1. CONTEXT

- Multi-objective optimization of the parameters of a planning strategy for the multi-year planning of the electricity distribution grid [DUT15]
- A stochastic black box provides noisy evaluation results of the objective functions $f_1, ..., f_d$ defined on a discrete search domain $X \subset \mathbb{R}^d$
- Previous $n$ evaluations at $X = \{X_1, ..., X_n\}$ assumed:
  - $Z_{i+1} = f_1(X_i) + \varepsilon_{i+1}$, ..., $Z_{i+q} = f_d(X_i) + \varepsilon_{i+q}$ where $\varepsilon_{i+q}$ are zero-mean random variables

2. OBJECTIVE

Estimate the Pareto-optimal solutions (or Pareto set $\Gamma$) of the problem:
$$x^* = \arg\min_{x \in X} f(x)$$

Defined as:
$$\Gamma = \{x \in X : \exists x' \in X \text{ such that } f(x') < f(x) \}$$

where $<$ stands for the Pareto domination rule:
$$y = (y_1, ..., y_d) < y' = (y'_1, ..., y'_d) \iff \forall i \leq q, y_i \leq y'_i \land \exists j \leq q, y_j < y'_j$$

Illustration of the Pareto domination rule:
- $y_1, y_2, y_3$ are non-dominated points
- $y_4$ is only dominated by $y_1$
- $y_5$ is only dominated by $y_2$
- $y_6$ is dominated by all other points

3. BAYESIAN OPTIMIZATION

- Define a probabilistic model for each $f$ conditional on previous observations
- Use a sampling criterion to select new evaluation points

Provides estimate of $f$ and a measure of uncertainty of the estimation

PARETO-OPTIMAL ESTIMATES
Built from the estimates of $f$

PROPOSED APPROACH

- Replace the multi-objective problem by the minimization of a single augmented Tchebycheff function [KNO06]:
$$f(x) = \max \{\omega f_i(x)\} + \rho \sum_{j} \omega_j f_j(x), \quad \sum_{j} \omega_j = 1, \rho > 0$$
- At each iteration, generate random weights $\omega_j$ and apply this function to the $n$ previous observations:
$$Z_{i+1} = f_1(X_i) + \varepsilon_{i+1}, ..., Z_{i+q} = f_d(X_i) + \varepsilon_{i+q}$$

Assume a homoscedastic Gaussian noise model and fit to $Z = (Z_1, ..., Z_n)$ a Gaussian Process model $\xi_n$ with parameters estimated by maximum likelihood
- We use the Knowledge Gradient (KG) criterion [FRA09] to select new point $X_{n+1}$ based on previous observations. The idea is to identify a point that is expected to reduce the minimum of the posterior mean of $\xi$:
$$X_{n+1} = \arg\max_{x \in X} KG(x)$$

with:
$$KG(x) = \min \{E[f(x')|\xi_n] - E[\min_{x \in X} E[f(x')|\xi_{n+1}] \}$$

where $\xi_{n+1}$ denotes a new observation of $\xi_n$ at $x$

4. NUMERICAL EXPERIMENTS

- Compare the proposed approach (PA) with random selection of points in a bi-dimensional bi-objective problem
- Compare use of batches of 1, 4 or 10 evaluations

5. OPEN QUESTIONS

- *Ideal* batch of evaluations?
- Performance comparison of the proposed approach to other methods in the literature?

REFERENCES


