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# NUMERICAL EVALUATION OF SOURCE AND SENSOR PLACEMENT METHODS FOR SOUND FIELD CONTROL

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## ABSTRACT

The quality of sound-field reproduction is strongly dependent on the placement of the secondary sources used to generate the desired sound-field. In this communication, we focus on the case of control by matching the generated sound-field with the desired values at particular points in the reproduction zone. Several loudspeaker and control point placements are compared, such as regular placements, FrameSense, Empirical Interpolation Method, etc. Numerical results show that regular placements are (relatively) inefficient, control points located inside the reproduction zone are necessary, and that best results are obtained by jointly optimizing the positions of the sources and control points with the Empirical Interpolation Method, which also minimizes the required output power of the loudspeakers. This method is also shown to be robust with respect to moderate position errors of the sources and control points.

## 1. INTRODUCTION

The issue of loudspeaker and control points placement is important in soundfield reproduction. Indeed, sub-optimal placement of loudspeakers can lead to inability to synthesize desired soundfields, or lead to excessive demands in loudspeaker output power. In addition, inadequate placement of control points can lead to poor soundfield control. Optimization of the placement is aimed at improving the reproduction performance, or at fixed quality, reduce the number of necessary loudspeakers and control points.

We compare several loudspeakers and control points placement methods, in terms of reproduction fidelity, necessary output power of the loudspeakers, and robustness to position errors. Our numerical results show that best performances are obtained with the Empirical Interpolation Method.

We briefly review soundfield control in section 2, and introduce the tested placement methods in section 3. Numerical results are given in section 4, and section 5 concludes the paper.

Extensive discussion of soundfield control, placement methods, and additional numerical results can be found in [1]<sup>1</sup>.

## 2. SOUNDFIELD CONTROL BY PRESSURE MATCHING

Control of a soundfield by pressure matching is achieved by estimating the activations of  $N$  loudspeakers, so that the error between the generated soundfield and the desired soundfield, measured at a finite number of control points, is minimized.

We assume that the control zone  $\Omega$  is enclosed in the reproduction zone  $D$ . Loudspeakers are placed on the boundary  $\partial D$  of  $D$ , and control points in  $\Omega$ .

A soundfield  $u(\mathbf{x}, \omega)$  at frequency  $\omega$  is a solution to the Helmholtz equation

$$\Delta u(\mathbf{x}, \omega) + k^2 u(\mathbf{x}, \omega) = 0. \quad (1)$$

Such solutions can be represented by the single layer boundary integral [2]

$$u(\mathbf{x}, \omega) = \int_{\mathbf{y} \in \partial D} \varphi(\mathbf{y}) G_{\text{m}}(\mathbf{x}|\mathbf{y}, \omega) d\mathbf{y}, \quad (2)$$

where  $G_{\text{m}}(\cdot, \omega)$  is the free-field Green function

$$G_{\text{m}}(\mathbf{x}|\mathbf{y}, \omega) = \frac{\exp(jk\|\mathbf{x} - \mathbf{y}\|_2)}{4\pi\|\mathbf{x} - \mathbf{y}\|_2}. \quad (3)$$

In this formulation, synthesis is achieved by a continuous distribution of monopoles. The first step towards practical application is the discretization of the integral as a sum over a finite number of sources. The synthesized soundfield  $u_{\text{syn}}$  is then a linear combination

$$u_{\text{syn}}(\mathbf{x}) = \sum_{l=1}^L d_l g_l(\mathbf{x}), \quad (4)$$

<sup>1</sup> Figures included in this article where reused from [1] under the CC BY 4.0 license <https://creativecommons.org/licenses/by/4.0/>

where  $g_l(\mathbf{x})$  is the value of the Green function of the  $l$ -th loudspeaker at point  $\mathbf{x}$ . and the driving signals are chosen to minimize the objective function

$$J = \int_{\mathbf{x} \in \Omega} \left| \sum_{l=1}^L d_l g_l(\mathbf{x}) - u_{\text{des}}(\mathbf{x}) \right|^2 d\mathbf{x}, \quad (5)$$

which is the mean squared reproduction error between the reproduced soundfield and the desired soundfield  $u_{\text{des}}(\mathbf{x})$ .

The source placement problem is twofold:

- ensure that the discrete set of sources can generate the desired soundfields up to a desired reproduction error,
- ensure that the amplitudes of the driving signals remain bounded.

The minimization of (5) is difficult in practice, as it implies measurement or computation of the synthesized soundfield in a continuous domain. We instead use a simpler objective

$$J_d = \|\mathbf{u}^{\text{des}} - \mathbf{G}\mathbf{d}\|_2^2, \quad (6)$$

where  $\mathbf{u}^{\text{des}}$  is a vector containing samples of the soundfield at  $M$  discrete control points, and  $\mathbf{G}$  is the transfer matrix between sources and control points.

The optimal driving signals  $\mathbf{d}$  are then found by

$$\mathbf{d} = \mathbf{G}^\dagger \mathbf{u}^{\text{des}}, \quad (7)$$

or, if Tikhonov regularization is used,

$$\mathbf{d} = (\mathbf{G}^H \mathbf{G} + \lambda \mathbf{I})^{-1} \mathbf{G}^H \mathbf{u}^{\text{des}}, \quad (8)$$

where  $\lambda$  is a user-defined regularization parameter.

The control points placement problem aims at choosing control points ensuring that the discrete error (6) is a good approximation of the continuous error (5).

In conclusion, the objective of joint loudspeaker and control point placement is twofold:

- firstly, to ensure that the loudspeakers are placed such that they can generate accurate approximations of the desired soundfields, and, moreover, that the amplitudes of their activations remain reasonable,
- secondly, to place the control points such that the norm of the error measured at the control points is an accurate approximation of the norm of the error over the entire reproduction domain.

### 3. ALGORITHMS FOR MICROPHONE AND LOUSPEAKER PLACEMENT

Several methods have been proposed to place microphones and/or loudspeakers. The methods tested in the numerical experiments are here briefly described. More extensive discussion of these methods can be found in [1].

#### 3.1 Reg: Regular placement

The simplest method, loudspeakers are placed regularly along the boundary of  $D$ , and control points regularly along the boundary of  $\Omega$ . A variant of this method is regular angular placement, with respect to the center of the reproduction zone.

#### 3.2 Rand: Random placement

In random placement, control points and loudspeaker positions are drawn from uniform probability density, resp. in the reproduction region and the loudspeaker candidate positions.

#### 3.3 GSO: Gram-Schmidt orthogonalization

In this loudspeaker placement algorithm [3], loudspeakers are iteratively chosen to maximize the angle between their Green vector and the space spanned by Green vectors of the previously chosen loudspeakers. The algorithm is initialized using an example of desired soundfield, and selecting the loudspeaker with Green vector closest to the desired soundfield. Here, control points are placed regularly in a double layer around the reproducing zone to avoid instabilities.

#### 3.4 Det: D-optimal design

Given a distribution of loudspeakers (e.g., regular), the objective of D-optimal sensor placement is to maximize the determinant of the Green matrix  $\mathbf{G}$ . This combinatorial problem can be approximated by a relaxed problem, by optimizing a probability distribution over the possible loudspeaker positions [4]. This relaxed problem is convex, and can be solved using standard convex optimization methods. The actual choice of control points is obtained by choosing the most probable points, and locally optimizing the set.

#### 3.5 MI: Mutual information

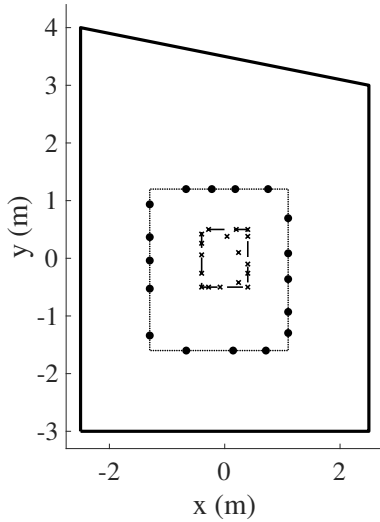
Sensor placement based on mutual information aims at maximizing the mutual information between the selected sensors and unselected sensors, ensuring that the informations measured by selected sensors and unselected sensors are similar [5]. This NP-hard problem can be approximately solved using a greedy algorithm.

#### 3.6 FS: FrameSense

The FrameSense algorithm [6] is based on the minimization of the frame potential of the Green matrix  $\mathbf{G}$ , defined by

$$FP(\mathbf{G}) = \sum_{m,m'} |\langle \mathbf{g}_m, \mathbf{g}_{m'} \rangle|^2 \quad (9)$$

where  $\mathbf{g}_m$  is the  $m$ -th column vector of  $\mathbf{G}$ . The frame potential is used as a proxy for the mean square error of the estimation of the coefficients of the soundfield decomposition. Minimization of the frame potential is approximately achieved by a greedy worst-out algorithm.



**Figure 1.** Room and reproducing zone geometry, and results of EIM placement at 400 Hz.

### 3.7 EIM: Empirical Interpolation Method

The Empirical Interpolation Method is an algorithm for joint placement of the control points and loudspeakers [7] [8]. It was introduced for applications in numerical analysis of partial differential equations. Its use for microphone array design was introduced in [9], and application to joint control point and source placement introduced in [10]. At each step, EIM selects the loudspeaker for which the approximation of its Green function by previously selected loudspeakers is the worst, and the control point that ensures that the linear system to be solved remains stable. The algorithm stops when the Green function of all remaining loudspeakers can be approximated with a predefined accuracy.

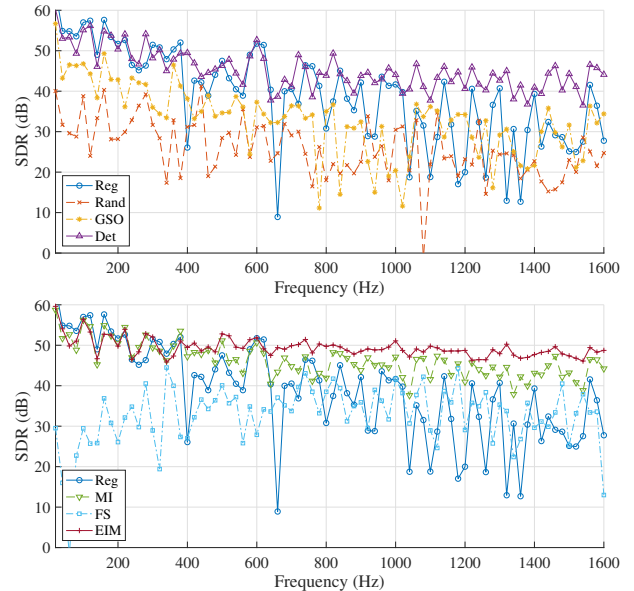
## 4. NUMERICAL RESULTS

The placement strategies are tested in a 2D setting. Sound propagation in a trapezoidal room (see figure 1) is simulated using FreeFem++, a finite element method solver. The absorption ratio of the walls is set at 0.5. Loudspeakers are placed on the boundary of a rectangle of dimensions  $2.4\text{m} \times 2.8\text{m}$ , discretized over 256 points. Soundfield reproduction is controlled in a rectangle region of dimensions  $0.8\text{m} \times 1\text{m}$ , discretized with a regular rectangular grid of pitch 0.04m.

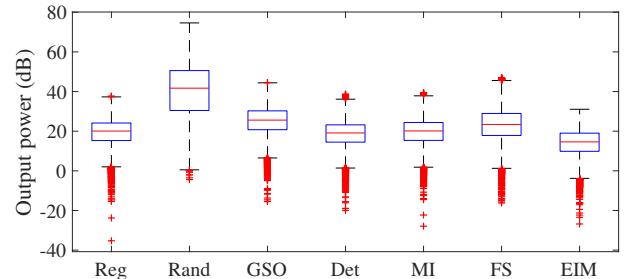
The methods are evaluated by reproducing plane waves, with directions sampled with a  $1^\circ$  interval, and computing the average signal-to-distortion ratio (SDR) over the reproduction zone.

### 4.1 Comparison of the methods

The placement is tested between 100Hz and 1600Hz. The placement algorithms are run at each frequencies. The number  $K$  of control points and loudspeakers is determined by the EIM algorithm. The driving signal of the loudspeakers are computed without regularization.



**Figure 2.** Mean SDR of narrowband soundfield reproduction.



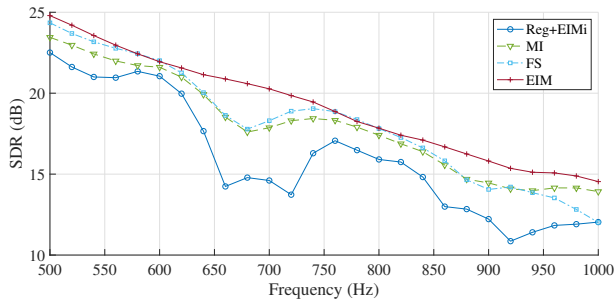
**Figure 3.** Power of the driving signals.

The SDR of the methods in function of the frequency are plotted on fig. 2. For regular placement, several dips in the SDR are visible, caused by instability of the soundfield control at the eigenfrequencies of the control region for Dirichlet boundary conditions. Indeed, at these frequencies, nonzero soundfields exist, that have zero values on the border of the region, and cannot be controlled by the control points placed on the border of the region. Higher SDRs are obtained by the EIM method.

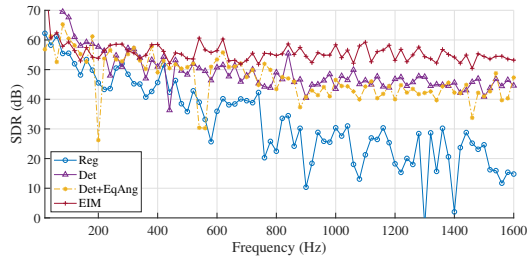
The distribution of the powers of the driving signals for the tested methods are plotted on figure 3. Among the methods, EIM has the lowest power requirements.

### 4.2 Robustness to position errors

In practice, loudspeakers and microphones cannot be exactly placed at the positions given by the placement algorithms. The robustness with respect to positioning errors is tested with a Gaussian error on the positions, with standard deviation of  $1.0 \times 10^{-2}\text{m}$ . Here, the driving signals are obtained using Tikhonov regularization. Results of regular loudspeaker placement combined with EIM control point placement, MI, FS and EIM are given on fig. 4 between 500 and 1000 Hz. Placement is here optimized for broadband reproduction. EIM exhibits better robustness.



**Figure 4.** Mean SDR of broadband soundfield reproduction with loudspeaker and control points position errors.



**Figure 5.** Mean SDR of narrowband soundfield reproduction. Comparison with regular and angle-regular placement of loudspeakers, and D-optimal control points placement.

### 4.3 Performances of regular placements

As the test soundfields are plane waves, and the loudspeaker can, in a farfield approximation, be considered as plane waves, one could postulate that angular uniform placement of the loudspeakers would be optimal, as this would minimize the maximal gap between the angle of a plane wave and the arrival angle of the closest loudspeaker. This loudspeaker placement is tested combined with Det and control points placement, and compared with Reg and EIM placements on fig. 5. The better performances of EIM shows that regular placement along the boundary, and angular uniform, are not optimal.

## 5. CONCLUSION

Several placement methods are tested. Numerical results show that regular placement of the loudspeakers and control points are not optimal, and that, among the tested methods, EIM exhibits the best performances in terms of reproduction errors, robustness to position errors, output power, and computational complexity. Future developments include experimental application, and further analysis of the broadband case.

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