



**HAL**  
open science

## **Igeood: An Information Geometry Approach to Out-of-Distribution Detection**

Eduardo Dadalto Câmara Gomes, Florence Alberge, Pierre Duhamel, Pablo Piantanida

### ► **To cite this version:**

Eduardo Dadalto Câmara Gomes, Florence Alberge, Pierre Duhamel, Pablo Piantanida. Igeood: An Information Geometry Approach to Out-of-Distribution Detection. NeurIPS DistShift Workshop 2021, Dec 2021, Virtual, France. <hal-03649034>

**HAL Id: hal-03649034**

**<https://centralesupelec.hal.science/hal-03649034v1>**

Submitted on 22 Apr 2022

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



HAL Authorization

# IGOOD: An Information Geometry Approach to Out-of-Distribution Detection

Eduardo D. C. Gomes<sup>1</sup>, Florence Alberge<sup>1</sup>, Pierre Duhamel<sup>1</sup> and Pablo Piantanida<sup>1</sup>



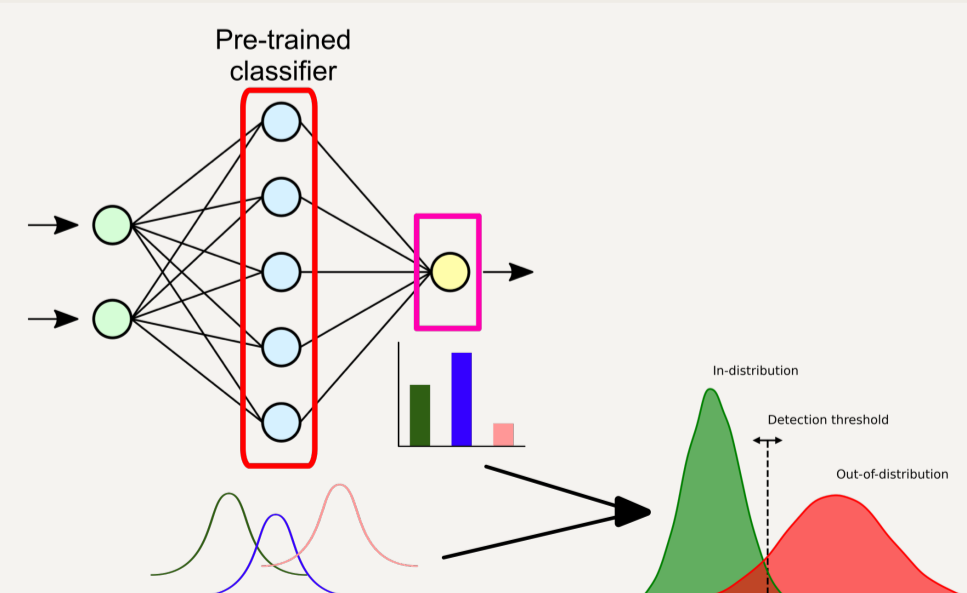
## Abstract

- In this paper, we introduce IGOOD, an effective method for detecting Out-of-Distribution (OOD) samples.
- IGOOD applies to any pre-trained neural network, works under different degrees of access to the ML model, does not require OOD samples or assumptions on the OOD data but can also benefit (if available) from OOD samples.
- By building on the geodesic (**Fisher-Rao**) distance between the underlying data distributions, our discriminator combines confidence scores from the logits outputs and the learned features of a deep neural network.

## Background

- Let  $\mathcal{X} \subseteq \mathbb{R}^d$  be the feature space and  $\mathcal{Y}$  a label space and let  $p_{XY}$  be the underlying unknown probability density function (pdf) over  $\mathcal{X} \times \mathcal{Y}$ .
- In order to model the underlying problem, we introduce an artificial binary random variable  $Z \in \{0, 1\}$  indicating with  $z = 1$  that the test sample  $\mathbf{x}$  is OOD and  $z = 0$  otherwise.
- The open-world data can then be modeled as a *mixture* distribution  $p_{X|Z}$  defined by
 
$$p_{X|Z}(\mathbf{x}|z=0) \triangleq p_X(\mathbf{x}), \quad p_{X|Z}(\mathbf{x}|z=1) \triangleq q_X(\mathbf{x}).$$
- The intrinsic difficulty arises from the fact that very little can be assumed about the unknown distributions  $p_X$  and  $q_X$ , in particular for out-of-distribution.
- **Alternative:** distance based criteria w.r.t an in-distribution probability reference.

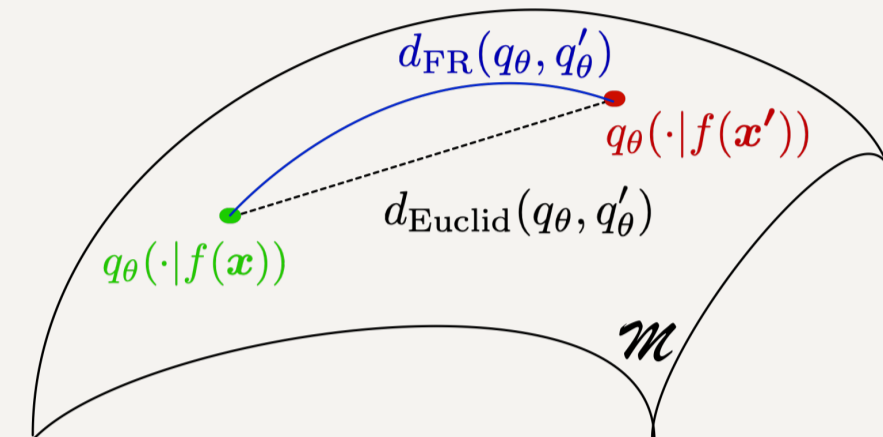
## Statistical Model



**Figure:** We model the hidden layers' outputs as class conditional Gaussian distributions and the DNN's outputs as softmax probability distributions.

## OOD Detection using the Fisher-Rao Distance

- **Fisher-Rao distance:** Let  $q_\theta(\cdot|\cdot)$  be a probability distribution with parameters  $\theta$ .



$$d_{FR}(q_\theta, q_{\theta'}) \triangleq \inf_{\gamma} \int_0^1 \sqrt{\frac{d\gamma^\top G(\gamma) d\gamma}{dt}} dt$$

**Figure:** Illustration of the shortest path between probability distributions without leaving the underlying statistical manifold.

Where  $\gamma(0) = \theta$ ,  $\gamma(1) = \theta'$ ,  $G$  is the Fisher Information Matrix and  $dt$  is the infinitesimal length element.

- **IGOOD score using the softmax probability:** Let  $q_\theta(\cdot|f(\mathbf{x}))$  be the softmax probability distribution of the outputs. We can define the Fisher-Rao distance between softmax distributions as:

$$d_{FR-Logits}(q_\theta(\cdot|f(\mathbf{x})), q_{\theta'}(\cdot|f(\mathbf{x}'))) \triangleq 2 \arccos \left( \sum_{y \in \mathcal{Y}} \sqrt{q_\theta(y|f(\mathbf{x})) q_{\theta'}(y|f(\mathbf{x}'))} \right)$$

- From which we derive the IGOOD score for the logits.

$$FR_0(\mathbf{x}) \triangleq \sum_{y \in \mathcal{Y}} d_{FR-Logits}(q_\theta(\cdot|f(\mathbf{x})), q_{\theta'}(\cdot|\mu_y))$$

- Where  $\mu_y$  are the class conditional centroids given by:

$$\mu_y \triangleq \min_{\mu \in \mathbb{R}^{|\mathcal{Y}|}} \frac{1}{N_y} \sum_{i: y_i=y} d_{FR-Logits}(q_\theta(\cdot|f(\mathbf{x}_i)), q_\theta(\cdot|\mu)).$$

- **IGOOD score leveraging latent features:** For each layer, we model the features as a set of class-conditional Gaussian distributions with diagonal standard deviation matrix. The distribution parameters are calculated accordingly.

$$\mu_y^{(\ell)} = \frac{1}{N_y} \sum_{i: y_i=y} f_j^{(\ell)}(\mathbf{x}_i), \quad \text{and} \quad \sigma^{(\ell)} = \text{diag} \left( \sqrt{\frac{1}{N} \sum_{y \in \mathcal{Y}} \sum_{i: y_i=y} (f_j^{(\ell)}(\mathbf{x}_i) - \mu_{y,j}^{(\ell)})^2} \right)$$

- We derive a confidence score by calculating the Fisher-Rao distance between the test sample  $\mathbf{x}$  and the closest class-conditional diagonal Gaussian.

$$FR_\ell(\mathbf{x}) = \min_{y \in \mathcal{Y}} d_{FR-Gauss} \left( (\mathbf{x}, \sigma^{(\ell)}), (\mu_y^{(\ell)}, \sigma^{(\ell)}) \right)$$

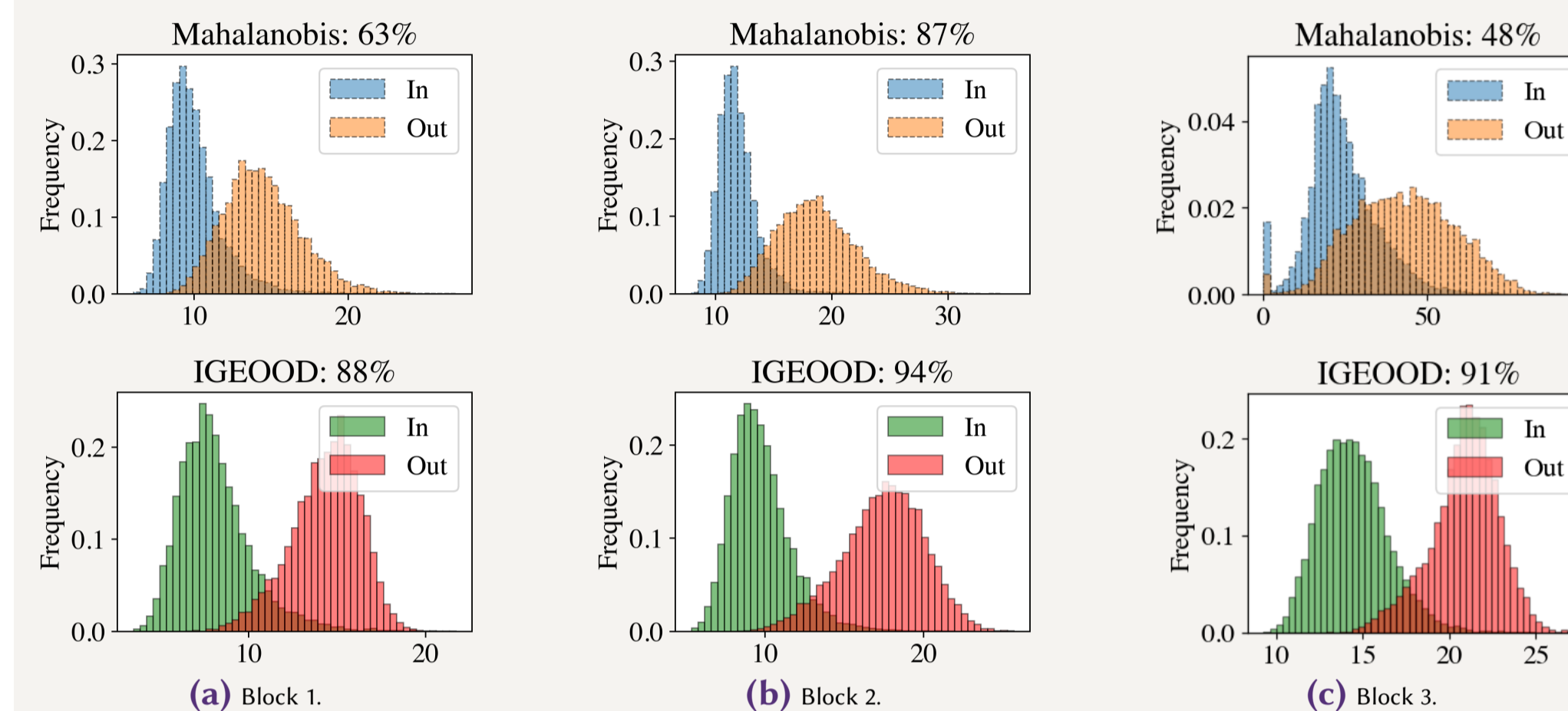
- **Feature ensemble:** we combine the confidence scores of the logits and low-level features through a linear combination. If OOD data is available, we can also calculate  $FR'_\ell(\mathbf{x}; \mu^{(\ell)'}, \sigma^{(\ell)'})$  with OOD statistics, obtaining IGOOD+.

$$FR(\mathbf{x}) \triangleq \alpha_0 FR_0(\mathbf{x}) + \sum_{\ell} \alpha_\ell \cdot FR_\ell(\mathbf{x}) + \alpha'_\ell \cdot FR'_\ell(\mathbf{x})$$

Therefore, we have derived a unified OOD detection framework that combines a single distance for both the softmax outputs and the latent features of a neural network.

## Experimental Results

- The IGOOD score increases the separation between in- and out-of-distribution data.



**Figure:** Histograms of the Mahalanobis and IGOOD scores for the outputs of each hidden block of a DenseNet model on CIFAR-10 (in-distribution) and SVHN (out-of-distribution) datasets.

- We increase the average TNR-95% by 11.8% and 2.5% with validation on OOD and adversarial data, respectively.

**Table:** Average and standard deviation of OOD detection performance for the White-Box settings. The abbreviation TNR-95%, C-10 and C-100 stands for TNR at TPR-95%, CIFAR-10 and CIFAR-100, respectively.

Model		Validation on OOD data		Validation on adversarial data	
		TNR-95%	AUROC	TNR-95%	AUROC
		Mahalanobis / IGOOD+ (ours)		Mahalanobis / IGOOD (ours)	
DenseNet	C-10	76.6±31/92.6±14	92.1±12/98.4±3.0	75.9±30/77.9±29	91.7±12/94.0±9.0
	C-100	67.2±28/90.2±21	90.2±13/97.7±5.0	60.4±34/70.9±35	85.3±19/90.8±13
	SVHN	93.3±8.0/98.0±2.0	98.6±1.0/99.6±0.1	93.7±10/92.2±9.0	98.6±2.0/98.4±1.0
ResNet	C-10	82.5±23/91.6±16	96.5±4.0/98.4±3.0	78.6±24/77.3±32	95.3±6.0/90.0±15
	C-100	70.4±30/86.4±23	91.9±10/97.1±5.0	57.4±36/65.1±33	86.9±13/88.6±15
	SVHN	96.8±6.0/98.9±2.0	99.2±1.0/99.7±0.1	96.3±8.0/93.6±14	99.1±1.0/98.4±3.0
Average and Std.		81.1±11/92.9±4.0	94.8±4.0/98.5±1.0	77.0±15/79.5±10	92.8±5.4/93.4±3.9

**Table:** TNR at TPR-95% (%) performance comparison in a White-Box setting considering the original results from [1,2,3,4]. Methods with a (\*) were tuned without OOD data.

OOD dataset		CIFAR-10		CIFAR-100		SVHN	
		Mahalanobis [1] / Gram Matrix* [2]	DeConf-C* [3] / Res-Flow [4]	IGOOD / IGOOD+	IGOOD / IGOOD+		
DenseNet	iSUN	95.3/99.0/ - / - /97.7/99.8	87.0/95.9/ - / - /93.8/99.7	99.9/99.4/ - / - /98.3/99.9			
	LSUN	97.2/99.5/99.4/98.2/98.5/99.9	91.4/97.2/98.7/96.3/95.2/99.9	99.9/99.5/ - /100/97.1/99.9			
	TinyImgNet	95.0/98.8/99.1/96.4/95.7/99.8	86.6/95.7/98.6/93.0/94.5/99.5	99.9/99.1/ - /100/98.2/99.9			
ResNet	SVHN/C-10	90.8/96.1/98.8/94.9/98.9/99.9	82.5/89.3/95.9/84.9/93.3/99.6	96.8/80.4/ - /99.0/91.6/98.3			
	iSUN	97.8/99.3/ - / - /97.2/99.9	89.9/94.8/ - / - /93.4/99.8	99.7/99.4/ - / - /99.8/100			
	LSUN	98.8/99.6/ - /99.0/98.4/100	90.9/96.6/ - /96.2/94.3/100	99.9/99.6/ - /100/99.7/99.9			
ResNet	TinyImgNet	97.1/98.7/ - /97.8/96.3/99.6	90.9/94.8/ - /94.6/90.1/99.6	99.9/99.3/ - /100/99.7/99.9			
	SVHN/C-10	87.8/97.6/ - /96.5/98.8/99.8	91.9/80.8/ - /93.0/91.6/99.7	98.4/85.8/ - /99.4/97.7/99.7			

[1] Kimin L. et al. A simple unified framework for detecting out-of-distribution samples and adversarial attacks, 2018.

[2] Sastry & Oore. Detecting out-of-distribution examples with Gram matrices, 2020.

[3] Hsu et al. Generalized ODIN: Detecting out-of-distribution image without learning from out-of-distribution data, 2020.

[4] Zisselman & Tamar. Deep residual flow for novelty detection, 2020.