in compensating the delay effect up to some upper limit. To enlarge the maximum time-delay, a set of predictors operating in cascade are implemented. First introduced in [10], this idea has been widely used in subsequent works [1], [5], [6], [7], [15].

In parallel with the above "finite-dimensional" research activity, the "infinite-dimensional" backstepping transformation based approach has been developed, see e.g. [16] and references therein. This approach consists in letting the output sensor delay be captured by a first-order hyperbolic PDE. Then, full-order observers are designed that estimate both the system (finite-dimensional) state and the sensor (infinite-dimensional) state.

In turn, the problem of designing sampled-output observers for continuous-time nonlinear systems is also of prime importance in regard of the fact that nowadays control systems are implemented using digital means. This problem has been investigated since the early nineties [8], but it has witnessed a notable renewed interest on recent years, e.g. [2], [12], [18]. Just as for the time-delay case, the main underlying idea in all proposed sampled-data observer methods is to start the design with an exponentially convergent observer for ODEs (without output sampling) and modify these observers so that exponential convergence is preserved in presence of output measurement sampling. In [18] the sampling effect has been accounted for by using a zero-order-hold (ZOH) sampling of the output estimation error as innovation term in the observer state equation. This approach has proved to work well when applied to linear observable systems that are disturbed by a globally Lipschitz function of the state vector. To enlarge the admissible maximum sampling interval, the observer gain is let to be inter-sample exponentially decaying in [2] where exponential convergence conditions are expressed in terms of LMIs involving the sampling interval and other design parameters. The time-varying delay effect caused by output sampling can also be compensated for by inserting inter-sample output predictors. This idea has first been introduced and illustrated for triangular Lipschitz systems in [12].

The problem of dealing simultaneously with both time-delay and output-sampling in observer design, has recently been investigated in [3], [4], [14]. The sampling and delay effects have been compensated for using inter-sample output predictors and state predictors. It was shown that the insertion of these predictors in any continuous-time observer, that is globally exponentially stable and robust with respect to output measurement errors, yields to an exponentially stable sampled-output observer. In the present work, sampling and delay effects are compensated for using only output predictors. Since no state predictors are involved, the new observer turns out to be much simpler, compared to those proposed in previously discussed works. Using the small gain

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method, sufficient conditions are established for the observer to be exponentially convergent. The sufficient conditions particularly involve the admissible maximum time-delay and sampling interval.

The paper is organized as follows: the observer problem is formulated in Section 2; the observer design and analysis are dealt with in Sections 3 and 4, respectively; a conclusion and reference list end the paper.

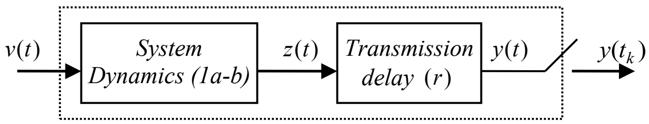

Fig. 1: System to be observed

II. Observer Problem Formulation

The continuous-time system under study is depicted by Fig. 1 and described by the following model:

$$\dot{x}(t) = A(v(t), z(t))x(t) + b(v(t), x(t)) \quad (1a)$$

$$z(t) = cx(t) \quad (1b)$$

$$y(t) = z(t-r) = cx(t-r) \quad (1c)$$

where $x(t) \in \mathbf{R}^n$ is the state vector; $z(t) \in \mathbf{R}$ and $y(t) \in \mathbf{R}$ are respectively the undelayed and delayed outputs; $v(t) \in \mathbf{R}^m$ is a known external input; the integer n and the delay $r > 0$ are respectively known integer and real; the raw vector $c \in \mathbf{R}^{1\times n}$ and the functions $A(v,z) \in \mathbf{R}^{n\times n}$, $b(v,x) \in \mathbf{R}^n$ are known and have the following triangular form:

$$A(v,z) = \begin{bmatrix} 0 & a_1(v,z) & 0 & \cdots & 0 \\ 0 & 0 & a_2(v,z) & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & a_{n-1}(v,z) \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \quad (2a)$$

$$b(v,x) = \begin{bmatrix} b_1(v,x_1) \\ b_2(v,x_1,x_2) \\ \\ b_n(v,x) \end{bmatrix} \quad (2b)$$

$$c = [1\, 0 \ldots 0] \in \mathbf{R}^{1\times n} \quad (2c)$$

where $a_i(v,z) \in \mathbf{R}$ and $b_i(v, x_1, ..., x_i) \in \mathbf{R}$ are known functions.

The problem under study is to design an observer providing online estimates $\hat{x}(t)$ of the state vector $x(t)$ such that the estimation error $\hat{x}(t) - x(t)$ converges exponentially to the origin. The observer must only make use of the external input $v(t) \in \mathbf{R}^m$ and the delayed output samples:

$$y(t_k) = z(t_k - r) = cx(t_k - r)$$

where the t_k's denote the sampling instants. The set $\{t_k\}$ is any partition of $\mathbf{R}_+$ i.e. an increasing sequence such that $t_0 = 0$, $t_k \to \infty$ as $k \to \infty$, and $0 < \tau < \infty$ with $\tau = \sup_{0\le k<\infty} (t_k - t_{k-1})$. The observation problem will be dealt with in the next section under the following assumptions:

A1. All system signals (v, x, y, z) are bounded and an upper bound y_M on the output amplitude $|y(t)|$ is known.

A2. The functions $a_i(v,z)$ $(i = 1 \cdots n)$ are class C^0 with respect to v, and C^1 with respect to z while the functions $b_i(v, x_1, ..., x_i)$ $(i = 1 \cdots n)$ are globally Lipschitz .

A3. The pair $(A(v,z), c)$ is uniformly observable i.e.

$$|\det(O(v,z))| \ge \varepsilon_o > 0, \ \forall v \in \mathbf{R}^m, \forall y \in \mathbf{R} \quad (3a)$$

for some (unknown) real constant ε_o, where $O(v,z)$ denotes the $n \times n$ matrix,

$$O(v,z) = \begin{bmatrix} c \\ cA(v,z) \\ \vdots \\ cA^{n-1}(v,z) \end{bmatrix} \quad (3b)$$

A4. There is a (unknown) real constant $0 < l_\Gamma < \infty$, such that:

$$\sup_{v\in\mathbf{R}^m, |z|\le y_M} \left|\dot{\Gamma}(v,z)\Gamma^{-1}(v,z)\right| \le l_\Gamma \quad (4a)$$

where $\Gamma(v,z) \in \mathbf{R}^{n\times n}$ is defined as follows:

$$\Gamma(v,z) = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & a_1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \prod_{i=1}^{n-1} a_i \end{bmatrix}, \ \forall v \in \mathbf{R}^m, z \in \mathbf{R} \quad (4b)$$

where the a_i's are as in (2a).

Remark 1. a) The complexity with the present observation problem lies, on one hand, in the interference of delay and sampling effects and, on the other hand, in the complexity of model (1a-d) involving the signal $z(t)$ (which is not accessible to measurements due to output delay) in the state matrix $A(v,z)$. This makes the first term on the right side of (1a) subject to a double uncertainty induced by the state vector $x(t)$ and the undelayed output $z(t)$. It turns out that the existing sampled-data observers (e.g. [2], [6], [12], [18]) are applicable to the system (1a-c).

b) In view of Assumption A1, it follows from A2 that the functions $a_i(v,z)$ $(i = 1 \cdots n)$ are also Lipschitz in z on the compact set $|z| \le y_M$. Since the input signal $v(t)$ is bounded, there exists a real constant l_a such that, for $i = 1 \cdots n$ and all real numbers z_1, z_2: $|a_i(v,z_1) - a_i(v,z_2)| \le l_a |z_1 - z_2|$, where l_a is only dependent on the functions $a_i(.,.)$ and the supremum of $|v(t)|$.

c) Similarly, it readily follows from A2 that, there exists a Lipschitz constant l_b such that, for z_1, z_2, $|b(v,z_1) - b(v,z_2)| \le l_b |z_1 - z_2|$, for some constant l_b that is only dependent on $b(.,.)$ and on the supremum of $|v(t)|$. In this respect, let us note that instead of the globally Lipschitz assumption on $b(.,.)$, one could alternately assume this function to be C^1 in x. Since the state vector trajectory $x(t)$ lies in a known compact domain, say D_x (by Assumption A1), one would conclude that $b(.)$ is Lipschitz on D_x. Then, the state estimation problem could be solved by replacing $b(v(t),\hat{x}(t))$, in the observer described in Section 2, by $b(v(t),P(\hat{x}(t)))$ where $P(.)$ denotes the orthogonal projection on the domain D_x. This alternative has been illustrated (in the absence of output sampling and time-delay) in [20].

d) Assumption A3 is also usual in observer design literature. Presently, that assumption amounts to assuming that, $\inf_{|z| \le y_M} |a_i(v,z)| \ge \varepsilon_a > 0$, for some real constant $\varepsilon_a > 0$.

e) Assumption A4 is a technical condition induced by the fact that the state matrix $A(v,z)$ is presently signal-dependent. A similar assumption was required in the non-delayed non-sampled case dealt with in [19].

f) The class of systems defined by (1a-c) is much wider than those in most existing works on sampled- and/or delayed-output based observer design, see e.g. [18], [12], [3], [2], [4], [6]. Indeed, in those works either the state matrix $A(v,z)$ is either constant or only dependent on the input signal v. In those works not involving output-delay, the signal z in the (corresponding) function $b(v,x)$ coincides with the output that is accessible to measurements (here denoted y). The fact that z is presently inaccessible to measurements entails an extra difficulty ■

III. Sampled-Output Observer Design

Since no existing sampled-output observer is applicable to system (1a-c), a new observer will now be constructed. To this end, some relevant expressions are immediately established from the problem formulation of Section II. In this respect, the following saturation operator, suggested by Assumption A1, will be used in the observer:

$$\sigma(\xi) = \mathrm{sgn}(\xi)\min(y_M, |\xi|) \tag{5}$$

where $\mathrm{sgn}(.)$ designates the sign function and y_M is as in Assumption A1. On the other hand, it follows from equations (1a-c) that the (delayed and non-delayed) outputs undergo the following ODEs, respectively:

$$\dot{y}(t) = cA\big(v(t-r), y(t)\big)x(t-r) + cb\big(v(t-r), x(t-r)\big) \tag{6a}$$

$$\dot{z}(t) = cA\big(v(t), z(t)\big)x(t) + cb\big(v(t), x(t)\big) \tag{6b}$$

In view of (1c), equation (6b) also rewrites in the integral form:

$$z(t) = y(t) + \int_{t-r}^{t} cA\big(v(s), z(s)\big)x(s) + cb\big(v(s), x(s)\big)ds \tag{6c}$$

Considering the above observations, the system model (1a-c) suggests the following sampled-output observer:

$$\dot{\hat{x}}(t) = A\big(v(t), \sigma(w_z(t))\big)\hat{x}(t) + b\big(v(t), \hat{x}(t)\big) - \Gamma^{-1}\big(v(t), \sigma(w_z)\big)\Delta^{-1}K\big(c\hat{x}(t) - w_z(t)\big) \tag{7a}$$

$$\dot{w}_y(t) = cA\big(v(t-r), \sigma(w_y(t))\big)\hat{x}(t-r) + cb\big(v(t-r), \hat{x}(t-r)\big) \quad \text{for } t_k < t < t_{k+1} \tag{7b}$$

$$w_y(t_k) = y(t_k) \tag{7c}$$

$$w_z(t) = w_y(t) + \int_{t-r}^{t} cA\big(v(s), \sigma(w_z(s))\big)\hat{x}(s) + cb\big(v(s), \hat{x}(s)\big)ds \tag{7d}$$

where

$$\Delta = diag\left[\frac{1}{\theta}, \dots, \frac{1}{\theta^n}\right] \in \mathbf{R}^{n \times n}, \text{ for any } \theta > 1 \tag{8}$$

and the gain $K \in \mathbf{R}^n$ is chosen such that the matrix $\overline{A} - Kc$ is Hurwitz with

$$\overline{A} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 \\ 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix} \tag{9}$$

Note that K exists because the pair $(\overline{A}, c)$ is observable. In (7a), the initial state estimates $\hat{x}(s)$ $(-r \le s < 0)$ are arbitrarily chosen. In (7d), one uses the initial conditions $w_z(s) = z(s) = y(s-r)$, $(-r \le s < 0)$. In view of (6a-c), the variables $w_y(t)$ and $w_z(t)$ are nothing other than estimates of the delayed and undelayed outputs $y(t)$ and $z(t)$, respectively. In fact, $w_y(t)$ is a prediction of $y(t)$ over the open intervals (t_k, t_{k+1}), it is generated by the predictor (7b-c) from the output samples $y(t_k), y(t_{k-1})\dots$ Then, the filter (7d) is resorted to get the estimate $w_z(t)$ of $z(t)$. Since none of $w_y(t)$ and $w_z(t)$ are a priori bounded, these signals enter through their saturated versions in the observer equations (7a-d). Without using this saturation, it will not be possible to ensure the boundedness of the various observer signals.

Remark 2. a) The observer (7a-d) is inspired by the high-gain observer proposed in (Schreier et al., 2001) for continuous-time systems with no output delay and no output sampling. The main novelty of the new observer is the inter-sample predictor (7b-d) providing the estimates $w_z(t)$ (of the non-delayed output $z(t)$) which is used in the innovation of the state equation (7a). Another new feature of the present observer, compared to [19], is the saturation operator $\sigma(.)$. These novel features will prove to be useful in getting rid of the delay and sampling effects.

b) Note that expression (7d) can be reformulated as follows:

$$w_z(t) = w_y(t) + \vartheta(t)$$

with $\vartheta(t)$ defined by the following ODE:

$$\dot{\vartheta}(t) = cA(v(t),\sigma(w_z(t)))\hat{x}(t) + cb(v(t),\hat{x}(t)) - cA(v(t-r),\sigma(w_z(t-r)))\hat{x}(t-r) + cb(v(t-r),\hat{x}(t-r))$$

$$\vartheta(0) = \int_{-r}^{0} cA(v(s),\sigma(y(s-r)))\hat{x}(s) + cb(v(s),\hat{x}(s))ds \quad \blacksquare$$

IV. Observer Analysis

The sampled-output observer defined by equations (7a-d) will now be analyzed. For analysis purpose, the following errors are introduced:

$$\tilde{x} = \hat{x} - x, \quad e_y = w_y - y, \quad e_z = w_z - z \tag{10}$$

Subtracting system equations (1a), (6a) and (6c) from the corresponding observer equations, i.e. (7a), (7b) and (7d), one gets using (10) and rearranging terms the following equations describing the error dynamics:

$$\begin{aligned}\dot{\tilde{x}} &= \left[A(v,\sigma(w_z)) - \Gamma^{-1}(v,\sigma(w_z))\Delta^{-1}Kc\right]\tilde{x} \\ &\quad + \left(A(v,\sigma(w_z)) - A(v,z)\right)x + \left(b(v,\hat{x}) - b(v,x)\right) \\ &\quad + \Gamma^{-1}(v,\sigma(w_z))\Delta^{-1}Ke_z \end{aligned} \tag{11a}$$

$$\begin{aligned}\dot{e}_y(t) &= cA(v(t-r),\sigma(w_y(t)))\tilde{x}(t-r) \\ &\quad + \left[cA(v(t-r),\sigma(w_y(t))) - cA(v(t-r),y(t))\right]x(t-r) \\ &\quad + \left[cb(v(t-r),\hat{x}(t-r)) - cb(v(t-r),x(t-r))\right] \\ &\qquad \text{for } t_k < t < t_{k+1}\end{aligned} \tag{11b}$$

$$e_y(t_k) = 0 \tag{11c}$$

$$\begin{aligned}e_z(t) &= e_y(t) + \int_{t-r}^{t} cA(v(s),\sigma(w_z(s)))\tilde{x}(s)ds \\ &\quad + \int_{t-r}^{t} c\left[A(v(s),\sigma(w_z(s))) - A(v(s),z(s))\right]x(s)ds \\ &\quad + \int_{t-r}^{t} c\left[b(v(s),\hat{x}(s)) - b(v(s),x(s))\right]ds\end{aligned} \tag{11d}$$

Note that the argument t has deliberately been omitted in (11a) to alleviate it. The main result is now stated in the following theorem:

Theorem 1. Let the sampled-output adaptive observer (7a-d) be applied to the system (1a-c), subject to Assumptions A1-A4. Then, there exists a real constant $1 < \theta^* < \infty$ such that if $\theta > \theta^*$ then, there exist real constant $0 < \tau^* < \infty$ and $0 < r^* < \infty$ so that, if $\tau < \tau^*$ and $r < r^*$ then, $\forall t \geq 0$:

$$\|\tilde{x}(t)\| \leq \rho_x e^{-\alpha t/2}, \ |e_y(t)| \leq \rho_y e^{-\alpha t/2}, \ |e_z(t)| \leq \rho_z e^{-\alpha t/2},$$

for some real constants $\alpha > 0, \rho_x > 0, \rho_y > 0, \rho_z > 0$ ■

Proof. The proof is partly inspired by [19], [12]. For

Remark 3. Admissible values of the delay r and the maximum sampling interval τ are those satisfying conditions (45), (51) and (57). Accordingly, the maximum admissible values, say r_M and α_M, depend on the free parameter α which has been introduced for analysis purpose. The smaller α the larger the maximum admissible values. It follows that r_M and α_M are obtained by letting $\alpha = 0$ in (45), (51) and (57). Doing so one gets:

$$r_M < \frac{1}{l_a\beta_7}, \tau_M < \frac{1}{l_a\beta_7}, \tag{58a}$$

$$\frac{\tau_M + r_M(1 - l_a\beta_7\tau_M)}{(1 - l_a\beta_7 r_M)(1 - l_a\beta_7\tau_M)} < \frac{1}{\gamma_1\theta(\theta + l_b)} \tag{58b}$$

Then, the sets of admissible delay and sampling interval are respectively defined by

$0 < r < r_M$ and $0 < \tau < \tau_M$ (59) ■

V. Conclusion

This study has addressed the problem of state estimation for the class of nonlinear systems (1a-c)-(2a-c) using sampled output measurements. The problem complexity lies, on one hand, in the interference of the output time-delay and sampling effects and, on the other hand, in the injection of the undelayed output (which is not accessible to measurements) in the state matrix $A(v(t),z(t))$. The proposed observer (7a-d) features a simpler output predictor defined by two ODEs (while previous observers involved output and state predictors defined by several ODEs). The maximum sampling interval and time-delay for the observer to be exponentially convergent are well defined by inequalities (58).

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