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BAYESIAN MULTI-OBJECTIVE OPTIMIZATION WITH NOISY EVALUATIONS

1. CONTEXT

- Multi-objective optimization of the parameters of a planning strategy for the multi-year planning of the electricity distribution grid [DUT15]
- A stochastic black box provides noisy evaluation results of the objective functions \( f_1, ..., f_d \) defined on a discrete search domain \( \mathbb{X} \subset \mathbb{R}^d \)
- Previous \( n \) evaluations at \( \mathbb{X} \subset \{X_1, ..., X_n\} \) assumed:
  - \( Z_{i1} = f_1(X_i) + \varepsilon_{i1}, ..., Z_{id} = f_d(X_i) + \varepsilon_{id} \)
  - where \( \varepsilon_{ij} \) are zero-mean random variables

2. OBJECTIVE

Estimate the Pareto-optimal solutions (or Pareto set \( \Gamma \)) of the problem:

\[
x^\star = \arg\min_{x \in \mathbb{X}} f(x), ..., f_d(x)
\]

Defined as:

\[
\Gamma = \{x \in \mathbb{X}: \exists x' \in \mathbb{X} \text{ such that } f(x') < f(x)\}
\]

where \( < \) stands for the Pareto domination rule:

\[
y = (y_1, ..., y_n) < y' = (y'_1, ..., y'_n) \iff \begin{cases} v_i \leq q_i, y_i \leq y'_i \\ b_j \leq q_j, y_j < y'_j \end{cases}
\]

Illustration of the Pareto domination rule:
- \( y_1, y_2, y_3 \) are non-dominated points
- \( y_4 \) is only dominated by \( y_3 \)
- \( y_5 \) is dominated by all other points

3. BAYESIAN OPTIMIZATION

- Define a probabilistic model for each \( f \) conditional on previous observations
- Use a sampling criterion to select new evaluation points

Provides estimate of \( f \) and a measure of uncertainty of the estimation

PARETO-OPTIMAL ESTIMATES

Built from the estimates of \( f \)

PROPOSED APPROACH

- Replace the multi-objective problem by the minimization of a single augmented Tchebycheff function [KNO06]:

\[
f(x) = \max_j \omega_j f_j(x) + \rho \sum_j \omega_j f_j(x), \quad \sum_j \omega_j = 1, \rho > 0\]

- At each iteration, generate random weights \( \omega_j \) and apply this function to the \( n \) previous observations:

\[
\hat{Z}_i = \max_j \omega_j Z_{ij}, \quad i = 1, ..., n
\]

- Assume a homoscedastic Gaussian noise model and fit to \( \hat{Z} = (\hat{Z}_1, ..., \hat{Z}_n) \) a Gaussian Process model \( \hat{f}_i \) with parameters estimated by maximum likelihood
- We use the Knowledge Gradient (KG) criterion [FRA09] to select new point \( X_{n+1} \) based on previous observations. The idea is to identify a point that is expected to reduce the minimum of the posterior mean of \( \hat{f} \):

\[
X_{n+1} = \arg\max_{x \in \mathbb{X}} KG(x)
\]

with:

\[
KG(x) = \min_{x \in \mathbb{X}} \mathbb{E}\left[\min_{x' \in \mathbb{X}} f(x')|Z_{n+1}\right] - \mathbb{E}\left[\min_{x' \in \mathbb{X}} f(x')|Z_{n+1}\right]
\]

where \( Z_{n+1} \) denotes a new observation of \( \hat{f}_i \) at \( x \)

- Update the model with new observation and iterate until stopping criterion is met

4. NUMERICAL EXPERIMENTS

- Compare the proposed approach (PA) with random selection of points in a bi-dimensional bi-objective problem
- Compare use of batches of 1, 4 or 10 evaluations

5. OPEN QUESTIONS

- ‘Ideal’ batch of evaluations?
- Performance comparison of the proposed approach to other methods in the literature?

REFERENCES