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# Bias Propagation and Estimation in Homogeneous Differentiators for a Class of Mechanical Systems

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**ABSTRACT** Motivated by non-anthropomorphic dynamic stabilization of a walking robot, we consider the bias propagation problem for a homogeneous nonlinear model-based differentiator applied to a reaction wheel pendulum with a biased position sensor. We show that the bias propagates through the velocity observer and compromises the vertical stabilization. To cancel the impact of the bias, we propose to augment the differentiator with a reduced-order bias observer. Local asymptotic stability of the augmented nonlinear observer is shown, where the observer gain can be tuned using matrix inequalities. Experimental results illustrate the applicability of the proposed solution.

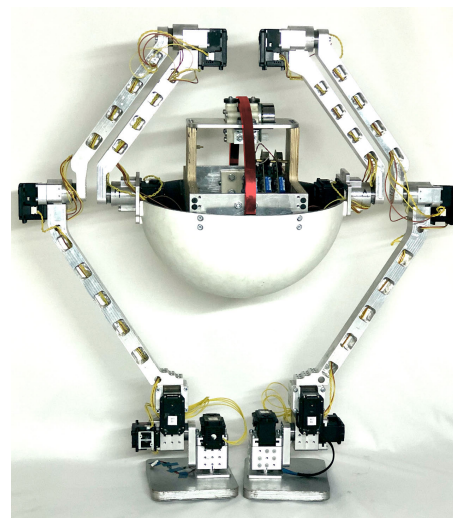
**INDEX TERMS** Sensor bias, velocity observer, homogeneous differentiator, inverted pendulum.

## I. INTRODUCTION

The research problem of this paper is motivated by the walking robot we are currently developing [1]. The robot uses an auxiliary non-anthropomorphic dynamic stabilization system that consists of two reaction wheels inside the robot's body (Figure 1). Since the flywheels are mutually orthogonal, the vertical stabilization of the robot can be considered for each axis separately, and a simplified model to study the stabilization problem is the model of an inverted reaction-wheel pendulum.

Stabilization of an inverted pendulum is well-studied, and nowadays, it is included as a scholar example in many graduate courses on control design. An in-depth analysis of pendulum motion, including linear and nonlinear models, is provided by Block *et al.* [2]. Spong *et al.* [3] have proposed a switching controller for swinging the pendulum upwards from the stable equilibrium, and an inspiring project has been proposed by the ETH Zürich team [4]–[6] that covers linear control for the one-dimensional case and nonlinear control for the three-dimensional balancing cube problem. However, it should be noted that pendulum stabilization algorithms

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**FIGURE 1.** AnyWalker robot uses reaction wheels (shown in red) as an auxiliary stabilization system, see [1].

typically implement a full-state control law that utilizes both position and velocity measurements of the pendulum. Thus, the velocity estimation becomes the critical element of the

vertical stabilization control design. Note also that the tilt angles [7] of the considered robot (Figure 1) are measured with multiple accelerometers, and we rely on soft sensors for velocity estimation.

One possible solution for velocity estimation is to design a full-state observer, where a model of system dynamics is used to estimate the state vector. For nonlinear systems, state estimation can be performed based on linearization (or linear time-varying representation) of the system dynamics yielding a standard linear observer, e.g., Kalman-Bucy filter, or a nonlinear observer can be proposed [8]; for discrete-time nonlinear systems, a Newton observer can be utilized, see, e.g., [9]. Notably, an exponential nonlinear velocity observer for a class of mechanical systems has been recently proposed in [10]. The drawback of the full-state observer design is that it typically requires an accurate model and estimates the coupled state vector of the whole system, which is not reasonable in the considered robotic design with multiple degrees of freedom.

Another approach that is widely used in engineering applications is to estimate velocity for each degree of freedom separately. From the signal processing point of view, this approach can be considered as numerical differentiation, where velocity estimation is seen as online differentiation of a measured position signal, e.g., a first-order difference used in [3], sliding-mode exact differentiators first proposed by Levant in [11], algebraic differentiators by Ushirobira in [12], or high-gain differentiators as described by Vasiljevic and Khalil in [13]. Whereas differentiator-based velocity observers can be designed model-free as in [3], better performance can be obtained when the observers use (at least partially) available model knowledge as in [11]–[13]. Unfortunately, if the pendulum's position measurements are biased, then the bias propagates through the model and can yield estimation errors. Particularly, for a legged robot, such a bias arises due to imprecisions in sensor placement and dependency of the center of mass point on the current robot's posture.

In this paper, we consider the homogeneous model-based nonlinear differentiators (HOMD) proposed by Perruquetti *et al.* [14]. We consider the scenario when position measurements are biased, and we estimate the bias simultaneously with the velocity estimation. A similar problem has been addressed by Gajamohan *et al.* [4], where the authors propose to use a low-pass filter for bias estimation. However, to the best of the authors' knowledge, the problem of bias propagation in the nonlinear model-based differentiator [14] has not been addressed before.

The contribution of this paper is twofold. First, we analyze the bias propagation in the nonlinear model-based differentiator and show that it yields a steady-state estimation error. Second, we propose to combine the differentiator with a reduced-order bias estimator. Our analysis shows that under a proper choice of the observer gain, the augmented system is locally asymptotically stable, and estimation errors converge to zero with a certain domain of attraction. The choice of

the gain can be performed using matrix inequalities. The proposed solution is illustrated in experimental studies and compared with the low-pass bias observer [4].

The preliminary results of this research are reported in [15]. Extending that work, this manuscript contains i) stability analysis of the low-pass bias observer, ii) stability convergence analysis of the proposed nonlinear observer, and iii) detailed and extended experimental studies.

The rest of the paper is organized as follows. In Section II we present a model of the considered system and a state-feedback controller capable of stabilizing the system if all states (including velocities) are measured. Next, in Section III we propose velocity observers based on homogeneous differentiators. We show that these observers ensure accurate finite-time velocity estimation in the absence of measurement bias, but the estimates are not accurate if the measurements are biased. The proposed observers are experimentally compared with a linearization-based Luenberger observer. In Section IV we consider bias estimation based on low-pass filtering and show the limitations of this approach. A reduced-order bias observer augmenting the previously designed control law is proposed and analyzed in Section V, where experimental results and comparison with the low-pass filtering are also given. Finally, possible further research directions are discussed in the concluding Section VI.

## II. MODEL DESCRIPTION AND STATE-FEEDBACK CONTROL LAW

The hardware for the tests (shown in Figure 2) is assembled from off-the-shelf components. A 70W Maxon EC 45 flat brushless DC motor is used to drive the reaction wheel (a bicycle brake rotor). The motor is controlled using Maxon EPOS2 50/5 controller running in torque mode; the controller measures the current as well as the rotor angle. A STM32F407 discovery board was chosen as the main computing unit, where the board communicates with the motor controller via CANopen protocol.

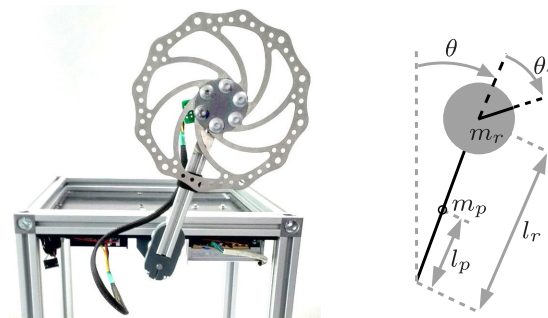


FIGURE 2. 1D pendulum hardware and corresponding notation.

### A. MODEL DESCRIPTION

In our model the main variables are  $\theta$  and  $\theta_r$ , where  $\theta$  is the angle between the pendulum and the vertical, and  $\theta_r$  is the angle of the reaction wheel with respect to the pendulum.

TABLE 1. Parameters of the experimental setup.

Description	Symbol	Value
Mass of the pendulum, kg	$m_p$	0.58
Pivot – pendulum’s center of mass distance, m	$l_p$	0.10
Moment of inertia of the pendulum around its center of mass, kg·m <sup>2</sup>	$J_p$	$3.8 \cdot 10^{-3}$
Mass of the reaction wheel, kg	$m_r$	0.35
Pivot – reaction wheel axis distance, m	$l_r$	0.22
Moment of inertia of the reaction wheel, kg·m <sup>2</sup>	$J_r$	$12.48 \cdot 10^{-4}$
Current-to-torque gain, N/A	$k$	$3.69 \cdot 10^{-2}$
Resolution of the reaction wheel angle measurement, rad	–	$6.54 \cdot 10^{-2}$
Resolution of the pendulum angle measurement, rad	–	$6.28 \cdot 10^{-4}$
Sampling frequency, Hz	–	500

It is worth noting that the notation follows [2] with one exception: we measure  $\theta_r$  with respect to the pendulum body and not with respect to the vertical. Refer to Figure 2 for an illustration; Table 1 provides all parameters and notation used in the paper. The values of the parameters were obtained by direct measurements.

We use Lagrange’s approach to derive the equations of motion. The Lagrangian is given by

$$\mathcal{L} = T_p + T_r - P = \frac{1}{2}J\dot{\theta}^2 + \frac{1}{2}J_r(\dot{\theta}_r + \dot{\theta})^2 - mlg \cos \theta,$$

where  $ml := m_p l_p + m_r l_r$ ,  $J := J_p + m_p l_p^2 + m_r l_r^2$ , and

$$\begin{aligned} T_p &= \frac{1}{2}(m_p l_p^2 + J_p)\dot{\theta}^2, \\ T_r &= \frac{1}{2}m_r l_r^2 \dot{\theta}^2 + \frac{1}{2}J_r(\dot{\theta}_r + \dot{\theta})^2, \\ P &= (m_p l_p + m_r l_r)g \cos \theta, \end{aligned}$$

are the kinetic energy of the pendulum, the kinetic energy of the reaction wheel, and the potential energy, respectively; all the symbols are as defined in Table 1.

Neglecting the friction, we derive the equations of motion:

$$\begin{aligned} J_r \ddot{\theta}_r + J_r \ddot{\theta} &= kI, \\ (J + J_r)\ddot{\theta} + J_r \ddot{\theta}_r &= mlg \sin \theta, \end{aligned} \quad (1)$$

where the symbols are as defined in Table 1, and  $I$  is the current in the motor windings. Assuming an internal fast-time-scale current loop, we consider  $I$  as our input control signal.

The model (1) can be rewritten for each degree of freedom as

$$\ddot{\theta} = -\frac{k}{J}I + \frac{mlg}{J} \sin(\theta) \quad (2)$$

and

$$\ddot{\theta}_r = \frac{(J + J_r)k}{JJ_r}I - \frac{mlg}{J} \sin(\theta). \quad (3)$$

The control goal is to locally stabilize the pendulum in the upper equilibrium, that is to drive the variables  $\theta$ ,  $\dot{\theta}$ , and  $\dot{\theta}_r$  to zero, while  $\theta_r \in \mathbb{R}$ . Let us further denote the upper equilibrium as the set

$$\Omega_0 := \{\theta = \dot{\theta} = \dot{\theta}_r = 0, \theta_r \in \mathbb{R}\} \subset \mathbb{R}^4,$$

which is an invariant set of the system (1).

*Definition 1:* We say that a control law (locally) stabilizes the system (1) if under this control law the set  $\Omega_0$  is (locally) attractive.

To simplify presentation, we note that the angle  $\theta_r$  does not affect the pendulum dynamics and define the state variable vector

$$x := [\theta \quad \dot{\theta} \quad \dot{\theta}_r]^\top.$$

Then the control goal is to construct the control input  $I$  such that the state  $x$  locally converges to the origin.

### B. STATE-FEEDBACK CONTROL LAW

To achieve the goal, the model (1) is linearized around the equilibrium  $x_{eq} := [0 \ 0 \ 0]^\top$  yielding the linearized model

$$\dot{x} = Ax + BI, \quad \theta = Cx, \quad (4)$$

where  $\theta$  is the measured output and

$$A := \begin{bmatrix} 0 & 1 & 0 \\ \frac{mlg}{J} & 0 & 0 \\ -\frac{mlg}{J} & 0 & 0 \end{bmatrix}, \quad B := \begin{bmatrix} 0 \\ \frac{k}{J} \\ \frac{k}{J} + \frac{k}{J_r} \end{bmatrix}, \quad C := \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}^\top.$$

To stabilize the pendulum, the state-feedback control law is proposed in the form

$$u = -Kx, \quad (5)$$

where  $K := [k_1 \ k_2 \ k_3]$  is the gain vector such that the matrix  $A - BK$  is Hurwitz, i.e., all its eigenvalues have negative real parts. Particularly, the gain vector  $K$  can be found as a solution of the optimal linear quadratic regulation (LQR) problem.

Note that if the zero reading of the sensor does not coincide with the equilibrium position and the angle  $\theta$  is measured with a certain constant offset, then the state-feedback control law does not achieve the control goal. This problem is well-known, see e.g., [2], [4]. Denote the constant offset as  $d$  and let the measured signal be

$$y = \theta + d. \quad (6)$$

Using  $y$  in place of the real value  $\theta$  in (5) yields

$$u = -K [y \quad \dot{\theta} \quad \dot{\theta}_r]^\top = -Kx - K [d \ 0 \ 0]^\top,$$

and the equilibrium of the linearized system (4) under this control law can be found as

$$x_{eq,d} = (A - BK)^{-1}BK \begin{bmatrix} d \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -d \frac{k_1}{k_3} \end{bmatrix}. \quad (7)$$

It means that under the measurements bias, the pendulum is stabilized at the upper position but the reaction wheel does not stop and  $\dot{\theta}_r$  converges to a non-zero constant value, i.e., the system is not stabilized in the sense of Definition 1.

*Remark 1:* It is worth noting that the problem of velocity estimation under biased measurement can be generalized,

e.g., instead of the model (2) with biased measurements (6) one can consider a more general class of systems

$$\ddot{y} = \beta(t) + \gamma(y - d),$$

where the scalar signal  $y$  is measured,  $d$  is a constant bias, the functions  $\beta : \mathbb{R}_+ \rightarrow \mathbb{R}$  and  $\gamma : \mathbb{R} \rightarrow \mathbb{R}$  are known, the function  $\gamma$  is differentiable, the approximation  $\gamma(y - d) \approx \gamma(y) - \gamma'(y)d$  can be used for sufficiently small  $d$ , and the goal is to estimate the derivative  $\dot{y}$ . However, the research problem of this paper has been motivated by a particular mechatronic system; thus, to keep the paper concise and illustrative, we present the results for the specific system (1).

### III. FINITE TIME VELOCITY OBSERVER

Let us put aside for a while the problem of biased measurements; under the control law (5), the upper equilibrium of the pendulum is locally asymptotically stable with a domain of attraction depending on the design parameters. However, to implement the law (5), measurements of the velocities  $\dot{\theta}$  and  $\dot{\theta}_r$  are required. If these variables are not measured directly, then a velocity observer should be used to generate the estimates  $\hat{\theta}$  and  $\hat{\theta}_r$ , and the law (5) takes the form

$$u = -K \begin{bmatrix} \theta & \hat{\theta} & \hat{\theta}_r \end{bmatrix}^\top. \quad (8)$$

One simple and convenient approach is to construct a Luenberger observer for the linearized system. To this end, instead of the reduced system (4) which does not include the angle  $\theta_r$ , we have to consider the full state vector that yields the observer

$$\frac{d}{dt} \begin{bmatrix} \hat{\theta} \\ \hat{\theta} \\ \hat{\theta}_r \\ \hat{\theta}_r \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{mlg}{J} & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{mlg}{J} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\theta} \\ \hat{\theta} \\ \hat{\theta}_r \\ \hat{\theta}_r \end{bmatrix} + \begin{bmatrix} 0 \\ -\frac{k}{J} \\ 0 \\ \frac{k}{J} + \frac{k}{J_r} \end{bmatrix} u + L \left( \begin{bmatrix} \theta \\ \theta_r \end{bmatrix} - \begin{bmatrix} \hat{\theta} \\ \hat{\theta}_r \end{bmatrix} \right), \quad (9)$$

where the gain vector  $L \in \mathbb{R}^{4 \times 2}$  is such that the estimation error dynamic matrix is Hurwitz. The drawback of this approach is that it is based on the linearization of the system, and therefore it is not valid when the angle  $\theta$  is not close to the desired position and the approximation  $\sin(\theta) \approx \theta$  does not hold. This drawback can be alleviated using linear observers with time-varying gains, e.g., with gains scheduling, or using nonlinear state observers for mechanical systems, see [10] and the references therein. However, such solutions are typically harder to design and implement.

Another approach to velocity estimation is based on differentiation [12], [14], [16]. Under this approach, the problem of velocity estimation is considered as the problem of online differentiation of a measured signal (angular position); therefore, each degree of freedom is considered separately and one observer is constructed to estimate  $\dot{\theta}$ , while another observer estimates  $\dot{\theta}_r$ . In what follows, we use this approach and

design nonlinear velocity observers based on homogeneous differentiators [14] to obtain velocity estimates with the finite time convergence.

*Remark 2:* To simplify and streamline the presentation, we omit here formal definitions of the homogeneity property and finite-time stability, which can be found in [14], [17].

#### A. PENDULUM VELOCITY OBSERVER

Let  $\hat{x}_p \in \mathbb{R}^2$  be an estimate of the vector  $x_p := [\theta \ \dot{\theta}]^\top$  and define the estimation error as  $e_p := \hat{x}_p - x_p$ , where we recall that  $e_{p,1} = \hat{\theta} - \theta$  is measured. Denote for any real numbers  $x$  and  $\alpha > 0$

$$[x]^\alpha := |x|^\alpha \operatorname{sgn}(x),$$

where  $\operatorname{sgn}(\cdot)$  is the sign function.

Recalling (2) and following [17], we construct the homogeneous velocity observer as a model-based differentiator of  $x_{p,1}$ :

$$\begin{aligned} \dot{\hat{x}}_{p,1} &= \hat{x}_{p,2} - k_{p,1} [e_{p,1}]^{\alpha_p}, \\ \dot{\hat{x}}_{p,2} &= -\frac{k}{J} I + \frac{mlg}{J} \sin(\theta) - k_{p,2} [e_{p,1}]^{2\alpha_p - 1}, \\ \hat{\theta} &= \hat{x}_{p,2}, \end{aligned} \quad (10)$$

where  $\alpha_p, k_{p,1}, k_{p,2}$  are the design parameters. Then the error dynamics is given by

$$\begin{aligned} \dot{e}_{p,1} &= e_{p,2} - k_{p,1} [e_{p,1}]^{\alpha_p}, \\ \dot{e}_{p,2} &= -k_{p,2} [e_{p,1}]^{2\alpha_p - 1}. \end{aligned} \quad (11)$$

Note that  $e_p = 0$  is the unique equilibrium of the system (11).

*Proposition 1:* Consider the system (11), where  $\alpha_p \in (\frac{1}{2}, 1)$  and  $k_{p,1}, k_{p,2}$  are chosen such that the polynomial  $s^2 + k_{p,1}s + k_{p,2}$  is Hurwitz. Let  $e_p(t)$  be the solution with the initial conditions  $e_p(0) \in \mathbb{R}^2 \setminus \{0\}$ . Then the origin is finite-time stable, i.e., there exists  $T = T(e_p(0)) > 0$  such that  $e_p(t)$  is defined and unique on  $[0, T)$ , bounded, and  $\lim_{t \rightarrow T} e_p(t) = 0$ .  $T$  is called the settling-time function of the system (11). Note that  $e_p(t) \equiv 0$  is the unique solution starting from  $e_p(0) = 0$ , and the settling-time function can be extended at the origin by  $T(0) = 0$ .

The proof of Proposition 1 follows applying Theorem 10 from [14] to the system (11).

#### B. REACTION WHEEL VELOCITY OBSERVER

Let  $\hat{x}_r \in \mathbb{R}^2$  be an estimate of the vector  $x_r := [\theta_r \ \dot{\theta}_r]^\top$  and define the estimation error as  $e_r := \hat{x}_r - x_r$ , where  $e_{r,1}$  is measured. Recalling (3) and similarly to (10), we construct the homogeneous velocity observer as

$$\begin{aligned} \dot{\hat{x}}_{r,1} &= \hat{x}_{r,2} - k_{r,1} [e_{r,1}]^{\alpha_r}, \\ \dot{\hat{x}}_{r,2} &= \frac{(J + J_r)k}{JJ_r} I - \frac{mlg}{J} \sin(\theta) - k_{r,2} [e_{r,1}]^{2\alpha_r - 1}, \\ \hat{\theta}_r &= \hat{x}_{r,2}, \end{aligned} \quad (12)$$



where  $\alpha_r, k_{r,1}, k_{r,2}$  are the design parameters. Then the error dynamics is given by

$$\begin{aligned} \dot{e}_{r,1} &= e_{r,2} - k_{r,1}[e_{r,1}]^{\alpha_r}, \\ \dot{e}_{r,2} &= -k_{r,2}[e_{r,1}]^{2\alpha_r-1}. \end{aligned} \quad (13)$$

The system (13) is similar to (11) and the finite-time stability of the equilibrium  $e_r = 0$  follows from Proposition 1 choosing  $\frac{1}{2} < \alpha_r < 1$  and  $k_{r,1}, k_{r,2}$  such that the polynomial  $s^2 + k_{r,1}s + k_{r,2}$  is Hurwitz.

### C. CLOSED-LOOP SYSTEM BEHAVIOR UNDER BIASED MEASUREMENTS

It can be shown that the control law (8) with the finite-time observers (10) and (12) locally stabilizes the system (1). Indeed, for the observers (10) and (12) there exists the common settling time  $T_{com} = T(e_p(0), e_r(0))$ , such that the control laws (5) and (8) are equivalent for  $t \geq T_{com}$ . Since the control law (5) is stabilizing for the linearized system (4), it is locally stabilizing for the nonlinear system (1) in a neighborhood of the desired equilibrium; define this neighborhood as  $\Omega_\alpha$ . Since the trajectories  $\hat{x}_p$  and  $\hat{x}_r$  are bounded, there exist constants  $\bar{e}_p > 0, \bar{e}_r > 0$ , and a set  $\Omega_\beta \subset \Omega_\alpha$ , such that for all initial conditions satisfying  $x(0) \in \Omega_\beta, \|e_p(0)\| < \bar{e}_p, \|e_r(0)\| < \bar{e}_r$  trajectories  $x(t)$  stay in  $\Omega_\alpha$  for  $t \in [0, T_{com}]$ . Then starting from  $t = T_{com}$  we can consider the system (1) under the control law (8), (10), (12) as the system (1) under the control law (5) with the initial condition  $x(T_{com}) \in \Omega_\alpha$  ensuring  $x(t) \rightarrow 0$ . It is worth noting that given  $e_p(0) = 0$  and  $e_r(0) = 0$  we have  $T_{com} = 0$  and the sets  $\Omega_\beta$  and  $\Omega_\alpha$  coincide.

Consider now behavior of the velocity observers (10), (12) under biased measurements, where we impose the following assumption.

*Assumption 1: The sensor offset  $d$  is sufficiently small and  $\sin(d) \approx d, \cos(d) \approx 1$ .*

Using  $y$  as a measurement of  $\theta$  in (10) and noting that  $\hat{\theta} - y = e_{p,1} - d$ , the error dynamics (11) takes the form

$$\begin{aligned} \dot{e}_{p,1} &= e_{p,2} - k_{p,1}[e_{p,1} - d]^{\alpha_p}, \\ \dot{e}_{p,2} &= a_1 \cos(\theta)d - k_{p,2}[e_{p,1} - d]^{2\alpha_p-1}, \end{aligned} \quad (14)$$

where we define  $a_1 := \frac{mgl}{J}$ . Obviously,  $e_p = 0$  is not an equilibrium of this system. Considering the pendulum in the upper half-plane where  $\cos(\theta) > 0$ , the equilibrium of the system (14) corresponds to the (steady-state for constant  $\theta$ ) velocity estimation error

$$e_{p,2}^0 := k_{p,1} \left( \frac{a_1 \cos(\theta)}{k_{p,2}} |d| \right)^{\frac{\alpha_p}{2\alpha_p-1}} \text{sgn}(d). \quad (15)$$

Analyzing the error dynamics of the reaction wheel observer, it can be shown that the steady-state (for constant  $\theta$ ) velocity estimation error is

$$e_{r,2}^0 = -e_{p,2}^0.$$

Applying these results to the control law (8), the closed-loop equilibrium is given by

$$\begin{aligned} x_{eq,d} &= (A - BK)^{-1}BK \begin{bmatrix} d \\ e_{p,2}^0 \\ e_{r,2}^0 \end{bmatrix} \\ &= \begin{bmatrix} 0 \\ 0 \\ e_{p,2}^0 \frac{k_3 - k_2}{k_3} - d \frac{k_1}{k_3} \end{bmatrix}. \end{aligned} \quad (16)$$

Thus, it follows that under biased measurements, the velocity estimates are not accurate and do not converge to the real values. The state-feedback control law with velocity observers drives the pendulum to the upper equilibrium but does not stabilize the system in the sense of Definition 1.

*Remark 3: It is worth noting that if the reaction wheel position  $\theta_r$  is measured with a constant bias  $d_r$  while the pendulum position  $\theta$  is measured without a distortion, then the error dynamics of the observer (12) becomes*

$$\begin{aligned} \dot{e}_{r,1} &= e_{r,2} - k_{r,1}[e_{r,1} - d_r]^{\alpha_r}, \\ \dot{e}_{r,2} &= -k_{r,2}[e_{r,1} - d_r]^{2\alpha_r-1}. \end{aligned}$$

*Then at the equilibrium we have  $e_{r,2} = 0$ , i.e., a bias of the reaction wheel sensor does not lead to a steady-state bias of the estimate  $\hat{x}_{r,2}$  of the reaction wheel velocity.*

### D. EXPERIMENTAL STUDIES

To illustrate the applicability of the proposed differentiator-based observer, we present here experimental results on stabilization for the hardware described in Section II. In this Section we consider the case when the optical encoder is perfectly adjusted, i.e.,  $d \approx 0$ , and compare performance of the homogeneous velocity observers (10) and (12) with the linear observer (9). Both observers are used with the feedback control law (8), and we consider the problem of the upright pendulum stabilization, where the initial position is in a neighborhood of the equilibrium. Controller gain parameters and the parameters of the homogeneous velocity observers (10) and (12) used in the experiment are given in Table 2. The gain matrix  $L$  of the observer (9) is chosen such that the observers have approximately equal rise time.

**TABLE 2. Controller parameters used in experiments.**

Description	Symbol	Value
LQR gains in (8)	$K$	[-582, -83, -1.2]
Parameters in (10)	$[k_{p,1}, k_{p,2}, \alpha_p]$	[5, 12, 0.9]
Parameters in (12)	$[k_{r,1}, k_{r,2}, \alpha_r]$	[5, 12, 0.9]

Experimental results are depicted in Fig. 3, where the initial position was set as  $\theta(0) \approx 0.1$ . It can be seen that for both types of observers the initial transients are close; however, the homogeneous differentiators provide faster stabilization with less position variations. Moreover, it was found that if the initial position is increased to  $\theta(0) \approx 0.15$ , the linearization-based observer does not

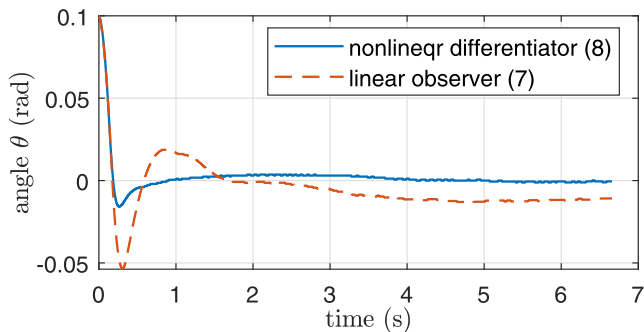


FIGURE 3. Closed-loop pendulum stabilization using observers (10), (12) and (9).

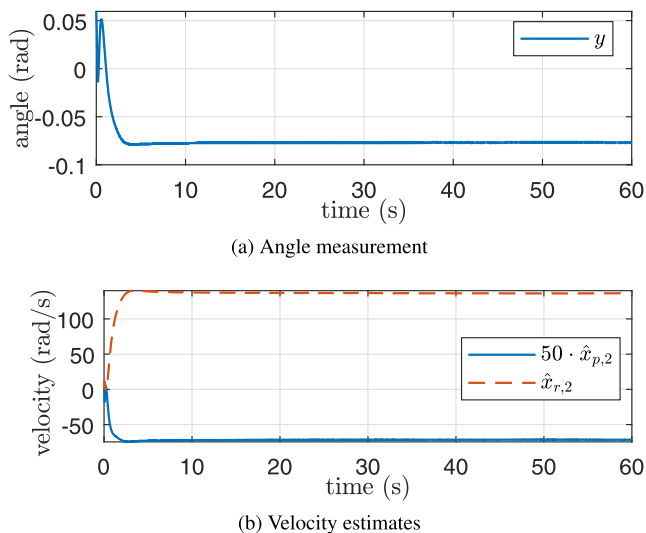


FIGURE 4. Control (8), (10), and (12) for  $d = -0.08$ .

provide system stabilization, and the pendulum falls, but the differentiator-based observer stabilizes the system.

Let us now show that the same control law (8), (10), and (12) cannot stabilize the pendulum for  $d \neq 0$ . As it is shown in (7), for  $d = -0.08$  the pendulum arrives to the upper position, but the reaction wheel does not stop and maintains a nonzero constant velocity. The experimental results are depicted in Fig. 4. Here the measured angle  $y$  converges to a nonzero value, such that  $y \approx d$ , and the physical angle  $\theta \approx 0$ . Velocity estimates  $\hat{x}_{p,2}$  and  $\hat{x}_{r,2}$  converge to nonzero values, where the value  $\hat{x}_{p,2} = e_{p,2}^0 \approx -1.43$  is predicted by (15). The physical velocities converge to  $\dot{\theta} \approx 0$  and  $\dot{\theta}_r \approx 138$  radians per second that corresponds to (16).

#### IV. LOW-PASS FILTER BIAS ESTIMATION

As it is shown in the previous sections, presence of a bias in the measurements precludes system stabilization, and one approach to cope with this is to design a bias observer. A common solution is based on low-pass filtering of the measurements as proposed in [4]. The key idea comes from the observation that in the closed loop (7), the output  $y$  tends to  $d$  as  $x_1$  tends to 0. Thus, one can estimate  $d$  observing the

steady-state value of  $y$ . However, such a procedure involves some decision making (to identify the end of transients) and switching behavior. To avoid stability issues, estimation of  $d$  can be performed in a slower time scale than stabilization implementing a low-pass linear filter [4]:

$$\frac{d}{dt} \hat{d}(t) = \gamma_d (y(t) - \hat{d}(t)), \quad (17)$$

where  $\gamma_d > 0$ . Next, the measurement  $y$  is replaced with  $y - \hat{d}$  and the control law

$$u = -K [y - \hat{d} \quad x_2 \quad x_3] \quad (18)$$

is applied.

The drawback of this approach is that the gain  $\gamma_d$  should be chosen sufficiently small, otherwise the closed-loop system may become unstable. This restriction is formulated in the following proposition.

*Proposition 2:* Consider system (4) with the biased measurement (6), bias observer (17), and the control law (18), where the gain vector  $K$  is such that the matrix  $A - BK$  is Hurwitz. Then there exists  $\tilde{\gamma}_d > 0$  such that the closed-loop system is stable for  $\gamma_d \in (0, \tilde{\gamma}_d)$  and not stable otherwise.

*Proof:* Define the bias estimation error  $\tilde{d} := d - \hat{d}$  and note that  $y - \hat{d} = x_1 + \tilde{d}$ . To study the stability of the closed-loop system, we note that from (17) the error dynamics is given by

$$\dot{\tilde{d}} = -\gamma_d \tilde{d} - \gamma_d x_1.$$

The control law (18) can be written as

$$u = -K [y - \hat{d} \quad x_2 \quad x_3] = -Kx - k_1 \tilde{d},$$

and the closed-loop system is thus given by

$$\begin{bmatrix} \dot{x} \\ \dot{\tilde{d}} \end{bmatrix} = \begin{bmatrix} A - BK & -k_1 B \\ -\gamma_d & 0 \end{bmatrix} \begin{bmatrix} x \\ \tilde{d} \end{bmatrix} =: F \begin{bmatrix} x \\ \tilde{d} \end{bmatrix}.$$

Therefore, the closed-loop system is stable if and only if the matrix  $F$  is Hurwitz. Next we find the admissible range of  $\gamma_d$  that guarantees this condition. Obviously, if  $\gamma_d = 0$  then the matrix  $F$  have the same (stable) eigenvalues as  $A - BK$  and one zero eigenvalue related to  $\tilde{d}$ .

To proceed, we first consider the characteristic polynomial of the matrix  $A - BK$  given by

$$\det(sI - (A - BK)) = s^3 + m_1 s^2 + m_2 s + m_3,$$

where

$$m_1 = \frac{kk_3}{J_r} + \frac{kk_3}{J} - \frac{kk_2}{J},$$

$$m_2 = -\frac{kk_1}{J} - \frac{gml}{J}, \quad m_3 = -\frac{gkk_3 ml}{JJ_r}.$$

Since the matrix  $A - BK$  is Hurwitz, the Routh's criterion states that

$$m_1 > 0, \quad m_2 > 0, \quad m_3 > 0, \quad m_1 m_2 > m_3. \quad (19)$$

It implies, particularly, that the matrix  $A - BK$  is Hurwitz only if  $k_1 < 0$  and  $k_2 < k_3 < 0$ .

On the other hand, the characteristic polynomial of the matrix  $F$  can be found as

$$\begin{aligned} \det(sI - F) &= (\gamma_d + s) \det(sI - (A - BK)) + \frac{kk_1}{J} \gamma_d s \\ &= s^4 + b_1 s^3 + b_2 s^2 + b_3 s + b_4, \end{aligned}$$

where

$$\begin{aligned} b_1 &:= \gamma_d + m_1, \quad b_2 := \gamma_d m_1 + m_2, \\ b_3 &:= \gamma_d m_2 + m_3 + \frac{kk_1}{J} \gamma_d, \quad b_4 = \gamma_d m_3. \end{aligned}$$

Routh's criterion states that the system is stable if and only if all coefficients  $b_i$ ,  $i = 1, \dots, 4$ , are positive, and

$$b_1 b_2 - b_3 > 0, \quad (b_1 b_2 - b_3) b_3 - b_1^2 b_4 > 0.$$

Due to (19), the inequality  $b_4 > 0$  implies the positivity of  $\gamma_d$ , whereas  $b_3 > 0$  implies  $\gamma_d < -\frac{kk_3}{Jr} =: \bar{\gamma}_{d,1}$ . Note that  $\bar{\gamma}_{d,1} > 0$  since  $k_3 < 0$  due to the positivity of  $m_3$ . The inequality  $b_1 b_2 - b_3 > 0$  is always satisfied for positive values of  $\gamma_d$ , since all the coefficients of the corresponding quadratic (in  $\gamma_d$ ) polynomial are positive.

The inequality  $(b_1 b_2 - b_3) b_3 - b_1^2 b_4 > 0$  corresponds to the positiveness of a cubic polynomial of  $\gamma_d$ . It is straightforward to verify that the highest order coefficient of this polynomial is negative, while the free coefficient is positive, and thus this cubic polynomial has at least one real positive root. Denote the smallest positive real root of the polynomial as  $\bar{\gamma}_{d,2}$ . Then the cubic inequality is obviously satisfied for all  $0 < \gamma < \bar{\gamma}_{d,2}$ .

Finally, define  $\bar{\gamma}_d := \min\{\bar{\gamma}_{d,1}, \bar{\gamma}_{d,2}\}$ . Then the matrix  $F$  is Hurwitz if and only if  $\gamma_d \in (0, \bar{\gamma}_d)$ .  $\square$

From Proposition 2, one can conclude that the drawback of the bias observer (17) is that if  $\gamma_d$  is small enough, then the stabilization goal is achieved with a long transient time; however, it is not possible to increase the value  $\gamma_d$  beyond the limit  $\bar{\gamma}_d$  to accelerate the transients without compromising the closed-loop stability.

## V. REDUCED-ORDER BIAS OBSERVER

In this section, we present a novel solution for bias estimation. More precisely, we show that a reduced-order observer can be combined with the nonlinear differentiators proposed in Section III, and that the resulting nonlinear observer ensures convergence of the estimates under certain conditions.

### A. REDUCED-ORDER OBSERVER DESIGN

Define the state vector

$$z := [x^\top \ d]^\top = [\theta \quad \dot{\theta} \quad d]^\top.$$

Then the measured signal is given by  $y = [1 \ 0 \ 1]z$ . Let  $\hat{z}$  be an estimate of  $z$  and define  $e := \hat{z} - z$ . Then  $e_3$  is the bias estimation error. Noting that  $z_1 = y - z_3$  and recalling Assumption 1, the following approximation holds:

$$\begin{aligned} \sin(z_1) &\approx \sin(y) - \cos(y)z_3 \\ &= \sin(y) - \cos(y)\hat{z}_3 + \cos(y)e_3. \end{aligned}$$

Then the dynamics of  $z$  can be written as

$$\begin{aligned} \dot{z} &= \begin{bmatrix} z_2 \\ -\frac{k}{J}I + a_1 \sin(z_1) \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} z_2 \\ a_1 \cos(y)e_3 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \beta_z \\ 0 \end{bmatrix}, \end{aligned} \quad (20)$$

where the signal  $\beta_z$  is available,

$$\beta_z := -\frac{k}{J}I + a_1 \sin(y) - a_1 \cos(y)\hat{z}_3.$$

If the velocity  $z_2$  is measured, then the reduced-order Luenberger-like linear observer of  $d$  can be constructed as

$$\begin{aligned} \dot{v} &= \beta_z, \\ \hat{z}_3 &= L_0(v - z_2), \\ \hat{d} &= \hat{z}_3, \end{aligned} \quad (21)$$

where  $L_0$  is the scalar design parameter. Then

$$\dot{e}_3 = -L_0 a_1 \cos(y) e_3. \quad (22)$$

As we consider the pendulum around the upper equilibrium, it is reasonable to impose the following assumption.

*Assumption 2: For trajectories of the system there exists  $c_0 > 0$  such that*

$$\min(\cos(\theta + d), \cos(\theta)) \geq c_0$$

*along these trajectories.*

Under Assumption 2, it is obvious that choosing  $L_0 > 0$  in (22) yields to exponential convergence of  $e_3$  to zero. With the estimator (21), the stabilizing control law (8) can be rewritten as

$$u = -K \begin{bmatrix} y - \hat{d} & \hat{\theta} & \hat{\theta}_r \end{bmatrix}^\top. \quad (23)$$

### B. CLOSED-LOOP CONVERGENCE

Let us now consider what happens when the reduced-order bias observer is in the loop with the homogeneous velocity observers proposed in Section III, where the state  $z_2$  in (21) is not measured directly but generated by the observer (10). At the same time, the estimate  $\hat{z}_3$  is used to compensate the bias, and we substitute  $y - \hat{z}_3$  in place of  $\theta$  in (10). The joint observers dynamics is now given by

$$\begin{aligned} \dot{\hat{z}}_1 &= \hat{z}_2 - k_{p,1} [\hat{z}_1 + \hat{z}_3 - y]^{\alpha_p}, \\ \dot{\hat{z}}_2 &= \beta_z - k_{p,2} [\hat{z}_1 + \hat{z}_3 - y]^{2\alpha_p - 1}, \\ \dot{v} &= \beta_z, \\ \hat{z}_3 &= L_0(v - \hat{z}_2). \end{aligned} \quad (24)$$

To proceed with the dynamics analysis, note that  $\hat{z}_1 + \hat{z}_3 - y = e_1 + e_3$ . Denote  $s := [e_1 + e_3 \ e_2 \ e_3]^\top$ . Then the error dynamics of the observer yields

$$\begin{aligned} \dot{s}_1 &= s_2 - k_{p,1} [s_1]^{\alpha_p} + L_0 k_{p,2} [s_1]^{2\alpha_p - 1}, \\ \dot{s}_2 &= -a_1 \cos(z_1) s_3 - k_{p,2} [s_1]^{2\alpha_p - 1}, \\ \dot{s}_3 &= L_0 k_{p,2} [s_1]^{2\alpha_p - 1}. \end{aligned} \quad (25)$$



Since  $\cos(z_1) \geq c_0$ , the only equilibrium of (25) is the origin  $s = e = 0$ . However, the error dynamics (25) is not homogeneous for  $\alpha_p < 1$  and thus it does not enjoy the finite-time stability property. An important observation is that for  $\alpha_p = 1$  the system (25) becomes a linear time-varying (due to  $\cos(z_1)$ ) system, for which stability analysis can be carried out using the standard arguments. In what follows, we study the behavior of the error dynamics for  $\alpha_p$  close to 1 and show that local asymptotic convergence can be analyzed using matrix inequalities, which also illustrate a trade-off between the domain of attraction and the convergence rate, see Remark 4 below.

Denote  $\Delta_\alpha := 1 - \alpha_p \geq 0$ . Then the first-order series expansion around  $\alpha_p = 1$  yields

$$\dot{s} = \left( A_0(z_1) - A_1 \ln(s_1^2) \Delta_\alpha \right) s \quad (26)$$

where

$$A_0(z_1) := \begin{bmatrix} -k_{p,1} + L_0 k_{p,2} & 1 & 0 \\ -k_{p,2} & 0 & -a_1 \cos(z_1) \\ L_0 k_{p,2} & 0 & 0 \end{bmatrix},$$

$$A_1 := \begin{bmatrix} L_0 k_{p,2} - \frac{1}{2} k_{p,1} & 0 & 0 \\ -k_{p,2} & 0 & 0 \\ L_0 k_{p,2} & 0 & 0 \end{bmatrix}.$$

Note also that

$$\lim_{s_1 \rightarrow 0} \ln(s_1^2) s_1 = 0.$$

Define  $A_m$  and  $A_M$  as the values of  $A_0(z_1)$  for  $\cos(z_1) = c_0$  and  $\cos(z_1) = 1$ , respectively, where  $c_0$  is defined in Assumption 2.

Suppose there exists  $P = P^\top > 0$  satisfying the following matrix inequalities for some  $\gamma > 0$  and  $\mu \in \mathbb{R}$ :

$$\begin{aligned} Q &:= -\left( PA_1 + A_1^\top P \right) \geq 0, \\ PA_m + A_m^\top P + \mu Q + \gamma P &\leq 0, \\ PA_M + A_M^\top P + \mu Q + \gamma P &\leq 0. \end{aligned} \quad (27)$$

If the inequalities (27) hold then for all  $z_1$  such that  $c_0 \leq \cos(z_1) \leq 1$  we have

$$PA_0(z_1) + A_0^\top(z_1)P \leq -\mu Q - \gamma P.$$

Then for the function  $V = s^\top P s$  we obtain

$$\dot{V} \leq -\gamma s^\top P s - \left( \mu - \ln(s_1^2) \Delta_\alpha \right) s^\top Q s.$$

Define

$$s_M := \exp\left(\frac{\mu}{2\Delta_\alpha}\right), \quad (28)$$

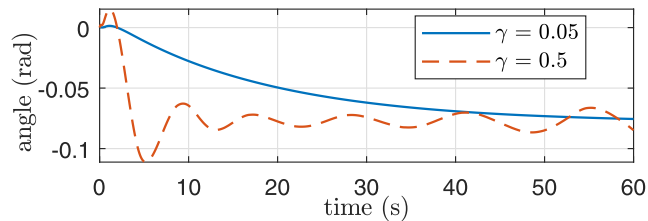
then it holds

$$s_1^2 < s_M^2 \Leftrightarrow \mu - \ln(s_1^2) \Delta_\alpha > 0.$$

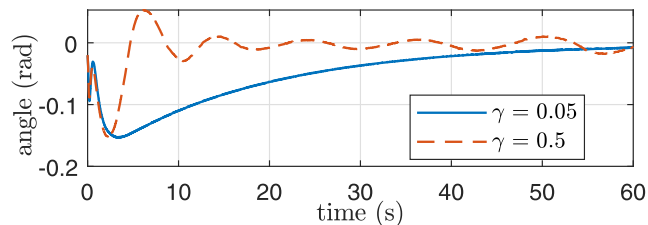
Since  $Q \geq 0$  due to (27), it follows that for  $s_1^2 \leq s_M^2$  we have

$$\gamma P + \left( \mu - \ln(s_1^2) \Delta_\alpha \right) Q > 0 \quad (29)$$

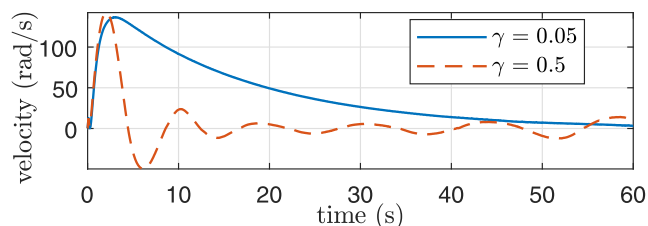
and  $\dot{V} < 0$  for  $s \neq 0$ .



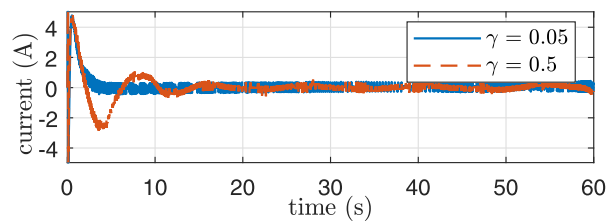
(a) Bias estimate  $\hat{d}$



(b) Estimate of the pendulum position  $\hat{\theta} = y - \hat{d}$



(c) Velocity estimate  $\hat{x}_{r,2}$



(d) Control current  $I(t)$

**FIGURE 5.** The control (10), (12), and (23) with the low-pass bias observer (17) for  $d = -0.08$ .

Let  $C > 0$  be the largest value such that

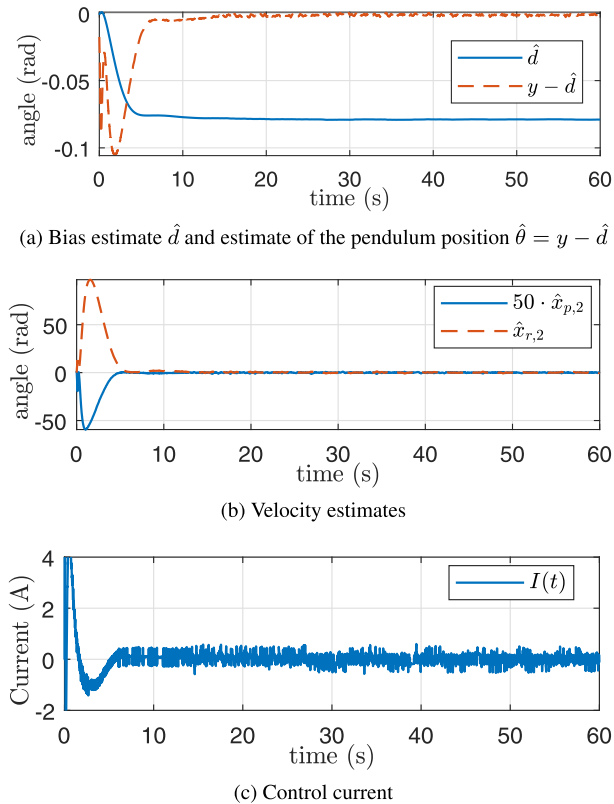
$$s^\top P s < C \Rightarrow s_1^2 < s_M^2.$$

Then the origin of (26) is asymptotically stable with the domain of attraction

$$\Omega := \{s : s^\top P s < C\}.$$

Indeed,  $\dot{V}$  is negative definite on  $\Omega$  and for any initial condition  $s(0) \in \Omega$  we have  $V(s(0)) < C$  and  $\dot{V} < 0$ ; hence, the trajectory  $s(t)$  remains in the set  $\Omega$  for all  $t$ . Recalling the Invariance-like theorem [18], we conclude that  $\dot{V} \rightarrow 0$ . Since for  $s \in \Omega$  the inequality (29) holds, the convergence  $\dot{V} \rightarrow 0$  implies  $s(t) \rightarrow 0$ .

The set  $\Omega$  is attractive for the first-order expansion system (26). By continuity, if  $\Delta_\alpha$  is small enough, then there exists a set  $\Omega_\alpha \subset \Omega$  that is attractive for the system (25).



**FIGURE 6.** The control (10), (12), and (23) with the reduced order bias observer (21) for  $d = -0.08$ .

These results are summarized in the following proposition, whose proof follows from the derivations above.

*Proposition 3:* Consider the observer (24) for the system (20) under Assumption 2. Choose parameters such that the LMIs (27) are feasible. Then there exist  $\varepsilon_\alpha > 0$  and a compact set  $\Omega_\alpha$ , such that for  $\alpha \in (1 - \varepsilon_\alpha, 1]$  and all initial conditions  $s(0) \in \Omega_\alpha$  it holds  $s \rightarrow 0$  and  $\lim_{t \rightarrow \infty} |\hat{z} - z| = 0$ .

*Remark 4:* The matrix inequalities (27) are linear for fixed  $\gamma > 0$  and  $\mu \in \mathbb{R}$ , thus they can be efficiently solved with the standard LMI tools, e.g., YALMIP [19].

### C. EXPERIMENTAL RESULTS UNDER BIASED MEASUREMENTS

In our experiments, we consider the same hardware as in Section III-D with the parameters listed in Table 1. As it is shown in Section III, if the optical encoder is perfectly adjusted and  $d = 0$  in (6), then the control law (8), (10), and (12) achieves the goal and stabilizes the pendulum; however, if the measurements are biased then the system does not converge to the desired equilibrium, see Fig. 4.

In this section, we present experimental results for the case when the differentiators-based observers (10), (12) are empowered with bias estimation. We compare both the low-pass bias observer (17) and the reduced-order observer (21); the control signal is computed as (23). The parameters of the differentiators and the state-feedback gains

are taken the same as in Section III-D, see Table 2, and parameters of the bias observers are given below. As in Section III-D, we consider the problem of the upright pendulum stabilization, where the initial position is in a neighborhood of the equilibrium and the sensor offset  $d$  in (6) is  $d = -0.08$ .

Results of the experiments with the velocity observers (10), (12), bias observer (17), and control law (23) are depicted in Figure 5 for  $\gamma_d = 0.5$  and  $\gamma_d = 0.05$  and illustrate the trade-off between the transient time and the closed-loop stability: for  $\gamma_d = 0.05$  the system is stabilized in approximately 60 seconds, and for  $\gamma_d = 0.5$  transients are faster but closed-loop oscillations appear.

Finally, we perform an experiment with the reduced-order bias observer (21). For this experiment, the gain  $L$  in (21) is chosen as  $L = 0.01$ . It can be numerically verified that for this value of  $L$  the matrix inequalities (27) are feasible for  $\mu \leq -0.2$  and  $\gamma = 0.62$  yielding  $s_M \approx 0.36$ . Results of the experiment are shown in Figure 6 and illustrate stabilization of the pendulum with fast transients.

### VI. CONCLUSION

Motivated by the vertical stabilization of a walking robot, we have studied the bias propagation in a nonlinear model-based differentiator applied to the reaction-wheel inverted pendulum. It has been shown that the bias leads to velocity estimation errors that compromises the stabilization.

To this end, we have proposed to combine the differentiator with a reduced-order bias observer. For the coupled bias and velocity observer, the local (in initial estimation errors) asymptotic convergence has been shown. The theoretical results of the paper are supported with the results of experimental studies.

One possible direction of further research is to address the robustness of the proposed approach with respect to time-varying parameters and model uncertainties, which are typical for advanced control applications, e.g., for walking robot stabilization.

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