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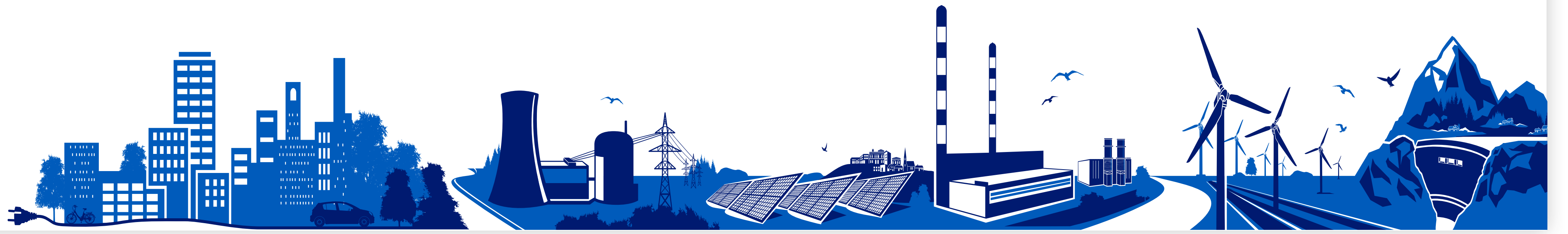
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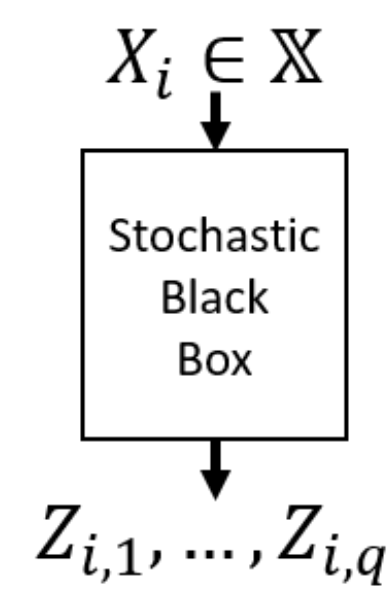
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# BAYESIAN MULTI-OBJECTIVE OPTIMIZATION WITH NOISY EVALUATIONS

## 1. CONTEXT

- **Multi-objective optimization** of the parameters of a planning strategy for the multi-year planning of the electricity distribution grid [DUT15]
- A stochastic black box provides **noisy evaluation results** of the objective functions  $f_1, \dots, f_q$  defined on a discrete search domain  $\mathbb{X} \subset \mathbb{R}^d$
- Previous  $n$  evaluations at  $X = (X_1, \dots, X_n)$  assumed:
  - $Z_{i,1} = f_1(X_i) + \varepsilon_{i,1}, \dots, Z_{i,q} = f_q(X_i) + \varepsilon_{i,q}$  where  $\varepsilon_{i,j}$ s are zero-mean random variables



## 2. OBJECTIVE

Estimate the **Pareto-optimal solutions** (or Pareto set  $\Gamma$ ) of the problem:

$$x^* = \operatorname{argmin}_{x \in \mathbb{X}} f_1(x), \dots, f_q(x)$$

Defined as:

$$\Gamma = \{x \in \mathbb{X} : \nexists x' \in \mathbb{X} \text{ such that } f(x') < f(x)\}$$

where  $<$  stands for the Pareto domination rule:

$$y = (y_1, \dots, y_q) < y' = (y'_1, \dots, y'_q) \Leftrightarrow \begin{cases} \forall i \leq q, y_i \leq y'_i \\ \exists j \leq q, y_j < y'_j \end{cases}$$

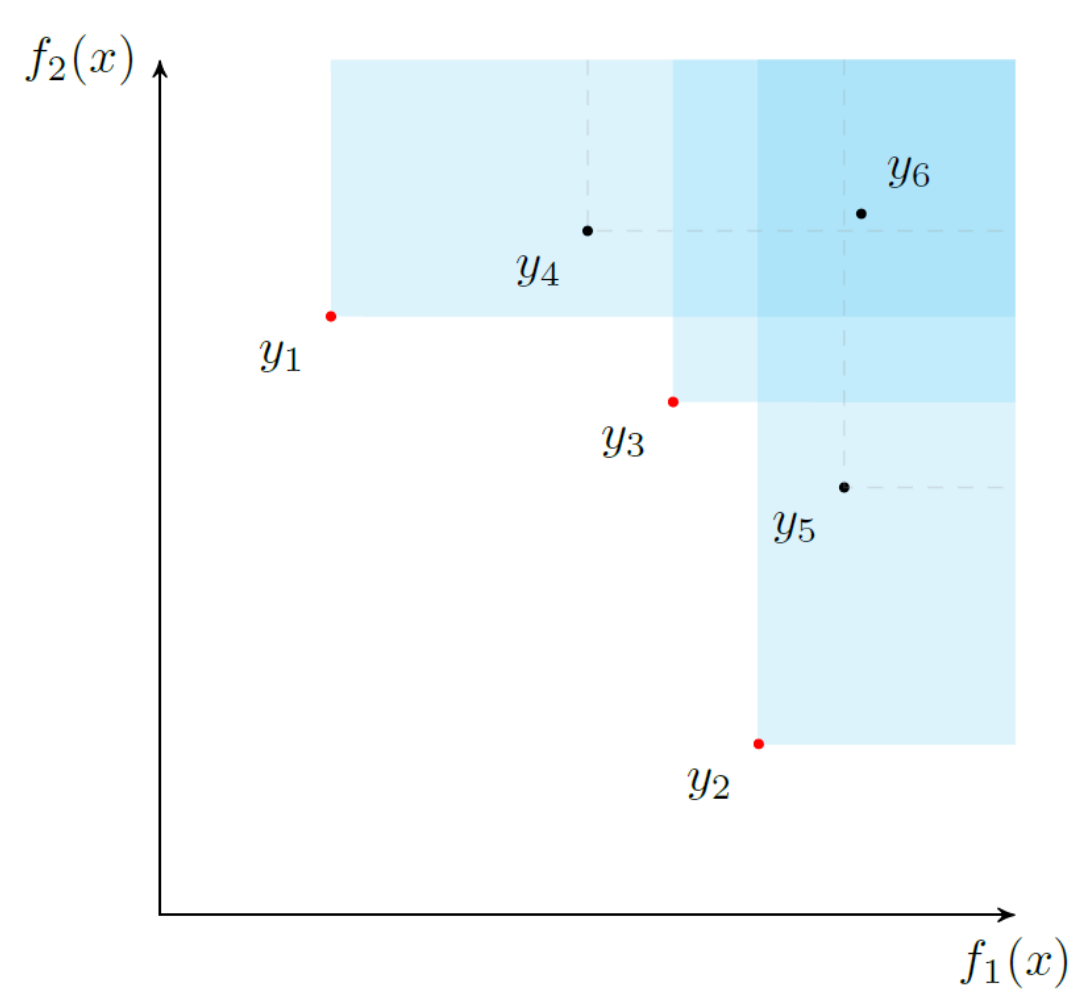
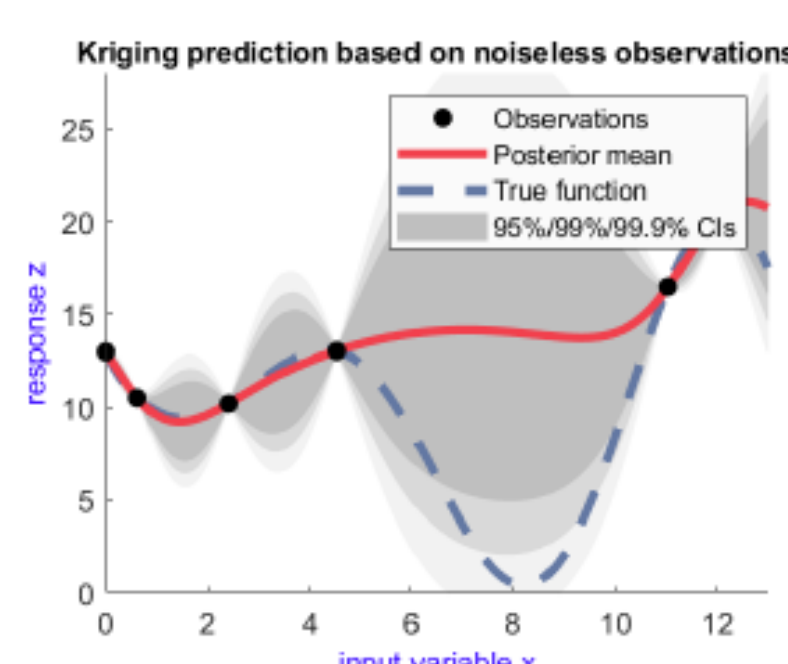


Illustration of the Pareto domination rule

- $y_1, y_2, y_3$  are non-dominated points
- $y_4$  is only dominated by  $y_1$
- $y_5$  is only dominated by  $y_2$
- $y_6$  is dominated by all other points

## 3. BAYESIAN OPTIMIZATION

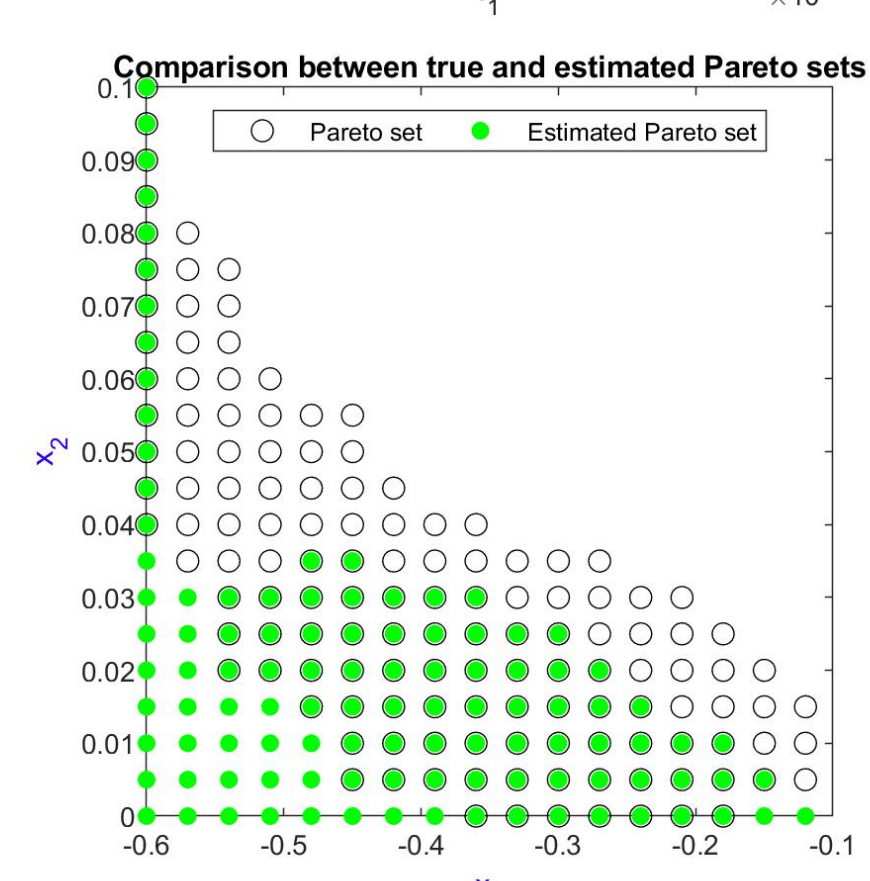
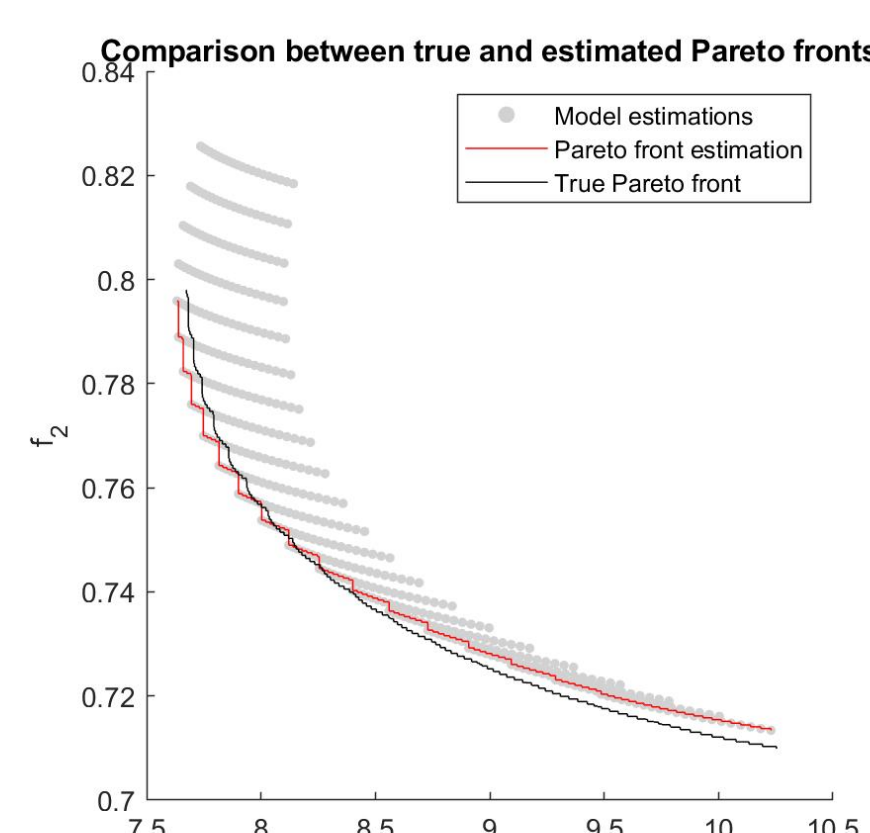
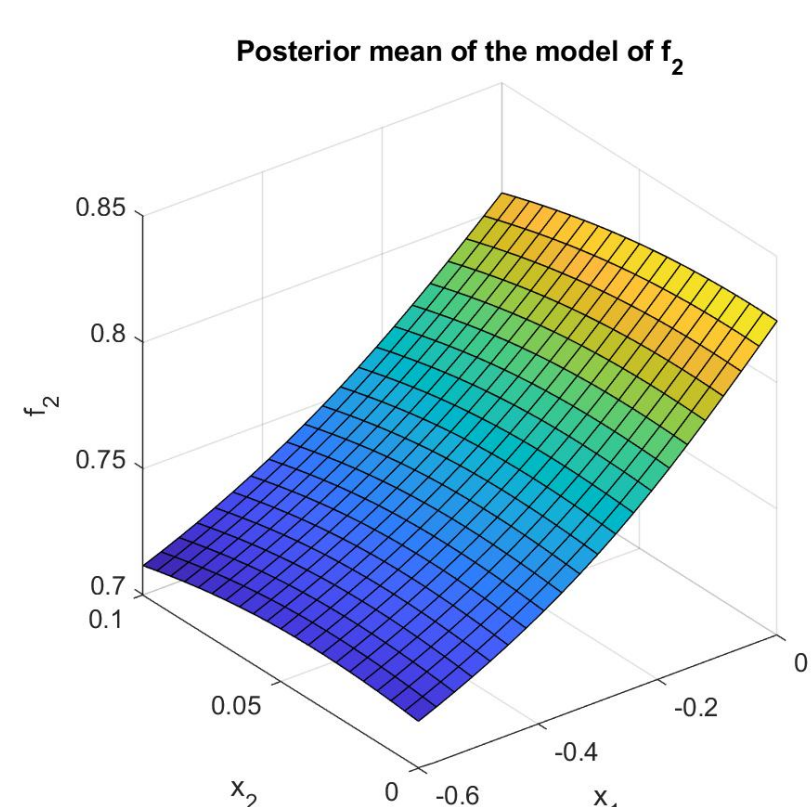
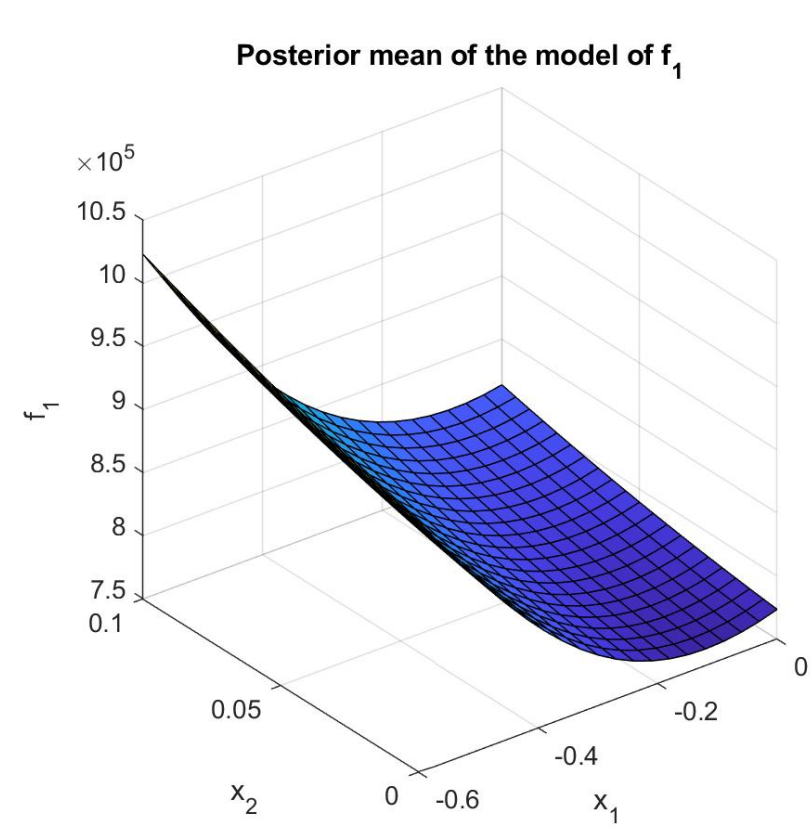
- Define a **probabilistic model** for each  $f$  conditional on previous observations
- Use a **sampling criterion** to select new evaluation points



Provides estimate of  $f$  and a measure of uncertainty of the estimation

### PARETO-OPTIMAL ESTIMATES

Built from the estimates of  $f$



## PROPOSED APPROACH

- Replace the multi-objective problem by the minimization of a single **augmented Tchebycheff function** [KNO06]:

$$\tilde{f}(x) = \max_j [\omega_j f_j(x)] + \rho \sum_j \omega_j f_j(x), \quad \sum_j \omega_j = 1, \rho > 0$$

- At each iteration, generate random weights  $\omega_j$  and apply this function to the  $n$  previous observations:

$$\tilde{Z}_i = \max_j [\omega_j Z_{i,j}] + \rho \sum_j \omega_j Z_{i,j}, \quad i = 1, \dots, n$$

- Assume a homoscedastic Gaussian noise model and fit to  $\tilde{Z} = (\tilde{Z}_1, \dots, \tilde{Z}_n)$  a Gaussian Process model  $\tilde{\xi}_n$  with parameters estimated by maximum likelihood

- We use the **Knowledge Gradient (KG)** criterion [FRA09] to select new point  $X_{n+1}$  based on previous observations. The idea is to identify a point that is expected to reduce the minimum of the posterior mean of  $\tilde{\xi}$ :

$$X_{n+1} = \operatorname{argmax}_{x \in \mathbb{X}} KG(x)$$

with:

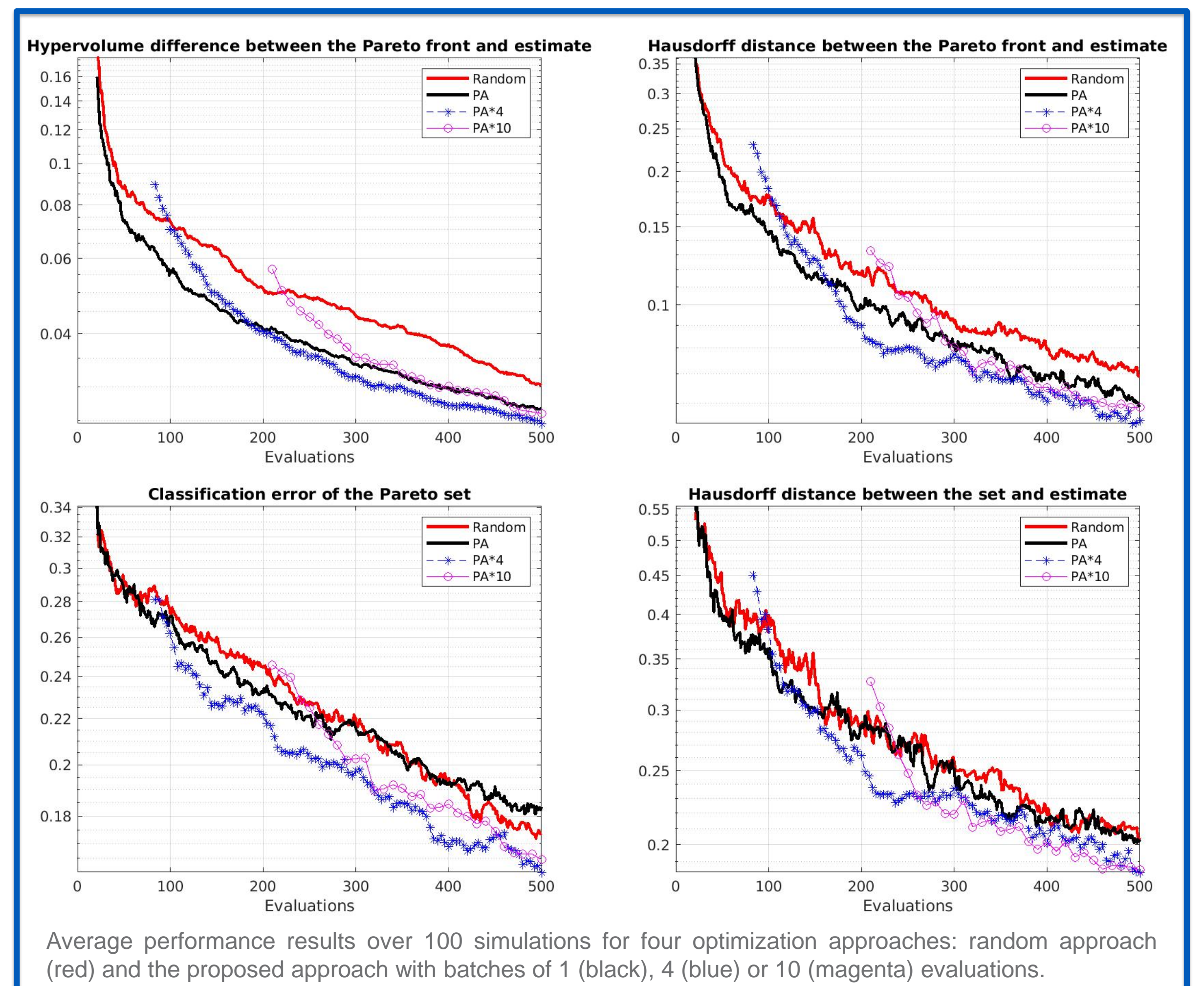
$$KG(x) = \min_{x' \in \mathbb{X}} \mathbb{E}[\tilde{\xi}_n(x')] - \mathbb{E} \left[ \min_{x' \in \mathbb{X}} \mathbb{E}[\tilde{\xi}_n(x') | \tilde{Z}_{n,x}] \right]$$

where  $\tilde{Z}_{n,x}$  denotes a new observation of  $\tilde{\xi}_n$  at  $x$

- Update the model with new observation and iterate until stopping criterion is met

## 4. NUMERICAL EXPERIMENTS

- Compare the proposed approach (PA) with random selection of points in a bi-dimensional bi-objective problem
- Compare use of batches of 1, 4 or 10 evaluations



Average performance results over 100 simulations for four optimization approaches: random approach (red) and the proposed approach with batches of 1 (black), 4 (blue) or 10 (magenta) evaluations.

## 5. OPEN QUESTIONS

- 'Ideal' batch of evaluations?
- Performance comparison of the proposed approach to other methods in the literature?

## REFERENCES

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