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BAYESIAN MULTI-OBJECTIVE OPTIMIZATION WITH NOISY EVALUATIONS

1. CONTEXT

- Multi-objective optimization of the parameters of a planning strategy for the multi-year planning of the electricity distribution grid [DUT15]
- A stochastic black box provides noisy evaluation results of the objective functions $f_1, ..., f_d$ defined on a discrete search domain $X \subset \mathbb{R}^d$
- Previous $n$ evaluations at $X = (X_1, ..., X_n)$ assumed:
  - $Z_i = f_i(X_i) + \varepsilon_i$, $i = 1, ..., n$
  - where $\varepsilon_i$ are zero-mean random variables

2. OBJECTIVE

Estimate the Pareto-optimal solutions (or Pareto set $\Gamma$) of the problem:

$$x^* = \arg\max_{x \in X} f(x), \ldots, f_d(x)$$

Defined as:

$$\Gamma = \{x \in X : \exists x' \in X \text{ such that } f(x') < f(x)\}$$

where $x$ stands for the Pareto domination rule:

$$y = (y_1, ..., y_d) < y' = (y'_1, ..., y'_d) \iff \begin{cases} v_i \leq q, y_i \leq y'_i \\ b_j \leq q, y_j < y'_j \end{cases}$$

Illustration of the Pareto domination rule:

- $y_1, y_2, y_3$ are non-dominated points
- $y_4$ is only dominated by $y_1$
- $y_5$ is only dominated by $y_2$
- $y_6$ is dominated by all other points

3. BAYESIAN OPTIMIZATION

- Define a probabilistic model for each $f$ conditional on previous observations
- Use a sampling criterion to select new evaluation points

Provides estimate of $f$ and a measure of uncertainty of the estimation

PARETO-OPTIMAL ESTIMATES

Built from the estimates of $f$

PROPOSED APPROACH

- Replace the multi-objective problem by the minimization of a single augmented Tchebycheff function [KNO06]:

$$\tilde{f}(x) = \max \{\omega f_j(x)\} + \rho \sum_{j=1} f_j(x), \quad \sum_{j=1} \omega_j = 1, \rho > 0$$

- At each iteration, generate random weights $\omega_j$ and apply this function to the $n$ previous observations:

$$\tilde{Z}_i = \max \{\omega_j Z_j\} + \rho \sum_{j=1} \omega_j Z_j, \quad i = 1, ..., n$$

- Assume a homoscedastic Gaussian noise model and fit to $Z = (\tilde{Z}_1, ..., \tilde{Z}_n)$ a Gaussian Process model $\mathcal{K}$ with parameters estimated by maximum likelihood
- We use the Knowledge Gradient (KG) criterion [FRA09] to select new point $X_{n+1}$ based on previous observations. The idea is to identify a point that is expected to reduce the minimum of the posterior mean of $\mathcal{K}$:

$$X_{n+1} = \arg\max_{x \in X} KG(x)$$

with:

$$KG(x) = \min_{x' \in X} E\left[F(x') \mid Z_{1:n}\right] - E\left[\min_{x' \in X} F(x') \mid Z_{1:n}\right]$$

where $Z_{1:n}$ denotes a new observation of $\mathcal{K}$ at $x$

Update the model with new observation and iterate until stopping criterion is met

4. NUMERICAL EXPERIMENTS

- Compare the proposed approach (PA) with random selection of points in a bi-dimensional bi-objective problem
- Compare use of batches of 1, 4 or 10 evaluations

5. OPEN QUESTIONS

- ‘Ideal’ batch of evaluations?
- Performance comparison of the proposed approach to other methods in the literature?

REFERENCES


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