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# Incipient Fault Detection and Estimation Based on Jensen-Shannon Divergence in a Data-Driven Approach

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## Abstract

Most data-driven diagnosis methods that are designed to detect faults, rely on measuring the mean and variation shifts. However, for incipient fault detection, these statistical criteria are slightly varying and are difficult to be accurately evaluated to reach good performances. Indeed, such faults are more likely to induce slight changes on the probability distribution rather than particular parametric changes. Therefore, the Jensen-Shannon Divergence (JSD), characterized by high sensitivity in measuring minor changes between probability distributions, is proposed in this paper. Its efficiency for detection and estimation is theoretically studied and validated by simulated data considering an auto-regressive (AR) system designing a multivariate data-driven process. The superior detection performances are demonstrated and compared with other more traditional statistical tests such as the Hotelling's  $T^2$  and the Squared Prediction Error (SPE) in the Principal Component Analysis (PCA) framework. Minor crack detection based on eddy-currents testing (ECT) experimental data are evaluated to highlight the performances of our proposal. The results show that JSD can detect minor cracks ( $0.01mm^2$  to  $0.04mm^2$ ) which were not possible when using the baseline impedance signal measurement. For the fault severity estimation, the accuracy of the theoretical model derived for Gaussian distributed signals is shown with an AR system. The maximum relative estimation error obtained in the worst faults severity conditions is smaller than 2.75% when the Signal to Noise Ratio (SNR) is larger than 25dB and smaller than 2.15%

when the Fault to Noise Ratio (FNR) is larger than  $-21\text{dB}$ . Application for the fault severity estimation on the ECT data validates the effectiveness of this fault estimation model.

*Keywords:* Incipient fault, Fault detection and estimation, Jensen Shannon Divergence, Data-driven process, Principal Component Analysis.

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## 1. Introduction

Incipient fault diagnosis plays a key role in the automation of inspection procedure and minimization of maintenance activities and costs [1, 2]. An incipient fault is mainly characterized by its slowly developing behavior and its barely noticeable effects [3, 4, 5]. Besides this type of defect is difficult to detect, it is dangerous for a system if its severity is underestimated. The early detection of the incipient fault can help to schedule preventive maintenance and prevent more serious failures [6]. For quantifying the fault severity, the estimation of the incipient fault is necessary.

The existing methods of fault detection and estimation can be generally classified as model-based and data-driven-based solutions [7, 8]. The model-based approach is related to the theoretical derivation of the system behavior. Its performances widely depend on the accuracy of the mathematical model which is degraded due to uncertainties, evolution of operating conditions, environmental changes, etc [9]. In contrast to the model-based approach, the process-history-based methods are based on the availability of a sufficient amount of historical process data to perfectly describe the process behavior using well-chosen descriptive features [5, 10, 11]. Basically, techniques that are often used for data-driven approaches are linked to the evaluation of statistical moments with order 1 to 4 (sample mean, variance, skewness, and kurtosis), Hotelling's  $T^2$ , squared prediction error (SPE), Cumulative Sum (CUSUM), ... [7, 12, 13]. These methods are efficient in measuring the parameter change of the data but are not accurate enough for incipient fault detection. These later seem more likely to cause a change of the probability distribution rather than obvious parameter change [14]. Particular techniques based on specific distance measures have then to be considered to detect such faults [15, 16]. Jensen-Shannon divergence (JSD) is a particular symmetrical operation of Kullback-Leibler information. It is a well-known tool for detecting the dissimilarities between probability distributions [17, 18]. It has been used in many domains such as image processing, text categoriza-

31 tion, and subject recognition. Its superior change detection capability has  
32 been well observed in [19, 20, 21, 22] for particular contexts. Nevertheless,  
33 to the best of our knowledge, the efficiency of this technique has never been  
34 considered in the particular working conditions of incipient fault that can be  
35 masked by the noise environment.

36 In this paper, JSD is proposed to detect incipient fault for different noise  
37 levels in a multivariate data-driven process. Firstly, a theoretical model is de-  
38 veloped to prove the detection efficiency of the technique. As a preprocessing  
39 and feature extraction method, Principal Component Analysis (PCA), which  
40 effectiveness in fault detection and estimation is well known [23, 24, 25, 26],  
41 is used to reduce the dimensionality of the processed variables [27] and ex-  
42 tract the most informative feature. Meanwhile, the maximum information  
43 is kept in this new feature space and the probability density functions of  
44 the system in healthy and faulty conditions can be derived. The detection  
45 performances are evaluated considering JSD but also typical statistics algo-  
46 rithms (Hotelling’s  $T^2$  and SPE) with well-proved detection capabilities in  
47 process health monitoring [7, 26, 28, 29]. These performances are shown us-  
48 ing a simulated AR process and compared to each other. To experimentally  
49 validate this detection step, minor material cracks based on Eddy Current  
50 Testing (ECT) data are considered.

51 The existence of material crack is harmful to the safety of an industrial  
52 system particularly in the transportation industry (planes, rockets, trains,  
53 vehicles,  $\dots$ ). In order to detect these cracks, a regular inspection of the  
54 system is required during all the production and maintenance process. Non-  
55 destructive evaluation (NDE) technique can be used to test the inside defects  
56 of materials and structures without affecting its operating conditions [30, 31].  
57 For conductive metallic materials, Eddy current based techniques such as  
58 Pulsed Eddy current (PEC) or Eddy Current Testing (ECT) are the main  
59 typical ones that are most often used as non-destructive testing methods  
60 for crack detection [32, 33, 34]. They are based on the principle that the  
61 inhomogeneity due to the defect will cause measurement shifts compared  
62 to the initial healthy conditions. To be applied in high-dimensional data  
63 conditions, fault feature extraction methods such as Principal Component  
64 Analysis (PCA) [35], Independent Component Analysis (ICA) [36], or Fisher  
65 Discriminant Analysis (FDA) [37] have been applied for the cracks diagnosis.  
66 Most of these works are focused on large cracks detection. This implies a  
67 significant visible signature due to the presence of the fault in the eddy  
68 current signals with a very low influence of the environmental noise. However,

69 in the case of minor cracks, which can be considered as incipient faults,  
70 the measurement variation due to the fault is weak. The fault signature  
71 compared to the roughness leads to a measured signal that does not clearly  
72 indicate the presence of the crack. Particularly, the measurement accuracy is  
73 largely affected by external measurement environment, the roughness of the  
74 surface, internal and sensor noise sensitivity,  $\dots$  [33, 38]. Thus, the detection  
75 and diagnosis for these incipient cracks in a high noise level are more tricky.  
76 In this paper, Jensen-Shannon divergence is proposed and used for incipient  
77 cracks detection in a high noise level based on the experimental ECT data.  
78 With our proposal, we show that it is possible to detect smaller cracks than  
79 those that have already been evaluated in the literature to the best of our  
80 knowledge.

81 The second aim of this paper concerns the fault severity estimation. Based  
82 on the JSD, we propose a theoretical model derived for Gaussian distributed  
83 data. The efficiency of this estimation process is validated using the simu-  
84 lated AR process and the experimental ECT data. As the incipient faults  
85 lead to very low fault severities they can be masked by the environmental  
86 noise of the process. Then, the accuracy of our estimation results is dis-  
87 cussed for several operating conditions corresponding to different Signal to  
88 Noise Ratio (SNR) and Fault to Noise Ratio (FNR). With our proposal this  
89 estimation is, to the best of our knowledge, the first that have been obtained  
90 for those crack size in noisy environment. Offering a slight overestimation of  
91 the fault severity and a very low error rate, this proposal allows to have a  
92 valuable safety margin in the health monitoring process.

93 This paper is organized as follows. The detailed incipient fault diagno-  
94 sis process and main notations are described in Section 2. Section 3 and  
95 4 respectively presents the derived incipient fault detection and estimation  
96 models with the evaluation of their performances and their application to  
97 ECT data. The last section 5 concludes the paper.

## 98 2. Incipient Fault Diagnosis Process

### 99 2.1. Main notations and fault model

100 Let's first define the main notations used throughout the paper:

- 101 •  $\mathbf{X}_{[N \times m]} = (\mathbf{x}_1, \dots, \mathbf{x}_j, \dots, \mathbf{x}_k, \dots, \mathbf{x}_m)$  represents original data matrix,  
102 in which  $m$  is the number of variables and  $N$  is the sample size.  $\mathbf{x}_k$  is the  
103  $k^{th}$  vector such as  $\mathbf{x}_k = [x_{1k}, \dots, x_{ik}, \dots, x_{Nk}]^\top$ , where  $i = [1, \dots, N]$

104 is the sample number.  $x_{ik}$  is then the  $i^{th}$  data sample of the  $k^{th}$  vari-  
 105 able.  $\mathbf{x}_j$  denotes the variable affected by the fault occurred within the  
 106 sampling interval  $[b, N]$ ,  $(\top)$  is the transpose matrix operator.

- 107 •  $\mathbf{S}$  is the sample data covariance matrix,
- 108 •  $\bar{\mathbf{X}}_{[N \times m]}$  is the centered and normalized matrix.
- 109 •  $l$  corresponds to the dimension of the principal subspace, leading to  
 110 the  $l$  first principal components. The  $m - l$  remaining ones give the  
 111 dimension of the residual subspace.
- 112 •  $\mathbf{P}_{[m \times m]}$  is the eigenvectors matrix (also denoted the loadings matrix) of  
 113  $\mathbf{S}$  associated to the eigenvalues  $\mathbf{\Lambda}$ , such as  $diag(\mathbf{\Lambda}) = [\lambda_1, \dots, \lambda_k, \dots, \lambda_m]$ .
- 114 •  $D_{JS}(\cdot)$  denotes Jensen-Shannon Divergence,  $D_{JS_{A_n}}$  is the approximated  
 115 theoretical equation of JSD derived for Gaussian distributed data.
- 116 •  $l$  is the crack length, and  $d$  is crack depth.
- 117 • pdf denotes the probability density function.
- 118 •  $P_{FA}$  refers to the probability of false alarm, and  $P_D$  is the probability  
 119 of detection.
- 120 • The mark  $(*)$  refers to faultless and noise-free data, mark  $(\sim)$  mentions  
 121 the faulty and noise-free data function,  $(\simeq)$  marks the faulty and noisy  
 122 data,  $(\hat{\cdot})$  marks the estimated function.

123 For our work, the fault model is based on the assumption that the fault  
 124 severity is constant. The fault occurs at instant time  $b$  on the  $j^{th}$  variable,  
 125 whereas its last  $(N - b)$  observations are affected by the fault.

126 A white Gaussian signal  $\mathbf{v}_k \sim \mathcal{N}(0, \sigma_v^2)$  is used to simulate the noise in  
 127 the environment. The noise matrix is then  $\mathbf{V}_{[N \times m]} = (\mathbf{v}_1, \dots, \mathbf{v}_k, \dots, \mathbf{v}_m)$ ,  
 128 where  $\mathbf{v}_j = [v_{1j}, \dots, v_{ij}, \dots, v_{Nj}]^\top$  is the noise vector of  $N$  samples that  
 129 affecting the  $j^{th}$  variable. Thus, the faulty variable  $\mathbf{x}_j$  can be written as:

$$\mathbf{x}_j = \mathbf{x}_j^* + \mathbf{F}_j + \mathbf{v}_j \quad (1)$$

130 where  $\mathbf{F}_j$  is the fault and its structure is denoted as:

$$\mathbf{F}_j = g \times [0 \dots 0 \dots x_{bj}^* \dots x_{Nj}^*]^\top \quad (2)$$

131 where  $g$  is the fault severity amplitude.

132 *2.2. Diagnosis process*

133 The diagnosis procedure proposed for fault detection and estimation of a  
 134 multivariate data-driven process in the Principal Component Analysis frame-  
 135 work is shown in Fig.1.

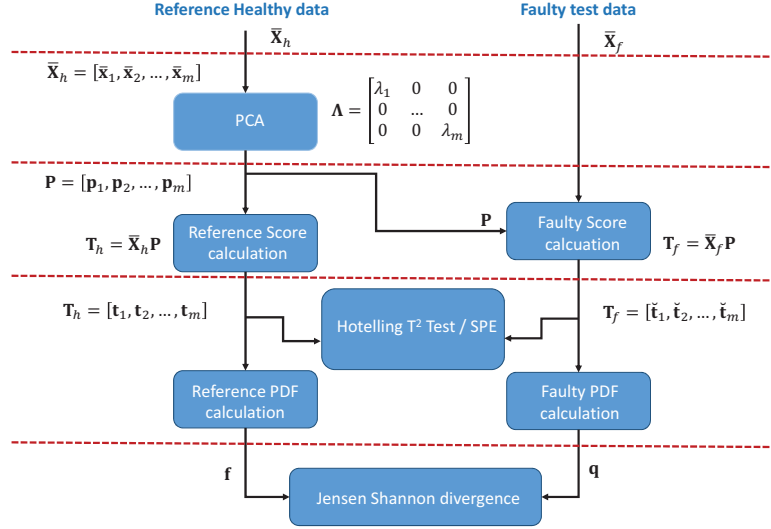


Figure 1: Proposed fault diagnosis procedure

136 The main operations of the diagnosis procedure based on JSD, T<sup>2</sup> and  
 137 SPE are described in the following items:

- 138 • The input data of PCA are first preprocessed and mean-centered. The  
 139 PCA model is then established based on the reference healthy data.  
 140 So, the eigenvalues and eigenvectors matrices  $\mathbf{\Lambda}$  and  $\mathbf{P}$  are derived  
 141 respectively.
- 142 • The reference score  $\mathbf{T}_h$  and the faulty score  $\mathbf{T}_f$  are obtained from the  
 143 data matrices using the same eigenvector matrix  $\mathbf{P}$ .
- 144 • The probability densities of each latent score of  $\mathbf{T}_h$  and each latent  
 145 score of  $\mathbf{T}_f$  are estimated using a normal kernel estimator.
- 146 • The diagnosis using Jensen-Shannon divergence is based on the pdfs  
 147 of the first principal component scores of  $\mathbf{T}_h$  and  $\mathbf{T}_f$ . The diagnosis  
 148 using Hotelling's T<sup>2</sup> is based on the principal component scores in the  
 149 principal subspace, and SPE uses the scores of the residual one.

150 *2.3. Materials and methods for our proposal*

151 *2.3.1. Principal Component Analysis*

152 In fault detection process for multivariate systems, Principal Component  
 153 Analysis (PCA) has been widely used as preprocessing (filtering and dimen-  
 154 sion reduction) or feature extraction (fault feature, component selection, . . . )  
 155 [24, 25, 26, 29, 30, 39]. In our study, PCA is used for dimension reduction  
 156 and component selection. Its major steps can be summarized as follows.

157 First, the covariance matrix  $\mathbf{S}$  of sample data can be defined as:

$$\mathbf{S} = \frac{1}{N-1} \bar{\mathbf{X}}^\top \bar{\mathbf{X}} \quad (3)$$

158 where  $\bar{\mathbf{X}}_{[N \times m]}$  is the centered and normalized matrix. Each vector of this  
 159 matrix  $\bar{\mathbf{X}}$  is written as:

$$\bar{\mathbf{x}}_k = \frac{\mathbf{x}_k - \mu_k}{\sqrt{\sigma_k^2}} \quad (4)$$

160 where  $\mu_k$  and  $\sigma_k^2$  are the mean and the variance for the  $k^{th}$  variable ( $k =$   
 161  $1, 2, \dots, m$ ). The Principal Component scores matrix  $\mathbf{T}_{[N \times m]}$  can be deter-  
 162 mined using the linear transformation:

$$\mathbf{T}_{[N \times m]} = \bar{\mathbf{X}}_{[N \times m]} \mathbf{P}_{[m \times m]} = (\mathbf{t}_1, \dots, \mathbf{t}_k \dots, \mathbf{t}_m) \quad (5)$$

163 where  $\mathbf{P} = (\mathbf{p}_1, \dots, \mathbf{p}_l, \dots, \mathbf{p}_m)$  is the eigenvectors matrix of  $\mathbf{S}$  associated to  
 164 the corresponding eigenvalues  $\lambda_1, \dots, \lambda_l, \dots, \lambda_m$ .

165 The principal subspace is defined by the first  $l$  principal components, and  
 166 the residual one is determined by the remaining  $(m-l)$  components. In our  
 167 paper, the number of principal components (PCs)  $l$  is obtained calculating  
 168 the Cumulative Percent of Variance (CPV) [39, 40].

$$\text{CPV}(l) = \frac{\sum_1^l \lambda_k}{\sum_1^m \lambda_k} \geq 90\% \quad (6)$$

169 The number of PCs cumulatively contributing to more than 90 percent  
 170 data variance will lead us to the principal subspace.

171 *2.3.2. Hotelling's  $T^2$  and SPE*

172 Typical detection indices are distance-based, aiming at evaluating how  
 173 much a new observation is away from each of the subspaces. The Hotelling's  
 174  $T^2$  is the typical statistic test based on the principal subspace, and SPE is the



175 typical statistic criterion based on the residual subspace. The observations  
 176 of  $T^2$  instant  $i$  is:

$$T^2 = \sum_{k=1}^l \frac{t_{ik}^2}{\lambda_k} \quad (7)$$

177 The threshold for  $T^2$  can be approximated by:

$$T_{l,\alpha}^2 = \frac{l(N^2 - 1)}{N(N - l)} \mathcal{F}_{l,N-l,\alpha} \quad (8)$$

178 where  $\mathcal{F}_{l,N-l,\alpha}$  is the Fisher distribution with two degrees of freedom,  $l$  and  
 179  $N - l$ .

180 The observations of SPE at instant  $i$  is:

$$SPE = \sum_{k=l+1}^m t_{ik}^2 \quad (9)$$

181 The theoretical threshold of SPE at significance level  $\alpha$  is:

$$\delta_\alpha^2 = \xi \chi_{h,\alpha}^2 \quad (10)$$

182 where  $\xi = \gamma_2/\gamma_1$ ,  $h = integer(\gamma_1^2/\gamma_2)$ ,  $integer(o)$  is the integer value of  $o$  and  
 183  $\chi_{h,\alpha}$  is the Chi-square distribution with  $h$  degrees of freedom. The constant  
 184  $\gamma_c$  is calculated as  $\gamma_c = \sum_{k=l+1}^m \lambda_k^c$  and  $\lambda_k^c$  is the  $k^{th}$  eigenvalue to the  $c^{th}$   
 185 power ( $c=\{1,2\}$ ).

### 186 2.3.3. Jensen-Shannon divergence

187 Jensen Shannon divergence (JSD) is a sensitive technique based on Shan-  
 188 non entropy excess of a couple of distributions in regard to the mixture of  
 189 their respective entropies [and without assumptions on their types](#) [41]. Con-  
 190 sidering  $\mathbf{f}(t)$  and  $\mathbf{q}(t)$  are continuous probability density functions (pdfs)  
 191 corresponding to the random variable  $t$ , JSD is the increment of the Shan-  
 192 non entropy. It is defined by a function of the Shannon entropy ( $S_E$ ) and its  
 193 value is denoted  $D_{JS}$  such as:

$$D_{JS}(\mathbf{f}, \mathbf{q}) = S_E \left[ \frac{\mathbf{f} + \mathbf{q}}{2} \right] - \frac{S_E(\mathbf{f}) + S_E(\mathbf{q})}{2} \quad (11)$$

194 The JSD can be also written as the symmetric operation of the Kullback-  
 195 Leibler information theoretical function denoted as:

$$D_{JS}(\mathbf{f}, \mathbf{q}) = \frac{1}{2} I(\mathbf{f}||M) + \frac{1}{2} I(\mathbf{q}||M) \quad (12)$$

196 where  $M = \frac{1}{2}(\mathbf{f} + \mathbf{q})$  is a mixture distribution,  $I$  is the Kullback-Leibler  
 197 information [42] defined as:

$$I(\mathbf{f}||\mathbf{q}) = \int \mathbf{f}(t) \log \frac{\mathbf{f}(t)}{\mathbf{q}(t)} dt \quad (13)$$

198 As the divergence has no closed-form, the integral function (13) is numer-  
 199 ically calculated using the Monte-Carlo approximation.

200 Otherwise, when the divergence value is theoretically equal to zero, it  
 201 means that the two considered probability density functions are exactly the  
 202 same: the reference one and the tested one are both obtained in healthy  
 203 condition. In real life applications and experimentally measured data, these  
 204 two distributions won't be exactly the same. Due to the environmental nui-  
 205 sances, the two functions will be slightly different: this will lead to a low  
 206 non-zero divergence due to the random noises. To proceed to an efficient  
 207 decision and accurately highlight whether the incipient crack exists, the de-  
 208 tection must be done with respect to a given threshold. This threshold is  
 209 settled by evaluating the JSD on the pdfs for the process data in healthy  
 210 conditions. Below this threshold, a minimized number of false alarms must  
 211 be considered meaning that there will be a reduced number of false informa-  
 212 tion about the presence of an incipient crack. At the same time, a maximized  
 213 number of true detection of a considered incipient crack should be obtained.  
 214 The evolution of this threshold is then carried out with the receiver operating  
 215 curve representation highlighting the detection performances.

### 216 3. Incipient Fault Detection

#### 217 3.1. Fault detection theoretical model

218 As mentioned in the previous section 2, PCA is first applied as the pre-  
 219 processing method and feature extraction method. Let's denote  $\mathbf{q}_k$  and  $\mathbf{f}_k$   
 220 the pdfs of the first  $l$  principal score obtained in faulty and healthy con-  
 221 ditions respectively. For Gaussian distributed signals, we can assume that  
 222 the principal scores are Gaussian distributed. Then,  $\mathbf{f}_k \sim \mathcal{N}(\mu_1, \sigma_1^2)$  and  
 223  $\mathbf{q}_k \sim \mathcal{N}(\mu_2, \sigma_2^2)$ , where  $\mu_1, \mu_2$  are the means and  $\sigma_1^2, \sigma_2^2$  are the variances.  
 224 The mixture distribution  $M_k$  is computed by  $M_k = \frac{1}{2}(\mathbf{q}_k + \mathbf{f}_k)$ .

225 In the PCA's model, the mean of the distribution is supposed unchanged  
 226 after the incipient fault occurrence [25, 43] and described in (14).

$$\mu_1 = \mu_2 \quad (14)$$

227 And for two normally distributed functions sharing the same mean, the com-  
 228 bined distribution is unimodal [44]. In the particular case of incipient fault  
 229 detection, the change on the pdf induced by the incipient fault is very slight.  
 230 Thus, we can consider the assumption that the mixture distribution  $M_k$  is  
 231 still normally distributed, such as  $M_k \sim \mathcal{N}(\mu_M, \sigma_M^2)$ . The mean  $\mu_M$  and the  
 232 variance  $\sigma_M^2$  of  $M_k$  can be calculated in (15) and (16):

$$\mu_M = \frac{1}{2}(\mu_1 + \mu_2) \quad (15)$$

233

$$\sigma_M^2 = \mathbb{E}(\mathbf{t}^2) - \mathbb{E}(\mathbf{t})^2 = \frac{1}{2}\sigma_1^2 + \frac{1}{2}\sigma_2^2 \quad (16)$$

234 The expression of the JSD under the Gaussian distribution assumption  
 235 is:

$$D_{JS_{An}}(\mathbf{f}, \mathbf{q}) = \frac{1}{4} \left[ \log \frac{\sigma_M^4}{\sigma_1^2 \sigma_2^2} + \frac{\sigma_1^2 + \sigma_2^2 + \frac{1}{2}(\mu_1 - \mu_2)^2}{\sigma_M^2} - 2 \right] \quad (17)$$

236 In the latter, the variances  $\sigma_1^2$  and  $\sigma_2^2$  in (17) can be rewritten considering  
 237 the component data variance and the additive noise ones such as:

$$\sigma_1^2 = \lambda_k^* + \sigma_v^2 \quad \sigma_2^2 = \tilde{\lambda}_k + \sigma_v^2 \quad (18)$$

238 where  $\lambda_k^*$  and  $\tilde{\lambda}_k$  are the eigenvalues of the  $k^{th}$  latent score under healthy  
 239 and faulty conditions without noise, respectively.  $\sigma_v^2$  is the variance of the  
 240 additive noise.

241 We assume that there is a relation between  $\tilde{\lambda}_k$  and  $\lambda_k^*$  :

$$\tilde{\lambda}_k = \lambda_k^* + \Delta\lambda_k \quad (19)$$

242 where  $\Delta\lambda_k$  is the eigenvalue bias due to incipient the fault occurrence.

243 Combining (16), (18) with (19), the variance of  $M_k$  is obtained:

$$\sigma_M^2 = \frac{1}{2}(2\lambda_k^* + \Delta\lambda_k + 2\sigma_v^2) \quad (20)$$

244 Based on (14), (16) and (20), equation (17) can be transformed into:

$$D_{JS_{An}}(\mathbf{f}, \mathbf{q}) = \frac{1}{4} \log \frac{(2\lambda_k^* + \Delta\lambda_k + 2\sigma_v^2)^2}{4(\lambda_k^* + \sigma_v^2)(\lambda_k^* + \Delta\lambda_k + \sigma_v^2)} \quad (21)$$

245 The obtained equation (21), clearly shows the relation between  $\Delta\lambda_k$  and the  
 246 JSD value.

247 *3.2. Incipient fault detection performances results and discussion*

248 The traditional multivariate AR system with Gaussian distributed data  
 249 is used as an example to evaluate the detection performance of  $T^2$ , SPE and  
 250 JSD. We consider:

$$\mathbf{x}(i) = \begin{bmatrix} 0.118 & -0.191 \\ 0.847 & 0.264 \end{bmatrix} \mathbf{x}(i-1) + \begin{bmatrix} 1 & 2 \\ 3 & -4 \end{bmatrix} \mathbf{u}(i-1) \quad (22)$$

251 
$$\mathbf{y}(i) = \mathbf{x}(i) + \mathbf{v}(i) \quad (23)$$

252 where  $\mathbf{u}$  is the correlated input

$$\mathbf{u}(i) = \begin{bmatrix} 0.811 & -0.226 \\ 0.477 & 0.415 \end{bmatrix} \mathbf{u}(i-1) + \begin{bmatrix} 0.193 & 0.689 \\ -0.320 & -0.749 \end{bmatrix} \mathbf{w}(i-1) \quad (24)$$

253  $\mathbf{w}(i) = [w_1(i) \ w_2(i)]^\top$  is the input vector built with two uncorrelated Gaus-  
 254 sian signals with zero mean and unit variance.  $\mathbf{u}(i) = [u_1(i) \ u_2(i)]^\top$  is  
 255 the measured input vector.  $\mathbf{y}(i) = [y_1(i) \ y_2(i)]^\top$  is the output vector and  
 256  $\mathbf{v}(i) = [v_1(i) \ v_2(i)]^\top$  is the noise vector which built with two uncorre-  
 257 lated Gaussian noise signals with zero mean and variance  $\sigma_v^2$ . The ma-  
 258 trix  $\mathbf{X}$  is formed with the AR measured inputs and outputs at instant  $i$   
 259 with  $i = 1 \dots N$ , i.e.  $\mathbf{X} = [\mathbf{y}_1 \ \mathbf{y}_2 \ \mathbf{u}_1 \ \mathbf{u}_2]^\top$ . Where each vectors  $\mathbf{y}_c$   
 260 and  $\mathbf{u}_c$  can be respectively written as  $\mathbf{y}_c = [y_c(1), \dots, y_c(i), \dots, y_c(N)]^\top$  and  
 261  $\mathbf{u}_c = [u_c(1), \dots, u_c(i), \dots, u_c(N)]^\top$  with  $c = \{1, 2\}$ .

262 After the application of PCA, we obtain four principal components with  
 263 the following eigenvalues  $diag(\mathbf{\Lambda}) = [40.26, 4.9, 1.14, 0.17]$ . The correspond-  
 264 ing percent of variance for the four PCs are then  $[86.64, 10.54, 2.45, 0.37]$ . So,  
 265 the first PC contains 86.64% of the original data overall information and the  
 266 CPV of the first two is 97.18%. With respect to equation (6), the principal  
 267 subspace is then composed of the first two principal components and the  
 268 residual one with the last two components. So, for this validation study, the  
 269 detection process using  $T^2$  is then based on the first two principal component  
 270 scores and SPE is evaluated using the last two components. For our proposal  
 271 using JSD, we will focus only on the first principal component which contains  
 272 a sufficiently significant part of the data information (86.64%).

273 Based on the mentioned fault model, assuming that the fault affects the  
 274 last 10% samples of  $\mathbf{y}_2$ , we have  $y_2(i) = (1+g)x_2(i) + v_2(i)$ . Thus, to highlight  
 275 the detection capabilities, we have affected  $\mathbf{x}_2$  with a fault corresponding to  
 276 80% bias on the signal amplitude in the last 100 samples of  $\mathbf{x}_2$  with  $N = 1000$

277 and  $SNR = 40dB$ . For the  $T^2$  test and SPE, the detection is evaluated on  
 278 each sample  $i$  but for JSD, the results are obtained by doing 900 realisations  
 279 of the healthy conditions and 100 realisations of the faulty ones. The results  
 280 are displayed in Fig.2. The obtained detection results can then be compared.

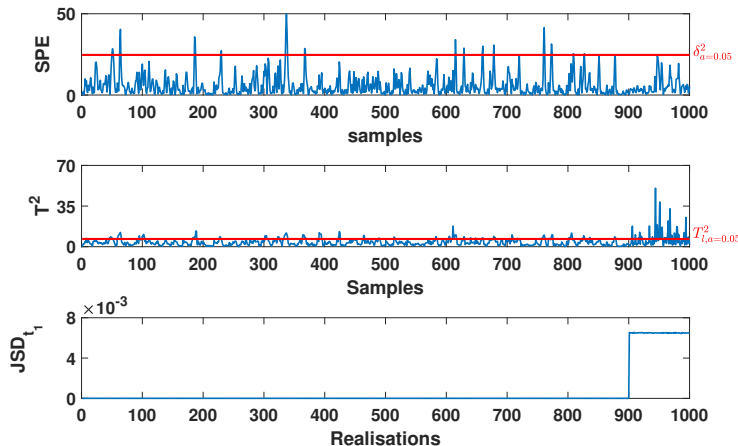


Figure 2: Fault detection performances of JSD,  $T^2$  and SPE

281 It is noted from this example that JSD is superior to the other criteria  $T^2$   
 282 and SPE. Even when the fault severity is very high, the detection with  $T^2$   
 283 and SPE reveals numerous false alarms and missed detections. For a given  
 284  $P_{FA} = 0.05$  the  $P_D$  evaluated for  $T^2$ , SPE and  $JSD$  are respectively 0.63,  
 285 0.07 and 1. Indeed for smallest fault severities,  $T^2$  and SPE will exhibit  
 286 poorer performances compared to JSD.

287 According to equation (21), the detection performances of the proposed  
 288 methodology are largely affected by the evolution of the noise level. To  
 289 highlight this purpose, we propose to evaluate these performances considering  
 290 the noise level compared to the original signal power (signal to noise ratio  
 291 i.e. SNR) or the fault power (fault to noise ratio i.e. FNR) defined as:

$$FNR = 10 \cdot \log_{10} \frac{\sigma_f^2}{\sigma_v^2} \quad (25)$$

$$SNR = 10 \cdot \log_{10} \frac{\sigma_s^2}{\sigma_v^2} \quad (26)$$

292 where  $\sigma_f^2$  is the fault power,  $\sigma_s^2$  is the signal power and  $\sigma_v^2$  is the noise power.

293 Considering that the noise can mask the incipient fault, the comparative  
294 study is first done by setting the SNR to 40dB and varying the fault severity.  
295 The detection performances results for the incipient fault of  $T^2$ , SPE, and  
296 JSD for different FNR are displayed in Fig.3.

297 It is clear that the detection performances of  $T^2$  and SPE are widely  
298 degraded with the decrease of FNR (i.e. lower fault severity can be obtained  
299 at a given noise level).

300 It is demonstrated that  $T^2$  and SPE are totally inefficient for incipient  
301 fault detection. Their detection performances are only acceptable for large  
302 FNR values (e.g. FNR is 65dB) that is corresponding to large fault severities.

303 Conversely, JSD shows excellent performances for incipient faults detec-  
304 tion. The probability of detection is  $P_D = 1$  even when  $FNR = 5$ dB. For  
305 lower fault severities the performances are still good for  $FNR = -5$ dB but for  
306 FNR values lower than  $-10$ dB the detection is more tedious.

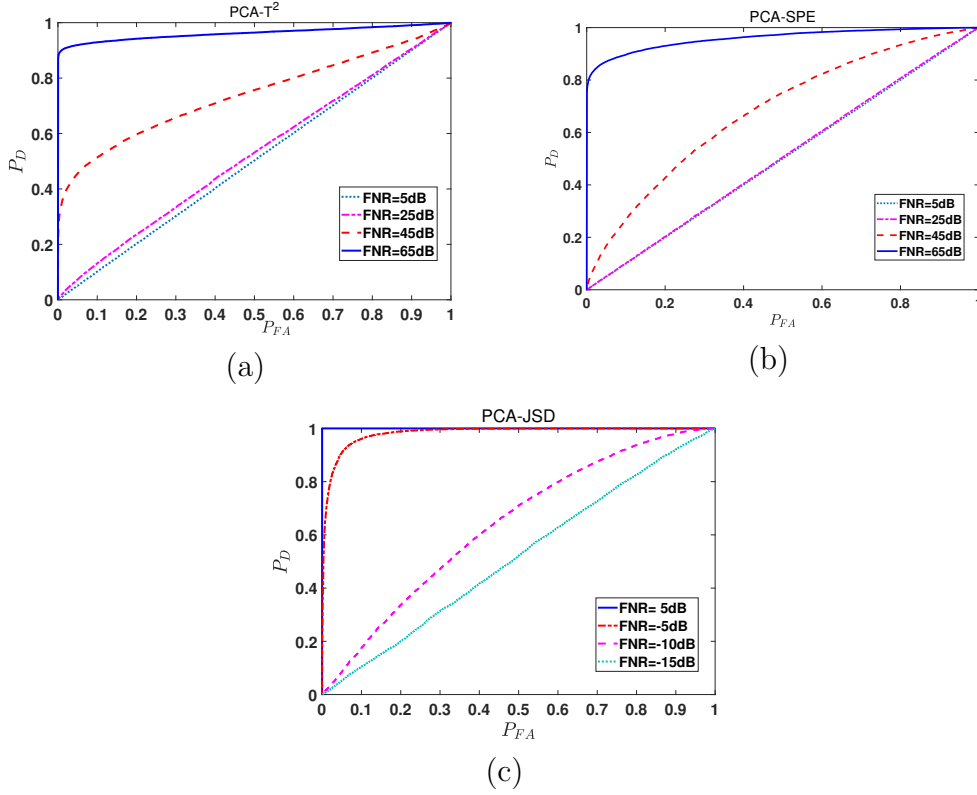


Figure 3: Fault detection performances considering different FNR with SNR=40dB for: (a) T<sup>2</sup>, (b) SPE and (c) JSD

307 In order to show the impact of noise factors on the detection capabilities of JSD, we consider a fault severity  $g = 0.02$ . The detection results  
 308 versus different SNR are given in Fig.4. JSD has high efficiency with 100%  
 309 detection capability for the low noise levels (SNR>35dB) with a very low  
 310 false alarm probability. For the high noise levels (SNR<30dB), the detection  
 311 performances of JSD are affected by the noise. The fault detection performances  
 312 of JSD decrease distinctly along with the SNR decreases (the noise  
 313 levels increase).  
 314

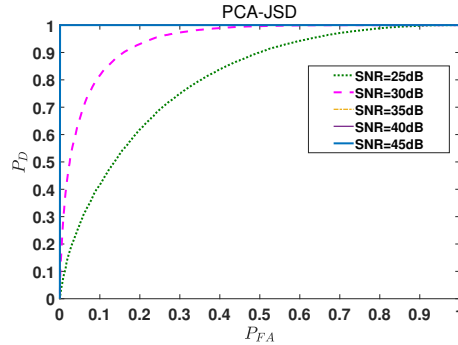


Figure 4: Fault detection results of JSD for different SNR,  $g = 0.02$

315 JSD is then efficient for the detection of incipient faults but its perfor-  
 316 mances are clearly affected when the noise level is very important compared  
 317 to the fault severity one.

### 318 3.3. Application to nondestructive incipient cracks detection

319 Material crack detection becomes more and more important for industrial  
 320 system and structures. The Eddy Current Testing (ECT) technique is a  
 321 widely used measurement approach. It is based on the measurement of the  
 322 magnetic field effects (induced voltage and current flowing in an excitation  
 323 coil) [31]. In Fig.5 we show an example of the change induced on the eddy  
 324 currents trajectory in a conductive material with the presence of a surface  
 325 crack.

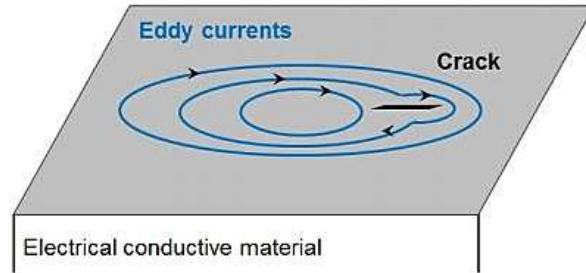


Figure 5: Effect of the presence of a crack on Eddy currents trajectory



326 Then, from these measurements, the evolution of the impedance with the  
 327 presence of the crack  $Z_f$  is obtained and can be written as:

$$Z_f = Z_h + \delta Z = R_f + jY_f \quad (27)$$

328 where the  $R_f$  and  $Y_f$  are respectively the real and imaginary parts of the  
 329 indexed impedance  $Z_f$ . The remaining  $\delta Z$  is the evolution of the healthy  
 330 impedance  $Z_h$  due to the presence of the crack. As an example, Fig.6 repre-  
 331 sents the evolution of the imaginary part of the impedance for a conductive  
 332 material containing cracks with  $d = 400\mu m$  depth and 2 different lengths  
 333  $l = 400\mu m$  and  $l = 600\mu m$  with a 2 MHz excitation frequency. The presence  
 334 of the crack is clearly visible through the variation of the impedance.

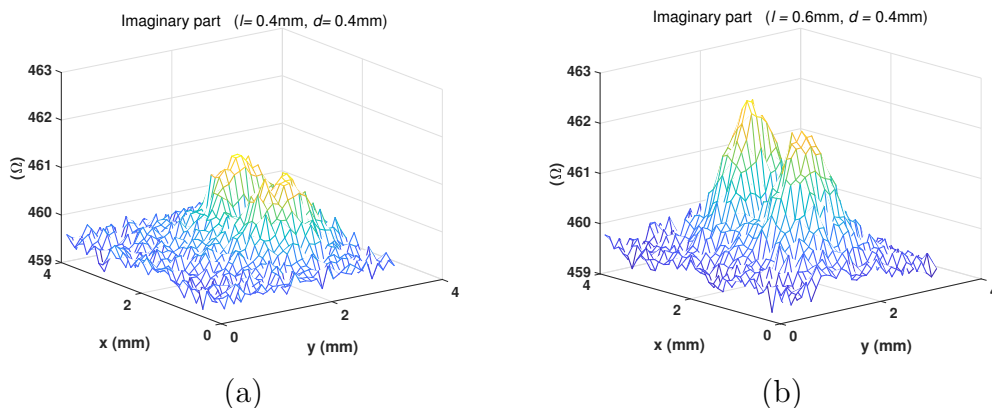


Figure 6: Imaginary part of the impedance of the ECT map for big cracks with sizes: (a)  $l = 0.4mm$  and  $d = 0.4mm$ , (b)  $l = 0.6mm$  and  $d = 0.4mm$

335 However, in the case of smaller cracks (incipient ones), these impedance  
 336 variations can be partially masked by the presence of environmental nuisance.  
 337 In Fig.7 we show the evolution of the imaginary part of the impedance in the  
 338 case of cracks with length or depth equal to  $100\mu m$  or  $200\mu m$ . The evolution  
 339 of the impedance value does not significantly disclose the presence of these  
 340 minor cracks due to the nuisances. Therefore, we propose, in this work,  
 341 to use of the Jensen-Shannon divergence to detect these incipient faults by  
 342 analysing these slight impedance variation. **These impedance variations will**  
 343 **cause slight changes in the faulty probability distribution compared to the**  
 344 **healthy reference one and are then evaluated by using the JSD.**

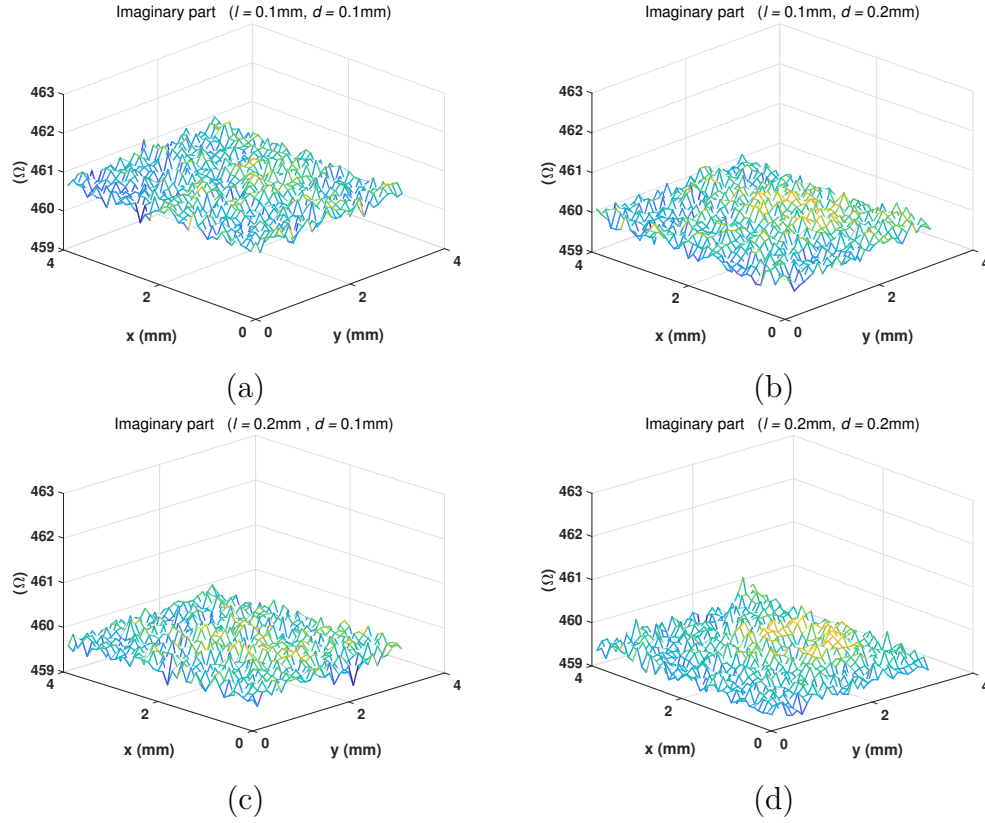


Figure 7: Imaginary part of the impedance of the ECT map for minor crack sizes: (a)  $l = 0.1mm$  and  $d = 0.1mm$ , (b)  $l = 0.1mm$  and  $d = 0.2mm$ , (c)  $l = 0.2mm$  and  $d = 0.1mm$ , (d)  $l = 0.2mm$  and  $d = 0.2mm$

345 In our work, we consider the complex impedance values obtained from an  
 346 ECT experimental system.

347 We mainly focus on incipient cracks such as their length ( $l$ ) or depth ( $d$ )  
 348 are equal to  $100\mu m$  and  $200\mu m$  in several combinations. The perturbation  
 349 level will be considered using several SNR values in the range  $[0, 20dB]$ . The  
 350 considered crack size for this study are then summarized in the following  
 351 table:

Cracks	length ( $l$ )	depth ( $d$ )
$C_1$	0.1mm	0.1mm
$C_2$	0.1mm	0.2mm
$C_3$	0.2mm	0.1mm
$C_4$	0.2mm	0.2mm

352 To apply our diagnosis procedure, complex impedance data obtained in  
353 the healthy and the faulty conditions and considered as impedance maps  
354 with size  $40 \times 32$ . In our study, we focus only the imaginary part of the  
355 impedance that are known to be more sensitive to the presence of the cracks.  
356 These values of the ECT map are arranged into a single row vector ( $1 \times 1280$ )  
357 and are then normalized. A reference map with healthy impedance values  
358 created. We enlarge each normalized ECT map ( $40 \times 32$ ) to a larger map  
359 ( $420 \times 100$ ) so that we can get the sufficient data and reduce the false alarms  
360 caused by the internal and external perturbations. For both healthy and  
361 faulty conditions 100 realisations were evaluated.

362 In Fig.8, we present the evolution of the JSD and the mean of the nor-  
363 malized impedance in healthy and faulty conditions for the crack sizes  $C_1$   
364 and  $C_2$  in a 20dB SNR environment. The dashed red lines are the thresholds  
365 of the JSD and the mean calculated at 99% of their maximum values in the  
366 healthy conditions.

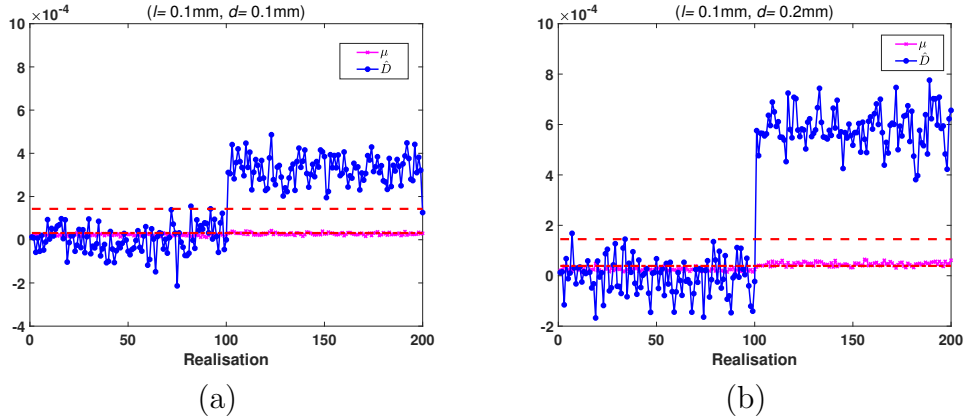


Figure 8: JSD and mean detection capabilities for the minor cracks size: (a)  $C_1$ , (b)  $C_2$

367 This result confirms that the mean fails to detect the crack with size  $C_1$   
368 ( $P_D = 0.24$  and  $P_{FA} = 0.01$ ). For the crack size  $C_2$ , lots of missed detections

369 are obtained ( $P_D = 0.84$  and  $P_{FA} = 0.01$ ). However, with the JSD, satisfying  
 370 detection capabilities are obtained for both considered crack sizes ( $P_D = 1$   
 371 and  $P_{FA} = 0.03$ ).

372 In Fig.9 and Fig.10 we present the detection performances of JSD for the  
 373 four considered incipient cracks sizes  $C_1, C_2, C_3, C_4$  with different perturba-  
 374 tion levels.

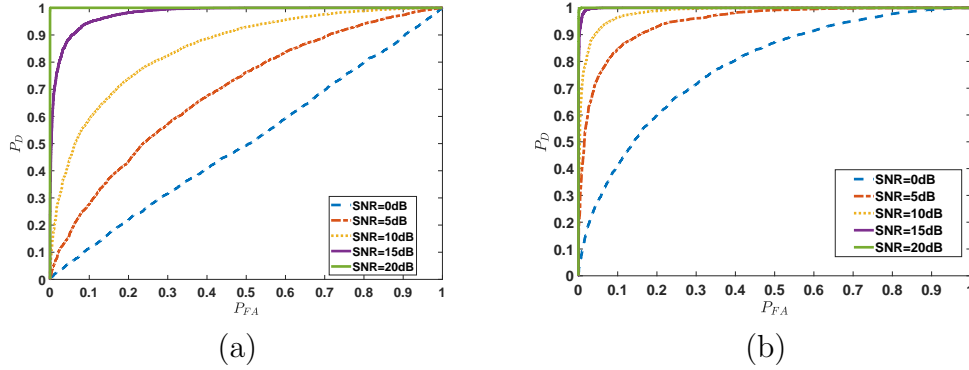


Figure 9: JSD detection performances for different perturbation level and for incipient cracks sizes: (a)  $C_1$  and (b)  $C_2$

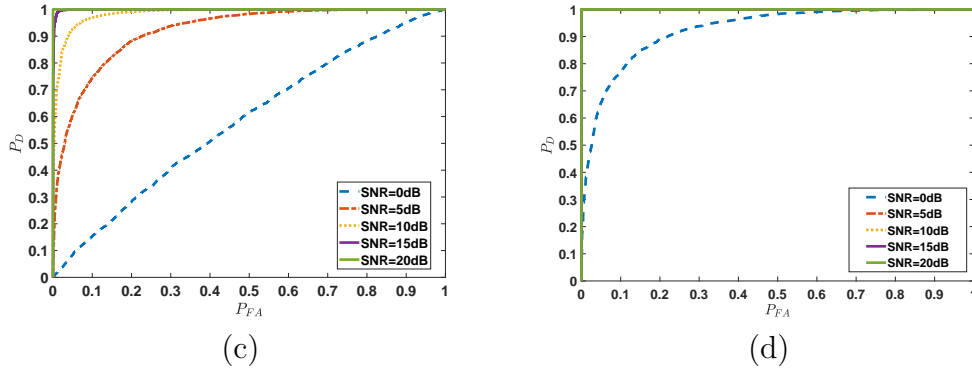


Figure 10: JSD detection performances for different perturbation levels and incipient cracks sizes: (c)  $C_3$  and (d)  $C_4$

375 The Figs clearly illustrate that the JSD has good detection performances  
 376 for these incipient cracks when the SNR is larger than 10dB even when  
 377 the crack size is  $l = 0.1mm, d = 0.1mm$  ( $C_1$ ). These detection capabilities

378 increase along with the SNR values. The JSD offers perfect detection perfor-  
 379 mances ( $P_D = 1$  with  $P_{FA} = 0$ ) even for the smallest considered crack size  
 380 ( $C_1$ ) when the SNR is 20dB. This particular SNR value corresponds to the  
 381 condition that the plate is smooth and the environmental perturbations are  
 382 well isolated.

383 The perturbations that can be caused by internal or external factors (low  
 384 SNR values) have a large effect on the JSD detection performances. It can be  
 385 clearly noticed from Fig.9-a that the JSD is not efficient for the detection of  
 386 the most incipient considered cracks size  $C_1$  when SNR=0dB. However, when  
 387 the crack size increases, the detection capabilities of JSD at 0dB increases  
 388 (see Fig.9-b, Fig.10-c and Fig.10-d).

#### 389 4. Incipient Fault Estimation

##### 390 4.1. Fault estimation theoretical model

391 Based on equation (19) the fault severity  $g$  can be determined as a func-  
 392 tion of  $\tilde{\lambda}_k$  and it is infinitely derivable in the neighbourhood of  $g \approx 0$ , The  
 393 Taylor development of  $\tilde{\lambda}_k$  can be written as:

$$\tilde{\lambda}_k = \lambda_k^* + \frac{\partial \tilde{\lambda}_k}{\partial g}(0)g + \frac{1}{2} \frac{\partial^2 \tilde{\lambda}_k}{\partial g^2}(0)g^2 + \dots \quad (28)$$

394 In our study, the PCA is based on the covariance matrix  $\mathbf{S}$  denoted as:

$$\mathbf{S} = \frac{1}{N-1} \bar{\mathbf{X}}^T \bar{\mathbf{X}} = \frac{1}{N-1} \begin{bmatrix} \bar{\mathbf{x}}_1^T \bar{\mathbf{x}}_1 & \cdots & \bar{\mathbf{x}}_1^T \bar{\mathbf{x}}_j & \cdots & \bar{\mathbf{x}}_1^T \bar{\mathbf{x}}_m \\ \vdots & & \vdots & & \vdots \\ \bar{\mathbf{x}}_j^T \bar{\mathbf{x}}_1 & \cdots & \bar{\mathbf{x}}_j^T \bar{\mathbf{x}}_j & \cdots & \bar{\mathbf{x}}_j^T \bar{\mathbf{x}}_m \\ \vdots & & \vdots & & \vdots \\ \bar{\mathbf{x}}_m^T \bar{\mathbf{x}}_1 & \cdots & \bar{\mathbf{x}}_m^T \bar{\mathbf{x}}_j & \cdots & \bar{\mathbf{x}}_m^T \bar{\mathbf{x}}_m \end{bmatrix} \quad (29)$$

395 where:

$$\bar{\mathbf{x}}_j = \mathbf{x}_j - \mu_j \mathbf{1} = (\mathbf{x}_j^* - \mu_j^* \mathbf{1}) + (\mathbf{F}_j - g \times \frac{1}{N} \sum_{i=b}^N x_{ij}^* \mathbf{1}) + \mathbf{v}_j \quad (30)$$

396 where  $\mathbf{1}$  is a column vector of  $N$  ones.

397 The first order derivative of the eigenvalue  $\tilde{\lambda}_k$  can be calculated as:

$$\begin{aligned} \frac{\partial \tilde{\lambda}_k}{\partial g} &= \mathbf{p}_k^{*\top} \frac{\partial \mathbf{S}}{\partial g} \mathbf{p}_k^* & (31) \\ &= \frac{2}{N-1} \left( p_{jk} \sum_{r=1}^m p_{rk} \sum_{i=b}^N (x_{ir}^* - \mu_r^*) x_{ij}^* + p_{jk}^2 \sum_{i=b}^N (x_{ij}^* - \frac{1}{N} \sum_{i=b}^N x_{ij}^*)^2 \times g \right) \end{aligned}$$

398 where  $\mathbf{p}_k^*$  is the eigenvector associated to  $\lambda_k^*$  in faultless and noise-free con-  
399 dition.

400 The second order derivative of  $\tilde{\lambda}_k$  can be obtained as:

$$\frac{\partial^2 \tilde{\lambda}_k}{\partial g^2} = \mathbf{p}_k^{*\top} \frac{\partial^2 \mathbf{S}}{\partial g^2} \mathbf{p}_k^* = \frac{2}{N-1} \left( p_{jk}^2 \sum_{i=b}^N (x_{ij}^* - \frac{1}{N} \sum_{i=b}^N x_{ij}^*)^2 \right) \quad (32)$$

401 if we denote that:

$$a_1 = p_{jk} \sum_{r=1}^m p_{rk} \left( \sum_{i=b}^{N-1} (x_{ir}^* - \mu_r^*) x_{ij}^* \right) \quad (33)$$

$$a_2 = 3p_{jk}^2 \sum_{i=b}^N \left( x_{ij}^* - \frac{1}{N} \sum_{i=b}^N x_{ij}^* \right)^2 \quad (34)$$

403 Choosing the first two derivative order of the equation (28) we obtain the  
404 following equation:

$$\Delta \lambda_k = \frac{2}{N-1} a_1 \times g + \frac{1}{N-1} a_2 \times g^2 \quad (35)$$

405 Then, the theoretical fault estimation expression can be obtained by find-  
406 ing the positive solution of (35):

$$\hat{g} = \frac{-a_1 + \sqrt{a_1^2 + (N-1)a_2 \Delta \lambda_k}}{a_2} \quad (36)$$

407 In order to obtain the fault amplitude estimation equation, we must cal-  
408 culate the equation of  $\Delta \lambda_k$  corresponding to the JSD values. As equation  
409 (21) can be seen as a function of  $\Delta \lambda_k$  and it is infinitely derivable in the  
410 neighborhood of zero, the Taylor development of  $D_{JS}$  can be given following  
411 (37).

$$D_{JS}(\Delta\lambda_k) = D_{JS}(0) + \frac{\partial D_{JS}(\Delta\lambda_k)}{\partial \Delta\lambda_k}(0)\Delta\lambda_k + \frac{1}{2} \frac{\partial^2 D_{JS}(\Delta\lambda_k)}{\partial \Delta\lambda_k^2}(0)\Delta\lambda_k^2 + \dots \quad (37)$$

412 We obtain its first order derivative (38) and second order derivative (39)  
413 based on (21).

$$a_3 = \frac{\partial D_{JS}(\Delta\lambda_k)}{\partial \Delta\lambda_k} = \frac{1}{4} \left[ \frac{2}{2\lambda_k^* + \Delta\lambda_k + 2\sigma_v^2} - \frac{1}{\lambda_k^* + \Delta\lambda_k + \sigma_v^2} \right] \quad (38)$$

$$a_4 = \frac{\partial^2 D_{JS}(\Delta\lambda_k)}{\partial \Delta\lambda_k^2} = \frac{1}{4} \left[ \frac{1}{(\lambda_k^* + \Delta\lambda_k + \sigma_v^2)^2} - \frac{2}{(2\lambda_k^* + \Delta\lambda_k + 2\sigma_v^2)^2} \right] \quad (39)$$

414 Choosing the first two order derivatives of Taylor equation to establish  
415 a quadratic equation (40) from which the resolution can derive the approxi-  
416 mated value of  $\Delta\lambda_k$ .

$$D_{JS}(\Delta\lambda_k) = D_{JS}(0) + a_3(0)\Delta\lambda_k + \frac{1}{2}a_4(0)\Delta\lambda_k^2 \quad (40)$$

417 In healthy conditions, the variance change  $\Delta\lambda_k$  is 0, and  $a_3(0)$  and  $a_4(0)$   
418 can be simplified as in (41). Then we find the solution given in (42).

$$a_3(0) = 0 \quad a_4(0) = \frac{1}{4} \left[ \frac{1}{(\lambda_k^* + \sigma_v^2)^2} - \frac{2}{(2\lambda_k^* + 2\sigma_v^2)^2} \right] \quad (41)$$

419

$$\Delta\lambda_k = \sqrt{\frac{2\hat{D}_{JS}}{a_4(0)}} \quad (42)$$

420 Finally, the approximated theoretical estimation of the fault severity  $\hat{g}$   
421 can be obtained by combining (36) and (42) such as:

$$\hat{g} = \frac{-a_1 + \sqrt{a_1^2 + (N-1)a_2(\sqrt{\frac{2\hat{D}_{JS}}{a_4(0)}})}}{a_2} \quad (43)$$

422 where  $\hat{D}_{JS}$  in (43) is calculated using Monte Carlo Simulation. Based on  
423 this equation (43), the fault severity can be estimated using the JSD value.

424 *4.2. Incipient Fault estimation performances results and discussion*

425 To verify the validity of the fault estimation model obtained in (43) for  
 426 incipient fault, we plot in Fig.11 the estimated fault severity ( $\hat{g}$ ) versus the  
 427 real one ( $g$ ) for different SNR values. For this work we have considered the  
 428 same AR Model that has been used in the previous section 3.2.

429 The results indicate that the fault estimation obtained with this model  
 430 leads to a little overestimation of the fault severity. This can be considered  
 431 as a safety margin in a sensible fault diagnosis context.

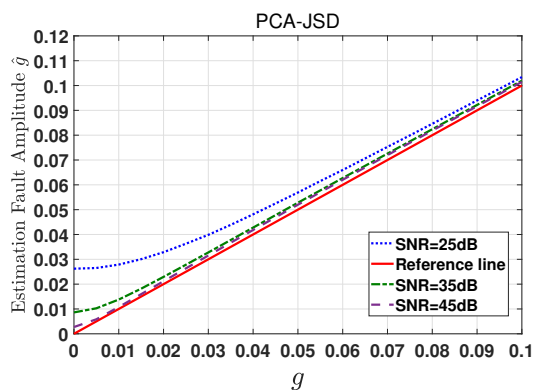


Figure 11: Fault estimation results at different noise levels

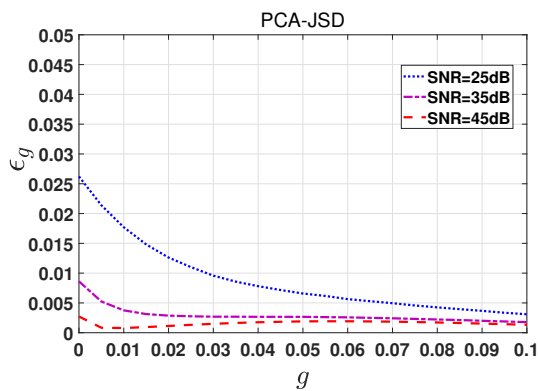


Figure 12: Estimation relative error at different noise levels

432 The relative error  $\epsilon_g$  is calculated as  $\epsilon_g = \frac{\hat{g}-g}{1+g}$  for quantifying the overes-  
 433 timation level. The results for different SNR are presented in Fig.12. The



434 relative error decreases either while the fault severity or the SNR increases.  
 435 For SNR = 25dB, the maximum error is 2.75%.

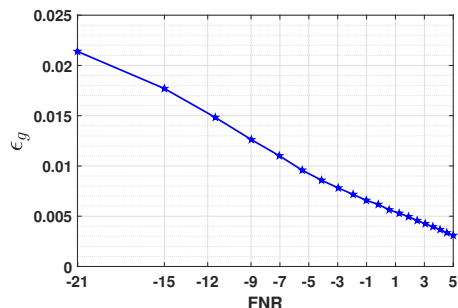


Figure 13: Estimation relative error with SNR=25dB for different FNR

436 The relative error results for different FNR (with SNR set at 25dB) is  
 437 displayed in Fig.13. The result highlights that the maximum error is 2.15%  
 438 when FNR is  $-21$ dB. Such a value is sufficiently low to be acceptable con-  
 439 sidering the incipient fault severity diagnosis context.

#### 440 4.3. Application to nondestructive incipient cracks severity estimation

441 In order to validate the proposed estimation process, we have considered  
 442 the evaluation of cracks based on experimental ECT data. The considered  
 443 ECT cracks considered here are the same as those described in section 3. The  
 444 fault estimation model is focused on the crack sizes given in Table 2 where the  
 445 crack area increases along with the length or depth values. **In this work only**  
 446 **incipient cracks evaluation are considered.** Then no cracks with an area larger  
 447 than  $0.24mm^2$  are evaluated here. These restrictions imply a more tedious  
 448 evaluation. For the estimation model, considering such constraint, we have  
 449 used the Taylor equation to be able to approximately derive some part of the  
 450 equations. This leads to the assumption that the estimated value should be  
 451 close to zero and not too far. While this assumption is not respected, the  
 452 estimation accuracy is decreased.

453 For each incipient crack size, we compute the FNR value. For this latter,  
 454 the noise level is related to the internal and external sources. It is obtained  
 455 considering the variance of the impedance signal in healthy conditions. As-  
 456 suming that the fault is independent from the noise, the fault level is con-  
 457 sidered as the main evolution between healthy and faulty signals. As shown

458 in Table 2, FNR for the smallest and biggest crack sizes are respectively  
 459  $-29.3\text{dB}$  and  $0.21\text{dB}$ .

Table 2: Crack sizes area and corresponding experimental FNR

Area ( $\text{mm}^2$ )	0.01	0.02	0.04	0.08	0.16	0.24
length ( $l$ ), depth ( $d$ )	0.1, 0.1	0.1, 0.2	0.2, 0.2	0.4, 0.2	0.4, 0.4	0.6, 0.4
Experimental FNR(dB)	$-29.3$	$-20.1$	$-16.8$	$-9.63$	$-4.5$	$0.21$

460 For our study, we consider the reference healthy data as the ECT matrix  
 461  $\mathbf{X}_h$  denoted as  $\mathbf{X}_h = (\mathbf{z}_1^h, \mathbf{z}_2^h, \mathbf{z}_3^h, \mathbf{z}_4^h)$ , where  $\mathbf{z}^h$  is the enlarged imaginary part  
 462 of the impedance signals acquired from the edge of the normalized map.  
 463 The faulty ECT matrix  $\mathbf{X}_f$  denoted as  $\mathbf{X}_f = (\mathbf{z}_1^f, \mathbf{z}_2^f, \mathbf{z}_3^f, \mathbf{z}_4^f)$ , where  $\mathbf{z}^f$  is the  
 464 enlarged imaginary impedance signals acquired from the faulty normalized  
 465 map. The number of samples is  $N = 4.2 * 10^4$ .

466 The PCA is applied to  $\mathbf{X}_h$  and  $\mathbf{X}_f$ . Then the reference distribution and  
 467 the faulty distribution of the first principal component scores are calculated.  
 468 Fig.14 presents the probability distribution for the first principal component  
 469 score  $\mathbf{t}_1$  corresponding to the crack size  $C_1$ . Twenty acquisitions of  $\mathbf{z}^f$   
 470 are done for each crack size. The Jensen-Shannon divergence values between a  
 471 faulty distribution and the reference one are computed according to equation  
 472 (12) using Monte carlo simulation. The estimated fault severities impedance  
 473 values are obtained based on equation (43). In Fig.15 we plot the average  
 474 estimation of the fault severity according to the crack size measured by FNR  
 475 value.

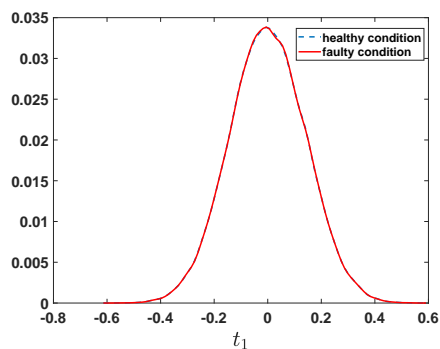


Figure 14: Pdfs of  $\mathbf{t}_1$  for the healthy and faulty component scores

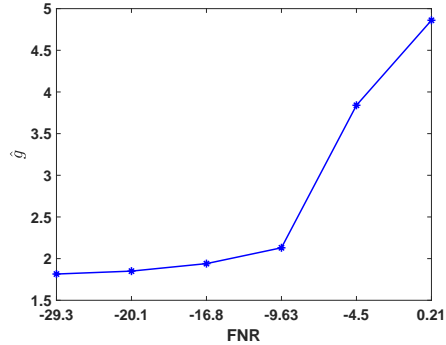


Figure 15: Fault severity estimation versus the FNR(dB)

476 Fig.15 clearly shows that the estimated fault severity grows monotonically  
 477 with the FNR. This result validates the effectiveness of the proposed fault  
 478 estimation model.

## 479 5. Conclusion

480 In this paper, we have proposed an incipient fault diagnosis scheme based  
 481 on the Jensen-Shannon Divergence for fault detection and estimation. For  
 482 both cases, a theoretical model is proposed and widely show the dependence  
 483 of the fault with the JSD and the noise level.

484 The effectiveness of the detection is first validated by using simulated  
 485 data from an AR system. The detection performances of JSD for incipient  
 486 fault are compared to the Hotelling's test  $T^2$  and the SPE ones. Using ECT  
 487 experimental data to detect incipient cracks in a conductive plate, JSD good  
 488 performances have been obtained in terms of  $P_D$  and  $P_{FA}$  even with SNR as  
 489 low as 20dB. The proposed approach allows to evaluate such incipient cracks  
 490 that was not able to be detected in the literature with such noisy ECT data.

491 Concerning the fault estimation, we prove, for Gaussian distributed data,  
 492 the efficiency and the accuracy of the proposal. For low noise conditions,  
 493 the estimation accuracy is good, the maximum relative error is 2.75% for  
 494 SNR=25dB and low fault severities (the maximum relative error is 2.15% for  
 495  $FNR = -21$ dB). With our proposal, the fault severity is slightly overesti-  
 496 mated particularly in high noise condition levels. This slight overestimation  
 497 is a valuable safety margin in health monitoring process. Finally, the appli-  
 498 cation for fault severity estimation on the ECT data proves the efficiency of

499 the derived model by providing an accurate estimation of the fault amplitude  
500 versus the FNR.

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