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An Interactive Game Theory-PSO Based Comprehensive Framework for Autonomous Vehicle Decision Making and Trajectory Planning

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Abstract:

The mutual dependence between autonomous vehicles and human drivers is an open problem for the safety and feasibility of autonomous driving. This paper introduces a game-theoretic trajectory planner and decision-maker for mixed-traffic environments. Our solution accounts for interaction with the surrounding vehicles while making decisions, and uses a clothoid interpolation method to generate human-like trajectories. The Particle Swarm Optimizer (PSO) used here bridges the decision-making and the trajectory generating processes for a joined execution. We chose an unsignalized intersection crossing scenarios to demonstrate the feasibility of our method. Testing results show that our approach reduces the dimension of the search space for the trajectory optimization problem and enforces geometric constraints on path curvature.

Keywords: Decision Making, Game Theory, Trajectory optimization, Particle Swarm Optimization.

1 INTRODUCTION

Among the main challenges that have emerged from autonomous driving, considering interactions with human drivers in a mixed traffic flow is of great importance [Jafary et al. (2018)]. However, the cohabitation between autonomous and human-driven vehicles might create complex interactions. Thus, the behavior of Autonomous Vehicles (AVs) must be understood by other traffic participants, as each agent's action is dependent and influenced by other participants' decisions [Olaverri-Monreal (2020)]. Based on the outputs from the decision-making systems, the motion planner determines a goal trajectory that the vehicle executes and, in turn, influences the evolution of the road scene. This makes us aware that, decision-making and trajectory generation are highly coupled and thus, require a joined execution.

Under this background, our work mainly focuses on modeling mixed traffic interactions between human-driven and autonomous vehicles, when an autonomous car behaves approximately like a human-driven one.

The outline of our paper is as follows. Section 1 starts with an introduction and a brief state-of-the-art. In section 2, we first present a human-like clothoid-based model to depict vehicles' trajectories. Next, we detail the proposed decision-making process, aiming to capture the social nature of strategic interactions via a game theoretic-based approach. Then, we devised a unified decision-making and route-planning framework, where the decision-maker evaluates trajectory feasibility and the trajec-

tory prompts the last token decision to update. The validation of the proposed model and case study simulation results are presented in section 3. Finally, some concluding remarks are given in Section 4.

1.1 RELATED WORK

Autonomous vehicle trajectory design: Curve interpolation techniques such as cubic splines, Bezier curves, and clothoids are frequently used for AV's trajectory modeling due to their intrinsic smoothness and parameterization through control points [González et al. (2015)]. In our work, we are interested in the last category, clothoids, also known as Euler curves. Clothoids are parametric curves whose curvature varies linearly to the arc length. This property counteracts curvature discontinuity, which prevents the undesirable jerk and allows smooth curvature transitions from straight to circular lines or vice versa [Lambert et al. (2019)]. Clothoid curves are already used by road network design standards. Therefore, the non-holonomic behavior is structurally taken into account in clothoid-based trajectory. Several studies explore the construction of clothoids for trajectory generation. For example, [Funke and Christian Gerdes (2016)] addresses the problem of path curvature jumps during lane-changing maneuvers via a path composed of clothoids. Furthermore, a roundabout path planner based on clothoid curves has been developed in [Silva and Grassi (2018)] to ensure passenger comfort through curvature continuity. Nevertheless, in these two studies, the authors focus on zero heading and zero curvature. In their study, [Alhajyaseen et al. (2013)] showed that turning trajectories can be approximated as a combination of clothoids, circular arcs, and straight lines.

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Vehicle interaction and decision-making: To tackle drivers' tactical decisions and model a human-like decision-making process, game theoretical frameworks have been explored in the context of transportation research [Zhang et al. (2010)]. Game theory is a paradigm that provides an effective framework to model, analyze, and solve social interaction between strategic decision-makers. The study in [Mandiau et al. (2008)] proposes a two-player normal-form game to describe the coordination of AVs within intersections. The authors handle the interactions between autonomous vehicles. However, vehicle dynamics were not considered here. This problem is addressed in the paper [Tian et al. (2018)] within autonomous roundabout crossing. Here, kinematics constraints are incorporated in the decision-making module. Finally, the paper [Cleac'h et al. (2022)] addresses the convergence problem and shows its complexity. The authors formulate a trajectory optimization problem in a Nash-style, dynamic game. The proposed approach shows a guaranteed local convergence, but not a global one.

2 JOINT PATH GENERATION AND DECISION MAKING

We consider the interactions between an autonomous car and a human-driven one in an unprotected left-turn driving scenario. Using the Nash equilibrium solution concept, we model a symmetrical, general-sum dynamic game. Communication between the two vehicles is not allowed. Only the starting location \mathbf{p}_s and target destination \mathbf{p}_g are observable. Our analysis focuses on the formulation from the Ego vehicle's standpoint.

2.1 Trajectory Generation Methodology

In this paper, we use the outcomes of the study [Abdeljaber et al. (2020)] to describe vehicles' trajectories. We simplify the linkage of two clothoids without using the arc of the circle, which does not affect the smoothness of the curve. We generate feasible trajectories from the starting location and orientation $\mathbf{p}_s(x_s, y_s, \theta_s)$ to $\mathbf{p}_g(x_g, y_g, \theta_g)$. Each trajectory \mathcal{T} is composed of straight and curve segments: $\mathcal{T} = \mathcal{C}_1 \oplus \mathcal{C}_2 \oplus \mathcal{C}_3$. The curve segment \mathcal{C}_2 comprises two clothoids $\mathcal{C}_2 = C_1 \oplus C_2$ determined through integration procedures as:

$$\begin{aligned} x(s) &= x_0 + \int_0^s \cos\left(\frac{1}{2}\kappa'\tau^2 + \kappa_0\tau + \theta_0\right) d\tau, \\ y(s) &= y_0 + \int_0^s \sin\left(\frac{1}{2}\kappa'\tau^2 + \kappa_0\tau + \theta_0\right) d\tau, \end{aligned} \quad (1)$$

Where (x_0, y_0) , θ_0 and κ_0 are respectively the coordinates, the orientation and the curvature of clothoid's base point. κ' and s represent respectively the sharpness and the curvilinear abscissa, linearly related according to $\kappa(s) = \kappa_0 + \kappa' \cdot s$. $\theta(s)$ is the tangent angle at s defined as :

$$\forall s, 0 \leq s \leq L_C, \theta(s) = \int_0^s \kappa(\tau) d\tau, \quad \theta(s) = \frac{1}{2}\kappa' \cdot s^2 + \kappa_0 \cdot s + \theta_0 \quad (2)$$

Where L_C denotes the length of the clothoid C . The four parts are joined via a set of points of interest, illustrated in figure (1). Optimizing multiple trajectories can be challenging depending on the problem formulation. In this work, we propose to simplify this problem by reducing the search space's dimension. Thus, we represent each trajectory through a set of knot points \mathbb{P} , then interpolate them using elementary clothoid equations, as described in equation (1). These knots are potentially discontinuity locations along the curvature profile $\kappa(s)$. Consequently, we enforce geometric and curvature continuities

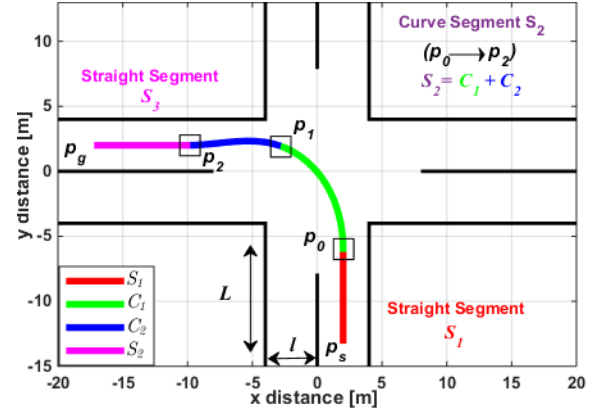


Fig. 1. Trajectory description

along the path length at each point $p_m(x_m, y_m, \theta_m)$ bridging two adjacent segments $S_1, S_2 \in \{\mathcal{C}_1, C_1, C_2, \mathcal{C}_3\}$ by holding the following [Bertolazzi and Frego (2018)]:

$$\forall p_m \in \mathbb{P}, \begin{cases} x_{S_2}(0) = x_{S_1}(L_{S_1}), & \theta_{S_2}(0) = \theta_{S_1}(L_{S_1}) \\ y_{S_2}(0) = y_{S_1}(L_{S_1}), & \kappa_{S_2}(0) = \kappa_{S_1}(L_{S_1}) \end{cases} \quad (3)$$

The uniqueness of the curve \mathcal{C}_2 is guaranteed using the outputs of the late study, as indicated below:

$$\begin{cases} x_{\mathcal{C}_2}(0) = x_0, & y_{\mathcal{C}_2}(0) = y_0, & \theta_{\mathcal{C}_2}(0) = \theta_0, & \kappa_{\mathcal{C}_2}(0) = \kappa_0 \\ x_{\mathcal{C}_2}(L_{\mathcal{C}_2}) = x_2, & y_{\mathcal{C}_2}(L_{\mathcal{C}_2}) = y_2, & \theta_{\mathcal{C}_2}(L_{\mathcal{C}_2}) = \theta_2, & \kappa_{\mathcal{C}_2}(L_{\mathcal{C}_2}) = \kappa_2 \end{cases} \quad (4)$$

For simplification, we fixed $p_0 = (\frac{l}{2}, -\frac{l}{3}, \frac{\pi}{2})$, here L predetermined distance upstream of the conflict zone, l represents the lane width. The orientations θ_1 and θ_2 at p_1 and p_2 respectively, are known based on vehicles' target destination. By making this specific choice, only 4 degrees of freedom $\mathbb{P} = (x_1, y_1, x_2, y_2) \in \mathbb{R}^4$ are required to completely design the trajectory \mathcal{T} .

Under these assumptions, we designed a decision-making algorithm that finds an optimal set \mathbb{P}^* , ensuring the generation of an efficient and conflict-free trajectory $\mathcal{T}(\mathbb{P}^*)$. The main advantage of our method is the combination of the search space for trajectory optimization with the strategic space that will be explored during the decision-making process.

2.2 Game theoretic decision-making process

This paper introduces a model that aims to improve social interactions at intersections. The model incorporates the decision-making process through Nash Equilibrium (NE), which ensures that all players are treated equally regardless of their type. However, in the context of intersection crossing, both player's objective function and constraints depend on the opponents' strategic space. Thus, our problem can be characterized as a specific instance of Nash equilibrium, referred to as Generalized Nash Equilibrium Problem (GNEP) [Facchinei and Kanzow (2010)].

Game structure:

$\mathcal{S}_v \subset \mathbb{R}^n$ and $\mathcal{S}_o \subset \mathbb{R}^m$ represent the sets of possible strategies available to player v , and its opponent o , respectively. Both players control their respective strategic spaces via n and m decision variables.

$\mathcal{S}^{game}(\mathcal{S}_v, \mathcal{S}_o) = \mathcal{S}_v \times \mathcal{S}_o \in \mathbb{R}^{n \times m}$ is the game strategic space.

$J^p(s^v, s^o) : \mathbb{R}^{n \times m} \rightarrow \mathbb{R}$ is the payoff function for player p .

The player v aims to solve the following optimization problem:

$$\underset{s^v \in \mathcal{S}_v}{\text{minimize}} \quad J^v(s^v, s^o) \quad \text{s.t.} \quad s^v, s^o \in \mathcal{S}^{game}(\mathcal{S}_v, \mathcal{S}_o) \quad (5)$$

The player O 's objective function J^o and its optimization problem are formulated similarly. Both players behave rationally in order to reach an equilibrium, following the "best answer" principle currently used in game theory. The equilibrium $(s^{*,v}, s^{*,o}) \in \mathcal{S}^{game}$ solves this problem respecting equation (6).

$$\forall s^v, \forall s^o \quad \begin{cases} J^v(s^{v*}, s^{o*}) \leq J^v(s^v, s^{o*}) \\ J^o(s^{v*}, s^{o*}) \leq J^o(s^{v*}, s^o) \end{cases} \quad (6)$$

We define a Generalized Nash Equilibrium problem for a two-player game as follows:

$$f(s^v, s^o) = \max \left\{ \sup_{\bar{s}^v \in \mathcal{S}_v} \{J^v(s^v, s^o) - J^v(\bar{s}^v, s^o), 0\} \right\} + \max \left\{ \sup_{\bar{s}^o \in \mathcal{S}_o} \{J^o(s^v, s^o) - J^o(s^v, \bar{s}^o), 0\} \right\} \quad (7)$$

In this work, a player strategy $s_i^v \in \mathcal{S}_v$, and $s_j^o \in \mathcal{S}_o$ encodes optimal knots $s_i^v = \mathbb{P}_i^v \in \mathbb{R}^4$, and $s_j^o = \mathbb{P}_j^o \in \mathbb{R}^4$, respectively, that will be interpolated following equation (1).

Players payoff design: The aggregation of cost indicators associated with player p 's trajectory \mathcal{T}_p , determine its payoff function $J^p(s^v, s^o)$.

- **Efficiency Awareness:** The AV is incited to closely match the speed limit as follows:

$$I_{(s^v)}^{eff} = \frac{|V_{max} - \bar{v}(s^v)|}{V_{max}} \quad (8)$$

where $\bar{v}(s^v)$ is the mean speed of a player executing a strategy s^v and V_{max} is the higher speed limit in the intersection.

- **Safety Enhancement:** Each trajectory is enclosed within an Oriented Bounding Box (**B**) representing the shape of the vehicle. We also define a safety measure based on the gap separating the two bounding boxes from a potential collision, noted Gap To Collide (**GTC**) as stated below:

$$I_{(s^v, s^o)}^{safe} = \min_{t_{min} < t < t_{max}} e^{-\left(\frac{GTC(t)}{G_{crit}}\right)} \quad (9)$$

$[t_{min}, t_{max}]$ is the interaction time span. G_{crit} is a critical threshold that ought to be permanently maintained between vehicles.

Constraints: Collision avoidance constraints are expressed in terms of an adaptive elliptic safety zone ψ as follows:

$$\forall t \in [t_{min}, t_{max}], \quad \psi(t) \cap B^o(t) = \emptyset \quad (10)$$

B^o is the bounding box representing the opponent vehicle.

To handle this constraint, we check that equation(10) is verified for each vertex $v_j^o(x_j^o, y_j^o) \in B^o$, $\forall j \in \{1, \dots, 4\}$ following:

$$\frac{\left[\cos(\theta) \cdot (x_j^o - x) + \sin(\theta) \cdot (y_j^o - y) \right]^2}{d_{maj}(s^v)^2} + \frac{\left[\sin(\theta) \cdot (x_j^o - x) - \cos(\theta) \cdot (y_j^o - y) \right]^2}{d_{min}^2} > 1 \quad (11)$$

With (x, y) , and θ are, respectively, the center coordinates and the orientation of the ellipse, which evolves according to $d_{maj}(s^v)$ and d_{min} , the major and minor elliptic semi-axes as follows:

$$d_{maj}(s^v) = \frac{1}{2}L_v + TTC \cdot v(s^v), \quad d_{min} = \frac{1}{2}l_w + d_{safe} \quad (12)$$

Where: $v(s^v)$ is the speed while executing strategy s^v . **TTC** is the time left before the collision. L_v and l_w are, respectively, the vehicle's length and wheelbase, d_{safe} represents a minimum lateral safety distance parameter.

Objective function formulation: The player's objective function incorporates both safety and efficiency features.

$$h(s^v, s^o) = \omega_1 \cdot I_{(s^v, s^o)}^{safe} + \omega_2 \cdot I_{(s^v)}^{eff}, \quad 0 \leq \omega_i \leq 1 \quad (13)$$

This function is constrained by non-linear constraints related to the left-turning maneuver on the one hand, and the opponent's vehicle strategies on the other. The optimization problem a player must solve is the following:

$$\min_{s^v, s^o} h(s^v, s^o), \quad \text{s.t.} \quad C(s^v, s^o) \leq 0, \quad (14)$$

Where $C(s^v, s^{oth})$ summarized the constraints expressed in equation (11).

2.3 Particle Swarm Optimization: A game-solving approach

Exhaustive exploration of all possible combinations in a search space is computationally intensive and time-consuming. Consequently, the optimization process requires sophisticated algorithms such as heuristic methods, and meta-heuristic algorithms, to attain satisfactory optimization results. Particle Swarm Optimization (PSO) is a meta-heuristic algorithm that mimics the collective swarm behavior. Recently, extensive applications of PSO can be found in the context of mobile robot's navigation Abdallaoui et al. (2022). In PSO, each random particle p represents a potential solution, has a fitness function $f(u)$ and searches for optima by adapting its position through its individual experience and knowledge exchange with neighboring particles. Given a \mathbf{P} -particle swarm, exploring in a D-dimensional search space, the notations are:

- $X_p^k = (x_{p,1}^k, \dots, x_{p,D}^k)$, $V_p^k = (v_{p,1}^k, \dots, v_{p,D}^k)$: position and respectively velocity of particle p at the k^{th} iteration.
- $p_{best}^k = \arg \min_{1 \leq i \leq k} f(X_p^i)$: best solution founded by particle p until the k^{th} iteration.
- $g_{best}^k = \arg \min_{\substack{1 \leq i \leq k \\ 1 \leq j \leq P}} f(X_j^i)$: best solution founded by the entire swarm \mathbf{P} until the k^{th} iteration.

The update of each particle's velocity and position from generation k to $k+1$ s governed by the following:

$$\begin{cases} V_p^{k+1} = \omega \cdot V_p^k + c_1 \cdot r_1 \cdot (p_{best}^k - X_p^k) + c_2 \cdot r_2 \cdot (g_{best}^k - X_p^k) \\ X_p^{k+1} = X_p^k + V_p^{k+1} \end{cases} \quad (17)$$

Here, the inertia weight ω governs the influence of the velocity. c_1 and c_2 represent cognition and social learning factors respectively Gou et al. (2017). The terms r_1 and r_2 correspond to random numbers uniformly generated in the range of $[0, 1]$.

In our research, each particle is a potential combination of both players strategies $(\mathcal{S}_v, \mathcal{S}_o)$, that explores the joined space $\mathcal{S}^{game}(\mathcal{S}_v, \mathcal{S}_o)$. Thus, the PSO-based algorithm is used to reach the Nash Equilibrium described in equation (7).

3 SIMULATION RESULTS

We validate our algorithm and demonstrate the performance of the proposed approach on two traffic scenarios commonly encountered in intersection crossing.

Particle Swarm Optimizer Performance

The convergence of the proposed PSO-based solver is depicted in the figures (2) and (3). From analyzing the best fitness value \mathbf{g}_{best} found over all the particles, we can see that Generalized Nash equilibrium can be obtained in less than 40 iterations in both scenarios. As iteration grows, the particles find a strategic combination (s^{v*}, s^{o*}) solving the two-player game in equation (7). Besides, we can see that the global best solution founded by the best particle in figure (2), converges faster and more smoothly than the average objective $\bar{C}^k = \frac{1}{P} \sum_{j=1}^P f(X_j^k)$ illustrated in figures (3). This indicates that we can avoid falling into a local optimum during the search process. From the above results, the proposed PSO-based solver is appropriate for finding Generalized Nash Equilibrium.

Scenario 1: This scenario is illustrated in the figure (4). We observe that both the autonomous vehicle and the human drivers proceed to cross the intersection concurrently. This synchronized action is based on their mutual anticipation that each of them will select a trajectory deemed safe. The Ego vehicle chose to proceed first at the intersection. The ego and the opponent vehicles successfully avoided collision and reached their target destinations.

Scenario 2: In the second case, presented in figure (5), the ego vehicle chose to cede the priority to the opponent vehicle, as it predicted that the conflict zone would not be cleared in time. This decision reflects the inability of ego vehicle to identify a feasible trajectory that avoids a potential collision with the trajectory being executed by its opponent, who would join the intersection ahead. Therefore, Ego vehicle's optimal response is to stop, ensuring that the opponent vehicle safely traverses the intersection.

4 CONCLUSIONS

This paper presents a comprehensive game theoretic approach for decision-making and trajectory optimization for autonomous vehicles. The primary goal of our study is to enhance interaction between autonomous and human-driven vehicles in mixed traffic scenario. We present a framework that allows autonomous vehicles to make decisions based on human drivers expectations and to analyze their decisions. We used a solver based on Particle Swarm Optimization to find Generalized Nash Equilibrium and optimize trajectories that satisfy the required safety constraints, while maintaining human-like behavior. The proposed approach underwent validations and showed its effectiveness, allowing AVs to maneuver through challenging driving situations and to effectively interact with other vehicles.

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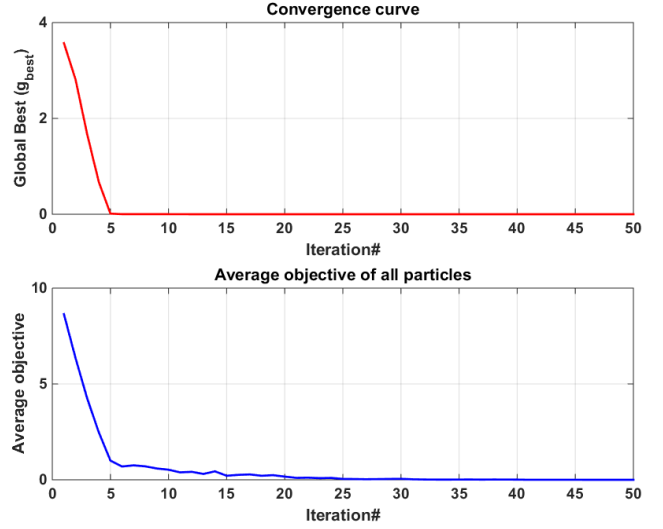


Fig. 2. Scenario 1: Convergence graph of PSO

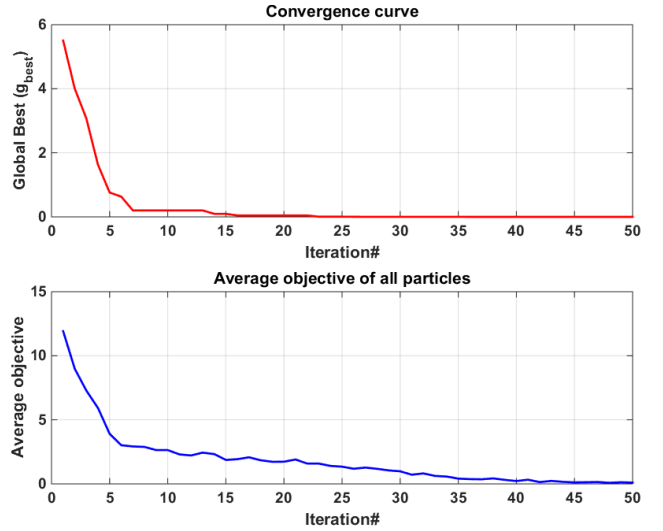


Fig. 3. Scenario 2: Convergence graph of PSO

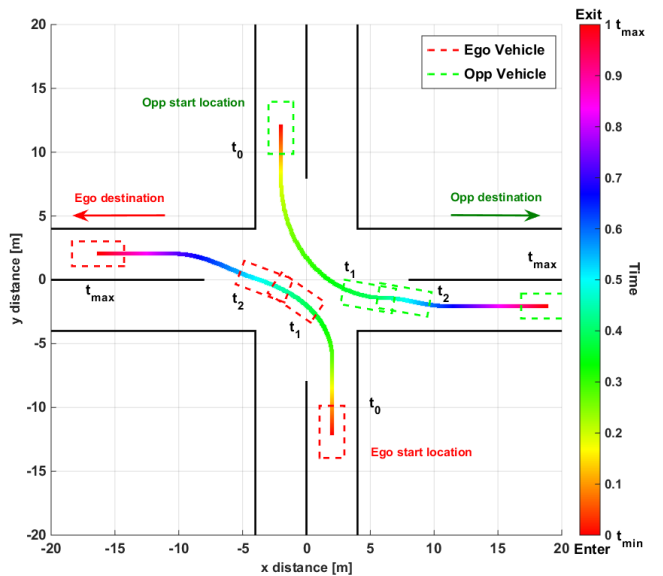


Fig. 4. Scenario 1: Ego vehicle and its Opponent (Opp) crossing the intersection

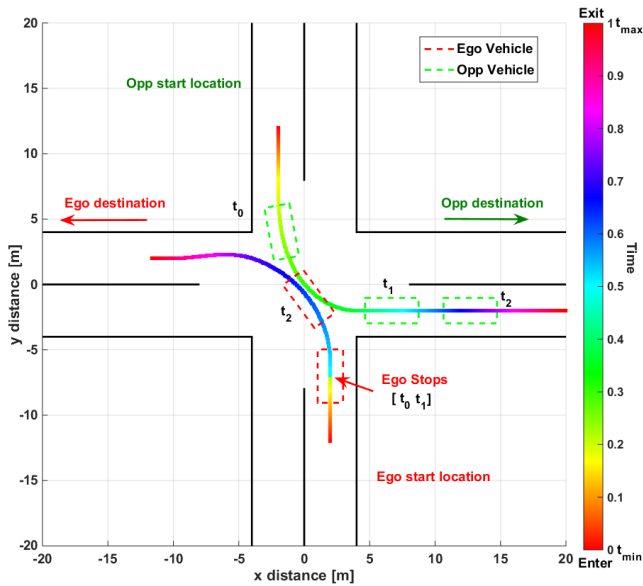


Fig. 5. Scenario 2: Ego vehicle stops and the Opponent (Opp) crosses first

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