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BAYESIAN MULTI-OBJECTIVE OPTIMIZATION WITH NOISY EVALUATIONS

1. CONTEXT

- Multi-objective optimization of the parameters of a planning strategy for the multi-year planning of the electricity distribution grid [DUT15]
- A stochastic black box provides noisy evaluation results of the objective functions $f_1, \ldots, f_d$ defined on a discrete search domain $X \subset \mathbb{R}^d$
- Previous $n$ evaluations at $X = (X_1, \ldots, X_n)$ assumed:
  * $Z_{i_1} = f_1(X_1) + \epsilon_{i_1}, \ldots, Z_{i_q} = f_q(X_q) + \epsilon_{i_q}$
  where $\epsilon_{i_j}$'s are zero-mean random variables

2. OBJECTIVE

Estimate the Pareto-optimal solutions (or Pareto set $\Gamma$) of the problem:

$$x^* = \arg\min_{x \in X} f(x), \ldots, f_d(x)$$

Defined as:

$$\Gamma = \{x \in X : \exists x' \in X \text{ such that } f(x') < f(x)\}$$

where $\prec$ stands for the Pareto domination rule:

$$y = (y_1, \ldots, y_d) \prec y' = (y'_1, \ldots, y'_d) \iff \begin{array}{ll}
  \forall i \leq q, y_i \leq y'_i \\
  \exists j \leq q, y_j < y'_j
\end{array}$$

Illustration of the Pareto domination rule:

- $y_1, y_2, y_3$ are non-dominated points
- $y_1$ is only dominated by $y_3$
- $y_3$ is dominated by all other points

3. BAYESIAN OPTIMIZATION

- Define a probabilistic model for each $f$ conditional on previous observations
- Use a sampling criterion to select new evaluation points

Provides estimate of $f$ and a measure of uncertainty of the estimation

**PARETO-OPTIMAL ESTIMATES**

Built from the estimates of $f$

**PROPOSED APPROACH**

- Replace the multi-objective problem by the minimization of a single augmented Tchebycheff function [KNO06]:
  $$f(x) = \max\{\omega_1 f_1(x) \} + \rho \sum_{j=2}^{d} \omega_j f_j(x)$$
  where $\omega_j = 1, \rho > 0$
- At each iteration, generate random weights $\omega_j$ and apply this function to the $n$ previous observations:
  $$Z_i = \max\{\omega_1 Z_{i_1} \} + \rho \sum_{j=2}^{d} \omega_j Z_{i_j}, \quad i = 1, \ldots, n$$
  - Assume a homoscedastic Gaussian noise model and fit to $Z = (Z_1, \ldots, Z_n)$ a Gaussian Process model $\hat{f}$ with parameters estimated by maximum likelihood
- We use the Knowledge Gradient (KG) criterion [FRA09] to select new point $x_{n+1}$ based on previous observations. The idea is to identify a point that is expected to reduce the minimum of the posterior mean of $\tilde{f}$:
  $$x_{n+1} = \arg\max_{x \in \mathbb{R}^d} KG(x)$$
  with:
  $$KG(x) = \min_{x' \in \mathbb{R}^d} \mathbb{E}[f(x'|Z)] - \mathbb{E}[\min_{x' \in \mathbb{R}^d} f(x') \mid Z_{n+1}]$$
  where $Z_{n+1}$ denotes a new observation of $\tilde{f}$ at $x$
  - Update the model with new observation and iterate until stopping criterion is met

4. NUMERICAL EXPERIMENTS

- Compare the proposed approach (PA) with random selection of points in a bi-dimensional bi-objective problem
- Compare use of batches of 1, 4 or 10 evaluations

5. OPEN QUESTIONS

- "Ideal" batch of evaluations?
- Performance comparison of the proposed approach to other methods in the literature?

**REFERENCES**


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