



**HAL**  
open science

## Structure and NO<sub>x</sub> Emissions of Stratified Hydrogen-Air Flames Stabilized on a Coaxial Injector

Maxime Leroy, Clément Mirat, Antoine Renaud, Stefano Puggelli, Stephan Zurbach, Ronan Vicquelin

► **To cite this version:**

Maxime Leroy, Clément Mirat, Antoine Renaud, Stefano Puggelli, Stephan Zurbach, et al.. Structure and NO<sub>x</sub> Emissions of Stratified Hydrogen-Air Flames Stabilized on a Coaxial Injector. *Journal of Engineering for Gas Turbines and Power*, 2023, 146 (3), 10.1115/1.4063579 . hal-04441509

**HAL Id: hal-04441509**

**<https://centralesupelec.hal.science/hal-04441509>**

Submitted on 6 Feb 2024

**HAL** is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

# Structure and NO<sub>x</sub> emissions of stratified hydrogen-air flames stabilized on a coaxial injector

## Maxime Leroy\*

Université Paris-Saclay,  
CNRS, CentraleSupélec, Laboratoire EM2C,  
91190, Gif-sur-Yvette, France.  
Safran Tech  
rue des Jeunes Bois, Châteaufort,  
78114, Magny-les-Hameaux, France  
Email: maxime.leroy@centralesupelec.fr

## Clément Mirat

Université Paris-Saclay,  
CNRS, CentraleSupélec, Laboratoire EM2C,  
91190, Gif-sur-Yvette, France.  
Email: clement.mirat@centralesupelec.fr

## Antoine Renaud

Université Paris-Saclay,  
CNRS, CentraleSupélec, Laboratoire EM2C,  
91190, Gif-sur-Yvette, France.  
Email: antoine.renaud@centralesupelec.fr

## Stefano Puggelli

Safran Tech  
rue des Jeunes Bois, Châteaufort,  
78114, Magny-les-Hameaux, France  
Email: stefano.puggelli@safrangroup.com

## Stephan Zurbach

Safran Tech  
rue des Jeunes Bois, Châteaufort,  
78114, Magny-les-Hameaux, France  
Email: stephan.zurbach@safrangroup.com

## Ronan Vicquelin

Université Paris-Saclay,  
CNRS, CentraleSupélec, Laboratoire EM2C,  
91190, Gif-sur-Yvette, France.  
Email: ronan.vicquelin@centralesupelec.fr

## ABSTRACT

*In recent years, the need for low-carbon power has seen hydrogen emerge as a potential fuel to replace conventional hydrocarbons in combustion to limit CO<sub>2</sub> emissions in several sectors, including aeronautics. The challenges posed by hydrogen combustion are similar to the issues of kerosene flames but more challenging, like nitrogen oxide (NO<sub>x</sub>) emissions and flame flashback. One potential solution to address these problems is to burn a rich mixture of hydrogen and air in globally lean conditions on a coaxial injector to*

---

\*Address all correspondence to this author

*obtain a stable and staged combustion and attempt to reduce emissions. In this article, the evolution of  $NO_x$  production as more air is mixed into the fuel is studied, as well as the changes in flame size and structure. In particular, the appearance of a secondary flame front is observed and increasing the proportion of air in the fuel mixture both shortens the flame and reduces the  $NO_x$  emission index. Additionally, the effect of the global equivalence ratio and flame thermal power are studied. Finally, existing models for  $NO_x$  emission of hydrogen flames on a coaxial injector based on average flame residence time and strain rate are tested and shown to have promising results.*

## **NOMENCLATURE**

- $_1$  Subscript for central (fuel) channel
- $_2$  Subscript for annular (air) channel
- $A$  Channel cross-sectional area ( $m^2$ )
- $d$  Channel diameter (mm)
- $e$  Inner channel lip thickness (mm)
- $J$  Momentum flux ratio
- $L_f$  Length of the diffusion flame front [m]
- $L_p$  Length of the premixed flame front [m]
- $\dot{m}$  Channel mass flow rate ( $kg \cdot s^{-1}$ )
- $\mathcal{P}_{th}$  Flame thermal power (kW)
- $Re$  Reynolds number  $Re = \frac{dU}{\nu}$
- $\rho$  Density ( $kg \cdot m^{-3}$ )
- $\Phi$  Equivalence ratio
- $u$  Channel bulk velocity ( $m \cdot s^{-1}$ )
- $NO_x$  Nitrogen oxides NO and  $NO_2$
- $EINO_x$   $NO_x$  emission index ( $g/kgH_2$ )
- $OH^*$  Excited OH radical

## INTRODUCTION

The drive to reduce CO<sub>2</sub> emissions of the aeronautical sector has seen hydrogen get increasing attraction for its potential of carbon-free combustion. However, the high combustion temperatures of hydrogen, as well as its high laminar flame speed, generate a few challenges to be tackled, such as nitrogen oxides (NO<sub>x</sub>) emission and stability. These challenges are not new, as burners using classical fuels such as natural gas or kerosene have required NO<sub>x</sub> reductions, while keeping stable flames. As a result, many concepts have been developed in the past, such as Lean Premixed [1], Lean Direct injection (LDI)[2] or Rich-Burn Quick-Mix Lean-Burn (RQL) [3]. These concepts can then be adapted for use in hydrogen combustion, and adaptations of LDI [4] and Lean Premixed [5, 6] have been studied, alongside new concepts such as micromix [7, 8]. Here, we will study a derivative of the RQL concept, which consists from a theoretical point of view of injecting first fuel and air to burn a rich mixture (in a technically premixed regime), then using a quick addition of air to finish combustion, and reach a globally lean mixture. The advantages of this technique for hydrogen are highlighted by the flame properties computed with the CRECK mechanism [9, 10] for standard pressure and temperature and shown in Fig. 1. RQL avoids the range of equivalence ratios around stoichiometry where NO<sub>x</sub> production is maximal because of the Zel'dovich or thermal reaction pathway. Additionally, the prompt NO pathway from CH radical in hydrocarbons flames, which produces an increasing amount of NO<sub>x</sub> with equivalence ratio, is not active in H<sub>2</sub> combustion [11]. It is also apparent that the maximum laminar burning velocity is located around the equivalence ratio of 2, making such rich premixed flames hard to blow off.

Hydrogen burners using a conventional RQL design have been studied by Göke et al.[12]. Taking advantage of the H<sub>2</sub> flame properties such as ease of ignition and strain rate resistance, a more compact combustor can be considered along with a different injector configuration such as a coaxial injector. Several works [13, 14] have recently shown various flame types stabilized on coaxial injectors with pure hydrogen injected in the central channel. The present work investigates using a coaxial burner as a RQL concept where a staged air injection with a rich premixed mixture and a coaxial stream is considered. Minakawa & Yuasa [15] studied a similar injector that showed



potential for low-NO<sub>x</sub> flames.

The observed flames stabilized on the coaxial injector share several features with non-premixed diffusion jet flames extensively studied in the literature. The seminal work of Peters and Donnerhack [16] provides analysis for NO<sub>x</sub> production from the Zel'dovich mechanism in unconfined jet diffusion flames, accounting for a flame residence time, which highlights the relationship between the flame length, the jet velocity, and NO<sub>x</sub> production. Their analytical model found the EINO<sub>x</sub> to scale as  $L_f^3/(u_1 d_1^2)$ , which can be interpreted as a residence time. Subsequent works have extended it to more complex cases, such as diluted or coaxial flames, while considering the effect of strain or improving the residence time modeling. Chen and Driscoll [17] have combined the analytical model from Peters and Donnerhack [16] with the flame length model from Dahm and Mayman [18] to explain how the addition of coaxial air achieved reductions of NO<sub>x</sub> emissions by shortening the flame. Taking into account the effect of the Damköhler number allowed to show that the normalized emissions  $EINO_x/(L_f^3/(u_1 d_1^2))$  scaled as the  $(u_1/d_1)^n$  with  $n = 1/2$ . Driscoll et al. [19] then investigated flame dilution with helium and compared the separate effect of coaxial air with that of rich premixing. This was further extended when Chen and Kollman [20] studied hydrogen flames with coaxial air, for moderate  $u_1/u_2$  ratios. They highlighted that in that case the flame volume  $V_f$  should be directly used instead of  $L_f^3$  to more accurately describe the flame zone. With this approach and replacing  $u_1/d_1$  with  $(u_1 - u_2)/d_1$  to take into account the effect of coaxial flow, they were again able to collapse their result on a single slope in a numerical study that appeared to follow another 1/2 power law. In addition, Weiland et al. [21] studied and modeled the effect of N<sub>2</sub> dilution on hydrogen coaxial flames, showing that combining coaxial air and dilution reduces flame length and emissions.

These studies provide an extensive basis for studying RQL-like jet flames stabilized on a coaxial injector. However, except for [15], no works could be found about the study of flame length and NO<sub>x</sub> emissions in staged hydrogen coaxial flames to tie them with existing knowledge on hydrogen flames. Furthermore, the evolution of the flame structure as the proportion of air in the central channel increases has yet to be studied. As a result, this work on rich-premixed hydrogen-air

coaxial flames focuses on the effect of air staging on the flame length and  $\text{NO}_x$  emission. The flame length is of prime importance in designing a compact aero-engine, but will also be shown to be an essential factor in  $\text{NO}_x$  production as in other jet flames studied in the literature.

The work is presented in the following sections. The second section presents the experimental setup used to produce and study hydrogen flames at a laboratory scale at ambient pressure and the post-processing method used for flame measurement. In Sec. 3, we first present the topology of the flames we obtain and describe the appearance of a secondary flame front due to the staged injection of air. Then, we study the evolution of flame lengths in unconfined jets under various degrees of staging. In the fourth section, the effect of staging on  $\text{NO}_x$  is investigated, and the observed evolution is compared to existing models. Finally, in Sec. 5, we study the effect of thermal power and global equivalence ratio variations and attempt to find a common model that can account for the change of all these parameters.

## 1 EXPERIMENTAL SETUP

This section presents the studied configuration (combustion chamber and injector), the controlling parameters, and the employed diagnostics.

### 1.1 Combustion chamber

The combustion chamber used in this study is the same as the one used in [14], and [22, 23], with the exception that the exit section is reduced to a 30 x 30 mm square in order to avoid recirculation of exterior air induced by the entrainment of the flame jet, as described in Fig. 2 (a). The gas analyzer used is the same as the one in [14], and the injector is a coaxial injector of external diameter  $d_2 = 20$  mm and two variants of internal diameter with either  $d_1 = 6$  mm or  $d_1 = 10$  mm. Using the  $d_1 = 6$  mm diameter allows to establish a fully developed flow while using the  $d_1 = 10$  mm does not, due to the change in section occurring near the end of the injector tube. In the annular channel, a centering piece located 76 mm upstream of the exit plane causes a developing flow profile.

## 1.2 Injector

### 1.2 Injector

The injection is controlled by three parameters: the flame thermal power  $\mathcal{P}_{th}$  which, sets the mass flow rate of hydrogen in the central channel  $\dot{m}_{1,H_2}$ , the rich central equivalence ratio  $\Phi_1$ , that determines the premixed mass flow rate of air in the central channel  $\dot{m}_{1,air}$  and the global equivalence ratio  $\Phi_{tot}$ , that sets the mass flow rate in the annular channel  $\dot{m}_2$ :

$$\Phi_1 = s \frac{\dot{m}_{1,H_2}}{Y_{2,O_2} \dot{m}_{1,air}}, \Phi_{tot} = s \frac{\dot{m}_{1,H_2}}{Y_{2,O_2} (\dot{m}_{1,air} + \dot{m}_2)} \quad (1)$$

with  $s = 8$ , the mass stoichiometric ratio and  $Y_{2,O_2} = 0.233$  the oxygen mass fraction in the oxidizer stream

Once these flow rates are set, we can compute the momentum flux ratio  $J$ :

$$J = \frac{\rho_2 U_2^2}{\rho_1 U_1^2} = \frac{A_1^2 \rho_1 \dot{m}_2^2}{A_2^2 \rho_2 \dot{m}_1^2} \quad (2)$$

An overview of the range of values for these parameters and others for is provided in Table 1.

### 1.3 Diagnostics

The flame structure is observed using OH\* chemiluminescence. A Princeton Instruments PI-MAX4 intensified CCD camera is equipped with a NIKKOR 105 mm UV objective and an Asahi SPECTRA XBPA310  $310 \pm 5$  nm bandpass filter. The aperture is fixed at f/8.0, and the exposure time is  $\Delta t = 15$  ms. Fifty images are averaged together to construct the mean images. Afterwards, the images are post-processed by inverse Abel transform to visualize the flame front, and a Gaussian filter of  $\sigma = 0.9$  pixels is applied to lessen the noise induced by this transform.

Exhaust gas composition analysis is performed using a HORIBA VA-5111 multi-component gas analyzer unit associated with a VS-5000 series sampling unit. The measured molar fraction of the species of interest are O<sub>2</sub>, using a paramagnetic sensor, and NO, using an NDIR sensor. Before being analyzed, the exhaust gases are dried through a condenser. Measurement of total

### 1.3 Diagnostics

NO<sub>x</sub> (NO+ NO<sub>2</sub>) is obtained using a NO<sub>2</sub> to NO converter upstream of the analyzer unit that a switch can toggle. The NO sensor has a range of 0-500 ppm and a manufacturer-specified repeatability of ± 2.5 ppm. The sensors are calibrated using calibration gas bottles with a 3% precision on composition.

In order to account for potential variations in the exit plane, three measurements of gas composition are taken, one at the center of the exit section, one at r = 6 mm, and one at r = 12 mm. The results are averaged to obtain the final value. The NO<sub>x</sub> emission index (EINO<sub>x</sub>) in g/kgH<sub>2</sub> is used to produce a metric under which all flames can be compared:

$$EINO_x = X_{NO_x, wet} \frac{M_{NO_2} \dot{m}_{tot}}{\bar{M}(\Phi_{tot}) \dot{m}_{H_2}} \quad (3)$$

As in [19], the molar mass of NO<sub>2</sub> is used to calculate EINO<sub>x</sub> as most NO is converted to NO<sub>2</sub> in the atmosphere. The average molar mass of the flue gas  $\bar{M}$  is obtained on the basis of a one-step reaction at the injected equivalence ratio. Furthermore,  $X_{NO_x, wet}$  is obtained from the dry NO<sub>x</sub> measurement using the expected water exhaust mole fraction from a one-step reaction at the given equivalence ratio, as it could not be measured.

$$X_{NO_x, wet} = X_{NO_x, dry} (1 - X_{H_2O}(\Phi_{tot})) \quad (4)$$

Gas molar fractions are measured once the chamber reaches thermal equilibrium, indicated by the stabilization of the wall temperature, to avoid possible transient effects. In addition, the mole fraction of unburned hydrogen is measured using a Madur SENMA electrochemical H<sub>2</sub> sensor with a 0-2000 ppm range. No significant levels (<30 ppm) of unburned hydrogen are detected in the flames in this study, indicating complete combustion.

## 1.4 Flame length measurement

### 1.4 Flame length measurement

Previous studies [18, 17, 19] have shown that flame length is of particular interest when studying  $\text{NO}_x$  in jet flames. We, therefore, estimate these flame lengths. We choose to use  $\text{OH}^*$  chemiluminescence imagery as the hot water vapor produced by  $\text{H}_2$  combustion radiates in the near-infrared region [24] and emits visible light above the effective reaction region of the flame. Our approach is similar to the one used in [25], and thresholding with Otsu's method [26] is employed.

Two different cases must be separated. In the first case, the flame displays one flame front, and the Otsu method is used on the  $\text{OH}^*$  image to separate the dark pixels in the background from the flame. This technique isolates the brighter pixels of the  $\text{OH}^*$  chemiluminescence image and allows us to capture the contour of the flame. The height of the highest pixel of this contour provides an estimate of the flame length. In the second case, two flame fronts are apparent and there is a need to separate the premixed inner flame front from the outer flame front. A multiple-threshold variant of the Otsu method is used to detect the two flames based on their brightness. Two thresholds are used, allowing us to obtain two different classes of pixels within the flames that capture the outer and inner flame front, respectively. The two flame front lengths are measured based on these thresholds. The effect of this thresholding method is illustrated in Figs. 3 and 4. Figure 3 shows the histogram of a flame with two flame fronts and the thresholds obtained by multiple Otsu thresholding. It shows three distinct categories of pixels: a sharp peak near zero corresponding to the background, a weaker peak of median intensity with a normalized intensity of around 0.4, and a smaller group of very high-intensity pixels centered around a normalized intensity of 0.7. The two thresholds obtained by the Otsu method are also traced in the histogram and separate the three different groups of pixels. Observation of the associated  $\text{OH}^*$  image in Fig. 4 allows us to verify that these groups of pixels correspond to the flame's discernible features. The contour drawn from the first threshold captures the longer diffusion flame, while the contour corresponding to the higher threshold captures the brighter pixels of the premixed flame front. As the brighter front can change as a function of the equivalence ratio, the attribution of each measure to each flame is based on its length: the shorter flame is the premixed one, and the longest is the diffusion flame.

Overall, these observations show that both flame fronts' lengths can be measured. As the Otsu thresholding is sensitive to the total number of pixels, the effect of cropping the background was studied and was found to diminish the flame length of the diffusion flame front by up to 5% after removing 90% of the width of the image where the flame was not present. In the rest of the study, the entire background is kept to maintain the same conditions for each image.

## 2 FLAME TOPOLOGY AND LENGTHS AT $\mathcal{P}_{th} = 10$ KW

### 2.1 RQL-like combustion stabilized above a coaxial injector

Using the coaxial injector described in Sec. 1, we obtain jet flames, of which an overview of the different visualizations is shown in Fig. 5. The particular example flame presents two flame fronts revealed by the inverse Abel transform. The shorter one is produced by the combustion of the rich premixed  $H_2$ /air mixture, while the longer one surrounds the first one and corresponds to a diffusion flame between burned gases from the first flame front and coflowing air. According to Fig. 1, the stoichiometric diffusion flame front is expected to be responsible for most of the  $NO_x$  emission.

The flames produced on this type of injector differ from the conventional and idealized RQL kind of flame as a hot diffusion flame front is still present after the cooler rich premixed flame. However, a quick transition from rich to lean burn should temper this high- $NO_x$  route. Hence, as in any practical RQL system, a rapid mixing with the added air is primordial to reduce the time in contact with the stoichiometry line.

### 2.2 Appearance of the secondary flame front

As shown in Fig. 6, when the fuel changes from non-premixed hydrogen to a fuel-air blend, the premixed flame front does not appear immediately. The appearance of the premixed flame front is described more finely in Fig. 7, which shows the Abel-inverted  $OH^*$  chemiluminescence images of flames between the central equivalence ratio  $\Phi_1 = 3.5$  where the first traces of the premixed flame front appear and  $\Phi_1 = 2$ . The shortening of the diffusion flame front is discussed in the next section. The images in Figs. 6 and 7 show that the premixed flame front appears and gradually strengthens.

### 2.3 Flame length of unconfined coaxial jets

This is due to two factors. The first is that the flame needs to have a sufficient velocity to stabilize at the exit of the central channel. As shown in Fig. 1, the laminar burning velocity (and therefore the turbulent flame speed) increases as  $\Phi_1$  approaches 2, which makes it possible for flames to stabilize. The other factor is that with growing staging, the fraction of fuel being burned in the premixed flame increases as opposed to the diffusion front. Therefore, the production of OH\* radical and the associated emission of light in the premixed flame front increases, making it more visible in OH\* imaging. In addition, turbulence might cause partial extinction of the richer flames, decreasing the light in the premixed flame front. As a result, only when  $\Phi_1$  is below 2.5 is the flame front established at the same level of luminosity as the base of the diffusion flame front. Above this value, the flame front is fainter, and our detection algorithms struggle to separate it from the background, which is why no length measurement for the premixed front is reported in Fig. 7 for  $\Phi_1 > 2.5$ .

Interestingly, the premixed flame front length in this series of flames remains somewhat constant. Assuming a conical Bunsen flame of base diameter  $d_1$  and of height  $L_p$ , we can write the turbulent flame speed:

$$S_T = u_1 \sin(\alpha) \rightarrow L_p \approx u_1 d_1 / (2S_T) \quad (5)$$

where  $\alpha$  is the flame half-angle. As  $\Phi_1$  increases, less air is conveyed in the central channel, and  $u_1$  decreases, which would shorten the premixed flame. However, simultaneously, the turbulent flame speed proportional to the laminar burning velocity also decreases. Both trends compensate and explain the similar premixed flame heights in Fig. 7.

### 2.3 Flame length of unconfined coaxial jets

Flame lengths have been measured for various conditions of central equivalence ratio  $\Phi_1$  and global equivalence ratio  $\Phi_{tot}$ . We report here the total flame length  $L_f$  corresponding to the diffusion flame front. The mass flow of hydrogen is fixed to yield a thermal power  $\mathcal{P}_{th} = 10$  kW, and  $\Phi_1$  is varied between infinity (non-premixed injection) and 2, for  $\Phi_{tot}$  between 0.3 and 0.9 by increasing

### 2.3 Flame length of unconfined coaxial jets

the flow rate of coaxial air. The coaxial jet flames are first observed in quiescent air with the combustion chamber removed to measure longer flames and without waiting for steady wall thermal conditions. With  $\Phi_{tot} < 1$ , the hydrogen combustion is complete with the injected air. To evaluate the potential influence of buoyancy, the dimensionless parameter  $\xi_L$  from Becker and Yamazaki [27] is calculated. Except for the longest non-premixed and  $\Phi_1 = 6$  flames, it is found to lie below 1.5, suggesting a negligible buoyancy effect.

Flame lengths normalized by internal diameter  $d_1$  are plotted in Fig. 8 as a function of the momentum flux ratio  $J^{-1/2}$ , based on the observations of a linear relationship, in some conditions, between  $L_f/d$  and  $J^{-1/2}$  by previous authors for various fuels and oxidizers [28, 25]. Isolines of global equivalence ratio  $\Phi_{tot}$  are drawn between points of the same internal diameter  $d_1$ . Following these isolines, it appears that under the same global equivalence ratio, premixing fuel and air first increases the length of the flame between  $\Phi_1 = \infty$  and  $\Phi_1 = 6$ . Then, between  $\Phi_1 = 6$  and  $\Phi_1 = 2$ , the flame length progressively becomes shorter. This evolution of flame length can be explained qualitatively by the evolution of both momentum flux ratio and stoichiometric mixture fraction  $Z_{st}$ , as described in [28]. As the staging is increased (reduction of  $\Phi_1$ ), the flow rate in the annular channel decreases while the flow rate in the central channel increases, which decreases the momentum flux ratio  $J$ , leading to less efficient mixing of air and fuel and, initially, a longer flame. However, as more air is mixed with the fuel in the central channel,  $Z_{st}$  drops as the amount of air from the coaxial flow needed to complete combustion decreases significantly, leading to a progressive shortening of the flame.

We can also observe the behavior of normalized flame length for a fixed value of  $\Phi_1$ . For each series of  $\Phi_1$  in fixed color,  $Z_{st}$  is constant, and the trend is only affected by the momentum flux ratio  $J$ . For  $J > 1$  in the leftmost part of Fig. 6, the annular flow momentum flux dominates, and a linear variation of the flame length is observed across all values of premixing equivalence ratio  $\Phi_1$ . This is consistent with coaxial jet scaling laws developed by [28]. The slope decreases with  $\Phi_1$ , with the non-premixed case  $\Phi_1 = \infty$  showing the steepest slope. Following [28], this change in slope is attributed to the variation in the stoichiometric mixture fraction  $Z_{st}$ , also yielding a variable effect of burnt gases in the jet development. However, applying those corrections in the present



case does not produce a good overlapping for all the data, suggesting that another mechanism could also participate. This linear trend for  $L_f$  remains up to  $J^{-1/2} \approx 2$ .

In contrast, for  $J \ll 1$  in the rightmost part of Fig. 6, the coaxial component becomes negligible, and a constant jet length is clearly retrieved for the lowest values of  $\Phi_1$ . For  $\Phi_1 = 4$ , the slope diminishes greatly, starting to resemble a plateau but still not flat, indicating that this might still be a transition zone. In the zone  $J^{-1/2} > 2$ , we observe this transition where the initial slopes progressively vary with  $J^{-1/2}$  and  $\Phi_{tot}$  getting larger.

From this, we can establish that varying staging heavily influences the flame's length. Based on previous literature, it is expected that  $\text{NO}_x$  emission also varies substantially. In addition, we see the most significant change in flame length occurs between  $\Phi_1 = 4$  and  $\Phi_1 = 2$ , which also coincides with the appearance of the premixed flame front. As a result, the evolution of  $\text{NO}_x$  emissions is studied more finely between these two limits in the next section.

### 3 $\text{NO}_x$ EMISSIONS: IMPACT OF STAGING AT $\mathcal{P}_{th} = 10$ KW.

#### 3.1 Measurements

To investigate the role of staging on  $\text{NO}_x$  emissions in coaxial flames, a series of flame with constant parameters  $\Phi_{tot} = 0.4$ ,  $d_1 = 6$  mm and  $\mathcal{P}_{th} = 10$  kW is produced on the injector. The central equivalence ratio ranges from  $\Phi_1 = \infty$  to  $\Phi_1 = 2$ , with an increased number of flames between  $\Phi_1 = 3.5$  and  $\Phi_1 = 2$  to allow for a finer observation as the secondary flame front appears. In addition, two flames with  $d_1 = 10$  mm at respectively  $\Phi_1 = 6$  and  $\Phi_1 = 2$  are produced to observe the effect of diameter under weak and strong staging conditions. In these experiments, the flames are confined in the combustion chamber described in Sec. 1.1. Both flame length and  $\text{NO}_x$  emissions are measured at every experimental point since emissions from hydrogen diffusion flames depend strongly on flame length and volume. As highlighted in Fig. 4, the flames in this work can display a secondary premixed flame front that could show a different behavior of  $\text{NO}_x$  production to the diffusion flame front. However, given the small  $\text{NO}_x$  production of premixed flames above an equivalence ratio of 2 in Fig. 1, we will neglect the rich premixed flame's contribution in our interpretation.

### 3.1 Measurements

Figure 9 shows the evolution of the  $\text{NO}_x$  emission index as a function of the inverse of the internal equivalence ratio  $\Phi_1$ . This quantity is proportional to the mass flow rate of air mixed with the fuel and represents the amount of air staging set in the injection. It shows that for  $d_1 = 6$  mm, the staging effect is quite gradual. From  $\Phi_1 = \infty$  to  $\Phi_1 = 3.5$ , the  $\text{EINO}_x$  decreases by about 0.1. From  $\Phi_1 = 3.5$  to  $\Phi_1 = 2$ , the  $\text{EINO}_x$  decreases by 0.5 while roughly the same amount of air has been added to the fuel. Low staging degrees are then not beneficial to reduce  $\text{NO}_x$  emissions. Two flames at  $d_1 = 10$  mm are also plotted in this graph, for  $\Phi_1 = 6$  and  $\Phi_1 = 2$ . No flame at  $\Phi_1 = \infty$  is represented due to the blowout encountered at that point. These two flames show that, for this larger diameter, staging still decreases  $\text{NO}_x$  emissions. The decrease observed between the two flames at  $\Phi_1 = 2$  and 6 is lower than that observed for flames with smaller internal diameters. Nevertheless, the overall level of  $\text{NO}_x$  emissions remains below that of the flames at  $d_1 = 6$  mm.

To understand more about the observed trends, Fig. 10 shows the plot of  $\text{EINO}_x$  as a function of the measured flame length  $L_f$  from this series of flames. We observe that the  $\text{NO}_x$  emission index overall increases with flame length. The only outlier in this trend is the point with  $\Phi_1 = \infty$  (non-premixed flame) that emits more  $\text{NO}_x$  than longer rich-premixed flames. At first, introducing a rich mixture instead of pure fuel appears to increase the flame length significantly, while at the same time,  $\text{NO}_x$  emissions remain quasi-constant. Increasing the proportion of air in the central channel further shortens the flame and reduces the  $\text{EINO}_x$  by about 20% compared to the non-premixed case. The additional points with  $d_1 = 10$  mm show a reduced flame length  $L_f$ . This is consistent with the observations on unconfined flames in Fig. 8 and explained by the lower  $J^{-1/2}$  with a larger diameter  $d_1$ . These shorter flame lengths yield the lower  $\text{NO}_x$  emission levels observed for flames with  $d_1 = 10$  mm in Fig. 9. Like the slighter decrease in  $\text{EINO}_x$  for  $d_1 = 10$  mm than for  $d_1 = 6$  mm in Fig. 9, the reduction in flame length between  $\Phi_1 = 6$  and  $\Phi_1 = 2$  when  $d_1 = 10$  mm is smaller. The flame length decreases by only around 1 cm with the larger diameter as opposed to 6 cm with the smaller diameter. The overall good correlation between  $\text{EINO}_x$  and the flame lengths confirms the necessity to quickly mix the secondary air with the premixed flame's rich products to attenuate  $\text{NO}_x$  emission as in RQL systems.

One may also note that the decrease in flame length with increased staging is more significant

### 3.2 Models

at lower values of  $\Phi_1$ , as  $L_f$  declines by around 2 cm between  $\Phi_1 = 2.25$  and  $\Phi_1 = 2$  while between  $\Phi_1 = 6$  and  $\Phi_1 = 3.5$ , this decrease is around 0.5 cm. This suggests that there could be a change in the mixing mechanism of the flame in this region. Figures 6 and 7 have highlighted the appearance and strengthening of the premixed flame front at the exit of the central channel. The nature of the diffusion flame front changes with air staging. In the non-premixed case, the flame front occurs between pure hydrogen and coaxial air. In the weakly staged case, the fuel side now includes oxygen due to the premixing, and the fresh gases' composition fed in the diffusion flame gradually changes as air staging is added. In the heavily staged case, the diffusion flame is supplied with a hot mixture of burned gases composed of hydrogen, water vapor, and nitrogen due to the premixed flame front. In this case, the new mixture condition can affect the mixing rules and could explain the observed further reduction in diffusion flame length for  $\Phi_1 = 2$ .

In addition, the behavior of this type of flame under conditions of increased pressure and temperature like that of a gas turbine combustion chamber would be interesting to investigate. These conditions increase the laminar flame speed and might shift the operability range of the injector to higher equivalence ratios to avoid flashback. Furthermore, it can be expected that the amount of  $\text{NO}_x$  produced by the non-premixed flame front increases due to the higher availability of oxygen radicals [29]. In contrast, if the main  $\text{NO}_x$  pathway in the premixed flame is the nitrous oxide pathway, its  $\text{NO}_x$  emissions should be pressure independent.

### 3.2 Models

As mentioned in the introduction, extensive work was done to model the evolution of the  $\text{NO}_x$  emission index of non-premixed hydrogen flames with various parameters. As a result, we evaluate the performance of these models on our flames.

We can first observe the behavior of  $\text{NO}_x$  emission under the first scaling developed by Peters and Donnerhack [16] for jet flames. Figure 11 shows the evolution of  $\text{EINO}_x$  with the residence time  $L^3_f/(d_1^2 u_1)$ . Hence, the jet velocity also affects the  $\text{NO}_x$  emission in addition to the flame length effect discussed above. The  $\text{EINO}_x$  profile is approaching proportionality with  $L^3_f/(d_1^2 u_1)$  across the various degrees of staging and the two diameters tested. The previous outlying point  $\Phi_1 = \infty$  in Fig. 10 is now closer to the trend followed by the other conditions. As the abscissa takes

into account the change in bulk velocity and diameter, the points with  $d_1 = 10$  mm follow closely the trend of the points with  $d_1 = 6$  mm compared to the previous figures. While the agreement is satisfactory, the works in [17, 19] have outlined the need to account for a Damköhler effect (usually considered as a global aerodynamic strain rate), which could explain the dropping slope of the trend depicted in Fig. 11. Additionally, this scaling does not account for coflowing air or premixing.

Figure 12 reproduces the same scaling as the one in [20] where the global strain rate  $(u_1 - u_2)/d_1$  in abscissa makes use of the presence of non-negligible coflow. The ordinate considers  $EINO_x/(V_f/(d_1^2 u_1))$  to remove the linear trend component due to the residence time. The volume of the flame  $V_f$  is calculated using the Otsu contour for the diffusion flame, with the assumption that the flames are axisymmetric. As the coflow velocity for the flame at  $d_1 = 10$  mm and  $\Phi_1 = 6$  is higher than the fuel flow velocity, the absolute value  $|u_1 - u_2|/d_1$  is retained. Two regimes can be distinguished in Fig. 12. First, at high strain, the scaled  $EINO_x$  appears to increase with strain. This is due to the increase in super-equilibrium radicals as described in [17]. Moreover, the data fit relatively well with the half power law found in [17, 19, 20]. However, an opposite trend is observed at lower strain, with the emission decreasing with strain. A similar effect is observed in [21]. However, the change of trend also coincides with the change of diameter, which may mean that the internal diameter is responsible for these effects. More data is needed to discriminate between these two possibilities. However, it shows these simple models are insufficient to describe entirely the present flames and other effects must be taken into account.

#### 4 ADDITIONAL PARAMETER VARIATIONS

Two additional parameters are studied: the flame thermal power and global equivalence ratio. Under constant  $\Phi_{tot}$  and  $\Phi_1$ , changing  $\mathcal{P}_{th}$  scales all the flow rates by a common factor, thereby adjusting the velocities by a common factor. This, in turn, affects the residence time of the gases in the flame, which is expected to affect  $NO_x$  emissions. In addition, variations in the bulk velocity in the central channel also affect the stabilization of the premixed flame front. On the other hand, the global equivalence ratio  $\Phi_{tot}$  is varied with  $\mathcal{P}_{th}$  and  $\Phi_1$  constant, which corresponds to changing

## 4.1 Thermal Power

the flow rate in the coflow.

### 4.1 Thermal Power

A study of thermal power is undertaken by examining two sets of three flames. In both sets, the global equivalence ratio  $\Phi_{tot}$  is 0.4 while the central equivalence ratio  $\Phi_1$  is chosen to be equal to 2 to have a premixed flame front present to observe the effects of variations of bulk velocity in the central channel. The two sets have different internal diameters  $d_1$ :  $d_1 = 6$  mm and  $d_1 = 10$  mm. Each set corresponds to a fixed momentum flux ratio.  $J = 0.09$  for the first set and 1.25 for the second.

The Abel-inverted OH\* chemiluminescence images from the first set are shown in Fig. 13. With increasing thermal power, the length of the premixed flame becomes higher in absolute value and when compared to the diffusion flame's height. Between the lowest and highest thermal power flame, there is a threefold increase in bulk velocity while a twofold rise in premixed flame length is measured. As momentum flux ratio  $J$  and equivalence ratios are fixed while varying the power, the diffusion flame length sees comparably smaller variations that may be linked to the changes in premixed flame length. Corresponding NO<sub>x</sub> emissions are reported in Table 2. EINO<sub>x</sub> decreases as thermal power increases with a stark decrease in NO<sub>x</sub> between 5 kW and 10 kW. Since the flame length only marginally changes, the residence time follows a threefold reduction with the increasing bulk velocity and explains mainly the drop in NO<sub>x</sub> at larger power. When analyzing the emissions scaled by the residence time in the flame (third row in Tab. 2), they increase with the velocity difference  $|u_1 - u_2|$  (sixth row), which is coherent with the observations in Fig. 12. Indeed, with higher thermal power, the strain increases, causing an increase in super-equilibrium radicals. This contribution slows down the EINO<sub>x</sub> decrease at 15 kW.

The results for the second set of flames with  $d_1 = 10$  mm are shown in Table 3. In this set of flame shown in Fig. 14, the difference in velocity between the internal channel and the coflow is smaller (around 7 m·s<sup>-1</sup> for the highest power case against 90 m·s<sup>-1</sup>) due to the increased internal diameter. The flames display a much shorter premixed flame front (see Fig. 14) across all values of thermal power due to the reduced bulk velocity with the larger diameter. One might note that the lowest thermal power used with this diameter is 6.5 kW instead of 5 kW. This is because the low

## 4.2 Global Equivalence Ratio

velocities at 5 kW caused a flashback of the flame. Other than this, flashback was not encountered when operating the burner. The same trend as the first set is observed with  $EINO_x$  decreasing with thermal power. However, it can be noticed that the flame gets here slightly longer with larger thermal power. The trend in scaled  $EINO_x$  is similar to the first set for  $d_1 = 6$  mm, as it increases with  $\mathcal{P}_{th}$ . The observations of Fig. 12 were in opposition where strain appeared to reduce the scaled  $EINO_x$  at lower strain rates. This suggests that there might be another mechanism involved when the change in lower strain rates is induced by  $\Phi_1$ .

## 4.2 Global Equivalence Ratio

The other study concerning the global equivalence ratio is undertaken by studying flames at  $\Phi_{tot} = 0.35$  and  $\Phi_{tot} = 0.45$  with a constant flame thermal power  $\mathcal{P}_{th} = 10$  kW for  $\Phi_1 = \infty$ , 6 and 2 when  $d_1 = 6$  mm, and  $\Phi_1 = 6$  and 2 when  $d_1 = 10$  mm (due to blow-off occurring when  $\Phi_1 = \infty$ ). To verify if the behavior observed in Fig. 11 is affected by coaxial air flow, these additional flames are plotted on the same  $EINO_x - L^3_f/(d_1^2 u_1)$  graph in Fig. 15. Furthermore, as  $d_1$  and  $u_1$  stay constant under variations of  $\Phi_{tot}$ , it allows us to observe the qualitative evolution of  $L_f$  with coaxial air. In all cases, increasing the global equivalence ratio increases flame length, consistently with the discussion of Fig. 8. This effect is the strongest for the non-premixed flames. Likewise, increasing the global equivalence ratio in all flames also increases the measured  $EINO_x$ . This follows the increase in residence time induced by the added flame length for fixed  $u_1$  but the trend of the added points step aside from the previous data series for  $\Phi_{tot} = 0.4$ . Clearly, the residence time alone cannot overlap the data. Updating Fig. 12, which already experienced some limitations at low strain rates, was inconclusive.

To better fit the trends observed for these flames, we turn to the work of Weiland et al. [21], who, with some intuition, devised a model that accurately represented nitrogen-diluted hydrogen flames with coaxial air. Using the far-field scaling law of Dahm and Mayman [18] for jet length as a function of fuel channel diameter, a scaling law of  $EINO_x/(V_f/(u_1 d_1^2))$  as a function of  $\frac{u_1}{d_1} \frac{d_1^*}{L_f} \approx \frac{u_1}{d_1} Z_{st} \left(1 + \frac{\dot{m}_2 u_2}{\dot{m}_1 u_1}\right)^{1/2}$  was devised. Where  $d_1^* = d_1 \sqrt{\frac{\rho_1}{\rho_2}}$ ,  $Z_{st}$  is the stoichiometric mixture fraction of the fuel mixture and  $\dot{m}_i u_i$  is the integrated momentum flux of a given channel. In our flames,  $Z_{st}$  ranges from 0.028 in the non-premixed case to 0.51 when  $\Phi_1=2$ . For nitrogen-diluted

hydrogen flames with coaxial air, this scaling was found to give a 1/2 power law, as observed previously in simple jet flame [17]. This suggests that it captures the effect of Damköhler number in that work. This scaling law is applied to all the studied flame conditions. Results are shown in Fig. 16. All the points do not lie on a single curve. Two groups of points, however, present a linear pattern of about the same slope in the logarithmic plot. The first group of points contains the non-premixed flames and the flames with  $d_1 = 10$  mm, while the other one includes the premixed flame with  $d_1 = 6$  mm. The main difference between these two groups is the momentum flux ratio:  $J > 1$  in the first group, while the other corresponds to flames with  $J < 1$ . This separation between the two sets of points suggests that the far-field scaling law from [18] may not be as good in both regimes. The data collapse is improved in Fig. 17 in which  $\frac{d_1^*}{L_f}$  is directly used instead. In this figure, all the points follow the same trend along a 1/2 power law. The result makes us believe that  $\frac{u_1}{L_f}$  can better represent the mixing time scale to account for the Damköhler-number effects. The reason why remains, however, still elusive as in [21] and will require further analysis.

## 5 CONCLUSION

Staged flames with rich premixing and coflowing air have been stabilized on a coaxial injector. The effect of air staging between the two channels on flame length, topology, and  $\text{NO}_x$  emission has been studied. It was first seen that introducing a rich mixture of fuel and air could reduce the total flame length compared to a non-premixed injection thanks to the modified stoichiometric mixture fraction between both streams. In addition, when nearing a central equivalence ratio of  $\Phi_1=2$ , a premixed flame front is stabilized and further reduces flame length. Air staging in the coaxial injector also reduces the  $\text{NO}_x$  emission index, primarily through its reduction of the diffusion flame height that is responsible for  $\text{NO}_x$  production. The variations in flame thermal power have shown that the flame global strain rate also significantly affect the  $\text{NO}_x$  emission. Models used in non-premixed flames have been tested and have shown to be ill-adapted to the studied flames. The model by [21] was applied and appeared to yield promising results for studied flames. Future work will include additional diagnostics on the instantaneous velocity field and the presence and position of flame fronts. This detailed knowledge will nourish modeling efforts of flame lengths and

## REFERENCES

emissions.

Further reduction in NO<sub>x</sub> emissions should also be sought. Increasing the internal diameter, in particular, would allow bulk velocities and momentum flux ratios that could not be reached with the existing design. Mixing enhancement with swirl to promote shorter flames is also envisioned.

In addition, study of the injector under high-pressure, as well as burning blends of hydrogen and natural gas would allow to evaluate practical applications of the injection method.

## ACKNOWLEDGEMENTS

This work was funded by CIFRE contract 2021/1697 from Agence Nationale de la Recherche et de la Technologie and Safran Tech.

## REFERENCES

- [1] Brewster, B. S., Cannon, S. M., Farmer, J. R. and Meng, F. "Modeling of lean premixed combustion in stationary gas turbines." *Progress in Energy and Combustion Science* Vol. 25 No. 4 (1999): pp. 353–385. 10.1016/S0360-1285(98)00014-8.
- [2] Ren, X., Xue, X., Brady, K. B., Sung, C.-J. and Mongia, H. C. "Fundamental investigations for lowering emissions and improving operability." *Propulsion and Power Research* Vol. 7 No. 3 (2018): pp. 197–204. 10.1016/j.jprr.2018.07.001.
- [3] Feitelberg, A. S. and Lacey, M. A. "The GE Rich-Quench-Lean Gas Turbine Combustor." *Journal of Engineering for Gas Turbines and Power* Vol. 120 No. 3 (1998): pp. 502–508. 10.1115/1.2818173.
- [4] Marek, C., Smith, T. and Kundu, K. "Low Emission Hydrogen Combustors for Gas Turbines Using Lean Direct Injection." *41st AIAA/ASME/SAE/ASEE Joint Propulsion Conference & Exhibit*. Joint Propulsion Conferences. American Institute of Aeronautics and Astronautics (2005). 10.2514/6.2005-3776.
- [5] Reichel, T. G., Goeckeler, K. and Paschereit, O. "Investigation of Lean Premixed Swirl-Stabilized Hydrogen Burner With Axial Air Injection Using OH-PLIF Imaging." *Journal of Engineering for Gas Turbines and Power* Vol. 137 No. 11 (2015): p. 111513. 10.1115/1.4031181.



## REFERENCES

- [6] Lee, T. and Kim, K. T. “Combustion dynamics of lean fully-premixed hydrogen-air flames in a mesoscale multinozzle array.” *Combustion and Flame* Vol. 218 (2020): pp. 234–246. 10.1016/j.combustflame.2020.04.024.
- [7] Funke, H. H.-W., Beckmann, N., Keinz, J. and Horikawa, A. “30 Years of Dry-Low-NO<sub>x</sub> Micromix Combustor Research for Hydrogen-Rich Fuels—An Overview of Past and Present Activities.” *Journal of Engineering for Gas Turbines and Power* Vol. 143 No. 7 (2021): p. 071002. 10.1115/1.4049764.
- [8] Agarwal, P., Sun, X., Gauthier, P. Q. and Sethi, V. “Injector Design Space Exploration for an Ultra-Low NO<sub>x</sub> Hydrogen Micromix Combustion System.” *Proceedings of the ASME Turbo Expo 2019: Turbomachinery Technical Conference and Exposition*. V003T03A013. Phoenix, AZ, June 17-21, 2019. 10.1115/GT2019-90833.
- [9] Ranzi, E., Frassoldati, A., Grana, R., Cuoci, A., Faravelli, T., Kelley, A. P. and Law, C. K. “Hierarchical and comparative kinetic modeling of laminar flame speeds of hydrocarbon and oxygenated fuels.” *Progress in Energy and Combustion Science* Vol. 38 No. 4 (2012): pp. 468–501. 10.1016/j.pecs.2012.03.004.
- [10] Song, Y., Marrodán, L., Vin, N., Herbinet, O., Assaf, E., Fittschen, C., Stagni, A., Faravelli, T., Alzueta, M. U. and Battin-Leclerc, F. “The sensitizing effects of NO<sub>2</sub> and NO on methane low temperature oxidation in a jet stirred reactor.” *Proceedings of the Combustion Institute* Vol. 37 No. 1 (2019): pp. 667–675. 10.1016/j.proci.2018.06.115.
- [11] Shudo, T., Omori, K. and Hiyama, O. “NO<sub>x</sub> reduction and NO<sub>2</sub> emission characteristics in rich-lean combustion of hydrogen.” *International Journal of Hydrogen Energy* Vol. 33 No. 17 (2008): pp. 4689–4693. 10.1016/j.ijhydene.2008.05.034.
- [12] Göke, S., Füre, M., Bourque, G., Bobusch, B., Göckeler, K., Krüger, O., Schimek, S., Terhaar, S. and Paschereit, C. O. “Influence of steam dilution on the combustion of natural gas and hydrogen in premixed and rich-quench-lean combustors.” *Fuel Processing Technology* Vol. 107 (2013): pp. 14–22. 10.1016/j.fuproc.2012.06.019.
- [13] Marragou, S., Magnes, H., Poinot, T., Selle, L. and Schuller, T. “Stabilization regimes and pollutant emissions from a dual fuel CH<sub>4</sub>/H<sub>2</sub> and dual swirl low NO<sub>x</sub> burner.” *International*

## REFERENCES

- Journal of Hydrogen Energy* Vol. 47 No. 44 (2022): pp. 19275–19288. 10.1016/j.ijhydene.2022.04.033.
- [14] Leroy, M., Mirat, C., Renaud, A. and Vicquelin, R. “Stabilization of Low-NO<sub>x</sub> Hydrogen Flames on a Dual-Swirl Coaxial Injector.” *Journal of Engineering for Gas Turbines and Power* Vol. 145 No. 2 (2022): p. 021021. 10.1115/1.4055711.
- [15] Minakawa, K. and Yuasa, S. “Study on Hydrogen Combustors with Two-Stage Combustion Method for Micro Gas Turbines.” *Journal of Environment and Engineering* Vol. 2 No. 3 (2007): pp. 590–600. 10.1299/jee.2.590.
- [16] Peters, N. and Donnerhack, S. “Structure and similarity of nitric oxide production in turbulent diffusion flames.” *Symposium (International) on Combustion* Vol. 18 No. 1 (1981): pp. 33–42. 10.1016/S0082-0784(81)80008-2.
- [17] Chen, R.-H. and Driscoll, J. F. “Nitric oxide levels of jet diffusion flames: Effects of coaxial air and other mixing parameters.” *Symposium (International) on Combustion* Vol. 23 No. 1 (1991): pp. 281–288. 10.1016/S0082-0784(06)80271-7.
- [18] Dahm, W. J. A. and Mayman, A. G. “Blowout limits of turbulent jet diffusion flames for arbitrary source conditions.” *AIAA Journal* Vol. 28 No. 7 (1990): pp. 1157–1162. 10.2514/3.25186.
- [19] Driscoll, J. F., Chen, R.-H. and Yoon, Y. “Nitric oxide levels of turbulent jet diffusion flames: Effects of residence time and Damkohler number.” *Combustion and Flame* Vol. 88 No. 1 (1992): pp. 37–49. 10.1016/0010-2180(92)90005-A.
- [20] Chen, J. Y. and Kollmann, W. “PDF modeling and analysis of thermal NO formation in turbulent nonpremixed hydrogen-air jet flames.” *Combustion and Flame* Vol. 88 No. 3 (1992): pp. 397–412. 10.1016/0010-2180(92)90042-N.
- [21] Weiland, N., Chen, R. H. and Strakey, P. “Effects of coaxial air on nitrogen-diluted hydrogen jet diffusion flame length and NO<sub>x</sub> emission.” *Proceedings of the Combustion Institute* Vol. 33 No. 2 (2011): pp. 2983–2989. 10.1016/j.proci.2010.06.075.
- [22] Jourdaine, P., Mirat, C., Caudal, J. and Schuller, T. “Stabilization Mechanisms of Swirling Premixed Flames With an Axial-Plus-Tangential Swirler.” *Journal of Engineering for Gas Turbines and Power* Vol. 140 No. 8 (2018): p. 081502. 10.1115/1.4038617.

## REFERENCES

- [23] Degenève, A., Jourdaine, P., Mirat, C., Caudal, J., Vicquelin, R. and Schuller, T. "Effects of a Diverging Cup on Swirl Number, Flow Pattern, and Topology of Premixed Flames." *Journal of Engineering for Gas Turbines and Power* Vol. 141 No. 3 (2018): p. 031022. 10.1115/1.4041518.
- [24] Schefer, R. W., Kulatilaka, W. D., Patterson, B. D. and Settersten, T. B. "Visible emission of hydrogen flames." *Combustion and Flame* Vol. 156 No. 6 (2009): pp. 1234–1241. 10.1016/j.combustflame.2009.01.011.
- [25] Degenève, A., Vicquelin, R., Mirat, C., Labégorre, B., Jourdaine, P., Caudal, J. and Schuller, T. "Scaling relations for the length of coaxial oxy-flames with and without swirl." *Proceedings of the Combustion Institute* Vol. 37 No. 4 (2019): pp. 4563–4570. 10.1016/j.proci.2018.06.032.
- [26] Otsu, N. "A Threshold Selection Method from Gray-Level Histograms." *IEEE Transactions on Systems, Man, and Cybernetics* Vol. 9 No. 1 (1979): pp. 62–66. 10.1109/TSMC.1979.4310076.
- [27] Becker, H. A. and Yamazaki, S. "Entrainment, momentum flux and temperature in vertical free turbulent diffusion flames." *Combustion and Flame* Vol. 33 (1978): pp. 123–149. 10.1016/0010-2180(78)90055-X.
- [28] Schumaker, S. A. and Driscoll, J. F. "Coaxial turbulent jet flames: Scaling relations for measured stoichiometric mixing lengths." *Proceedings of the Combustion Institute* Vol. 32 No. 2 (2009): pp. 1655–1662. 10.1016/j.proci.2008.06.051.
- [29] Correa, S. M. "A Review of NO<sub>x</sub> Formation Under Gas-Turbine Combustion Conditions." *Combustion Science and Technology* Vol. 87 No. 1-6 (1993): pp. 329–362. 10.1080/00102209208947221.

## REFERENCES

Table 1. PARAMETER VALUES FOR FLAMES IN FIG. 10

Parameter	Range	Parameter	Range
J [-]	0.09-7.2	$\Phi_{tot}$ [-]	0.4
$Re_1$ [-]	2000 - 18000	$u_1$ [ $m \cdot s^{-1}$ ]	18.2-73.9
$Re_2$ [-]	12500-17000	$u_2$ [ $m \cdot s^{-1}$ ]	18.3-28.2
$\mathcal{P}_{th}$ [kW]	10	Pressure [atm]	1

Table 2. NO<sub>x</sub> EMISSIONS AT VARIOUS THERMAL POWERS FOR  $\Phi_1=2$ ,  $\Phi_{tot}=0.4$ ,  $d_1=6$  mm.

$\mathcal{P}_{th}$ [kW]	5	10	15
$EINO_x$ [g/kgH <sub>2</sub> ]	4.47	2.52	1.99
$EINO_x/(V_f/(u_1d_1^2))$ [g/(kgH <sub>2</sub> ·s)]	161	178	236
$L_f$ [cm]	20.4	18.4	18.8
$L_p$ [cm]	3.4	5.4	7.7
$ u_1 - u_2 $ [ $m \cdot s^{-1}$ ]	30.0	60.3	90.0

Table 3. NO<sub>x</sub> EMISSIONS AT VARIOUS THERMAL POWERS FOR  $\Phi_1=2$ ,  $\Phi_{tot}=0.4$ ,  $d_1=10$  mm.

$\mathcal{P}_{th}$ [kW]	6.5	10	15
$EINO_x$ [g/kgH <sub>2</sub> ]	2.78	2.32	1.96
$EINO_x/(V_f/(u_1d_1^2))$ [g/(kgH <sub>2</sub> ·s)]	122	135	167
$L_f$ [cm]	13.3	14.4	14.9
$L_p$ [cm]	1.9	2.5	3.1
$ u_1 - u_2 $ [ $m \cdot s^{-1}$ ]	2.6	4.3	6.6

## REFERENCES

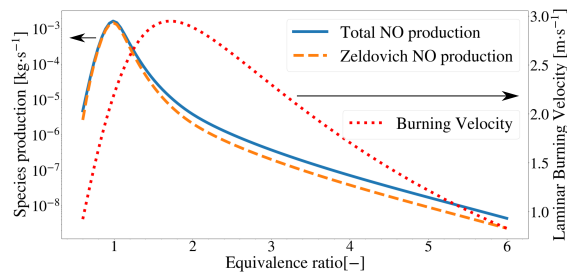


Fig. 1.  $\text{NO}_x$  production (premixed flames with a 15-cm long domain) and laminar burning velocity of hydrogen-air flames as a function of equivalence ratio under standard pressure and temperature.

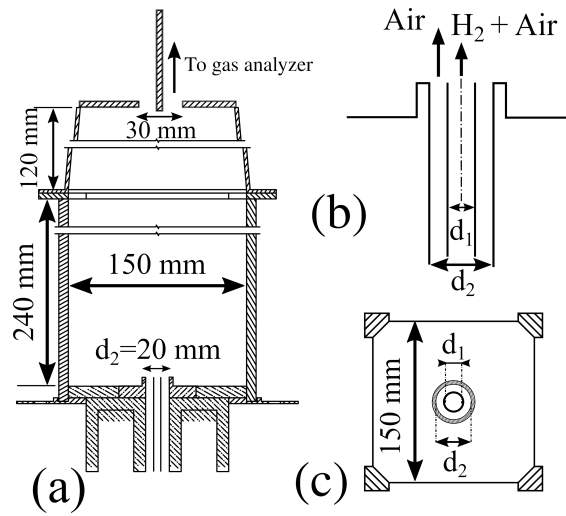


Fig. 2. (a) Sketch of the square-section combustion chamber. (b) Sketch of the coaxial injector. (c) Sketch of the injector exit plane.

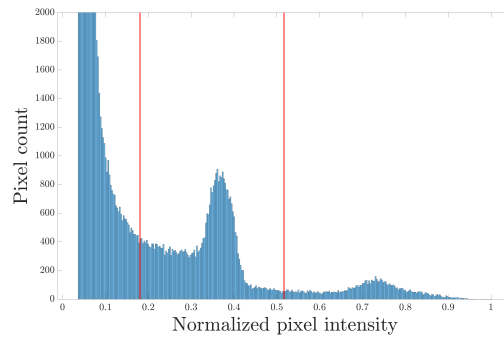


Fig. 3. Histograms of an  $\text{OH}^*$  image.  $\Phi_{tot} = 0.4$ ,  $\Phi_1 = 2$ ,  $\mathcal{P}_{th} = 15$  kW.

REFERENCES

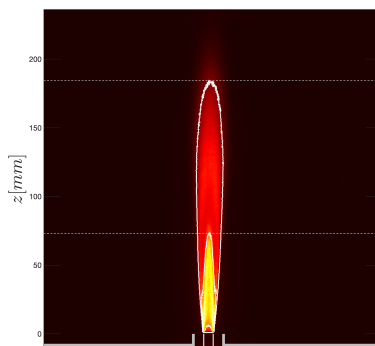


Fig. 4. Flame OH\* image (line-of-sight integrated) for  $\Phi_{tot} = 0.4$ ,  $\Phi_1 = 2$ ,  $\mathcal{P}_{th} = 15$  kW, associated Otsu contours (plain white lines) and corresponding flame lengths (horizontal dashed lines).

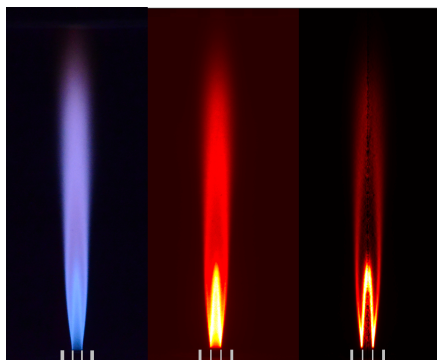


Fig. 5. Dual flame fronts visualization. From left to right: Visible range, OH\* chemiluminescence, Abel-inverted OH\* chemiluminescence.  $\Phi_{tot} = 0.35$ ,  $\Phi_1 = 2$ ,  $\mathcal{P}_{th} = 10$  kW.

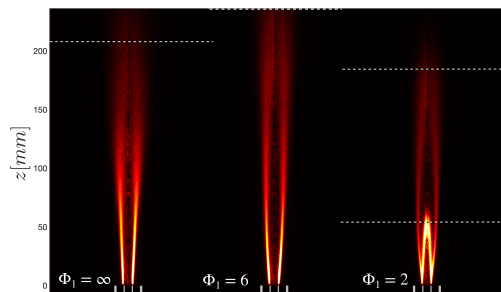


Fig. 6. Abel-inverted OH\* chemiluminescence images under varying degrees of air staging.  $\Phi_{tot} = 0.4$ ,  $\mathcal{P}_{th} = 10$  kW.

REFERENCES

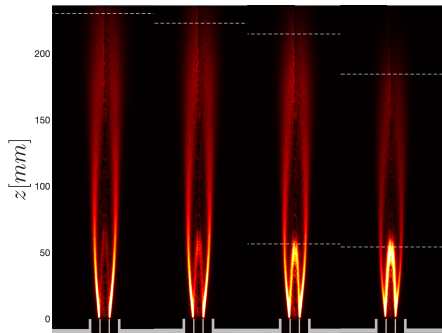


Fig. 7. Abel-inverted OH\* chemiluminescence images at various values of  $\Phi_1$  with  $d_1 = 6$  mm,  $\Phi_{tot}=0.4$  and  $\mathcal{P}_{th}=10$  kW. From left to right:  $\Phi_1=3.5, 3, 2.5, 2$ .

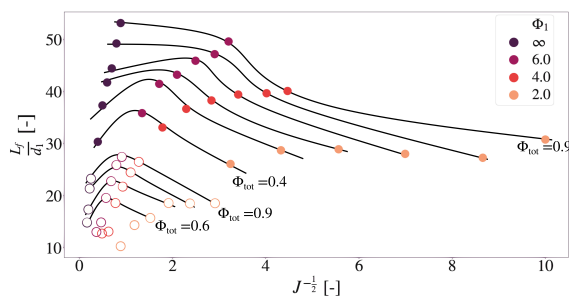


Fig. 8. Flame length  $L_f$  as a function of  $J^{-\frac{1}{2}}$  for different air staging conditions indicated by  $\Phi_1$ : closed symbols  $\bullet$ :  $d_1 = 6$  mm; open symbols  $\circ$ :  $d_1 = 10$  mm.

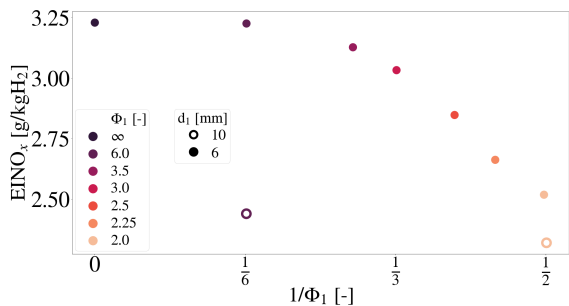


Fig. 9.  $EINO_x$  for various air staging conditions with  $d_1 = 6$  mm,  $\Phi_{tot}=0.4$  and  $\mathcal{P}_{th}=10$  kW.

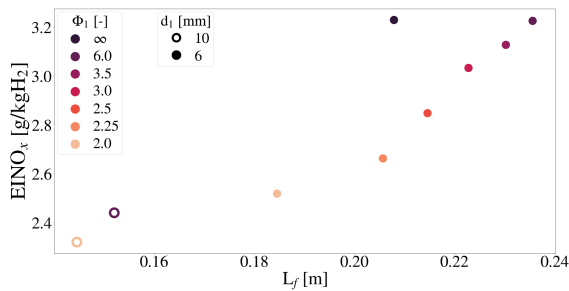


Fig. 10.  $EINO_x$  as a function of flame length for the same operating conditions as in Fig. 9.

REFERENCES

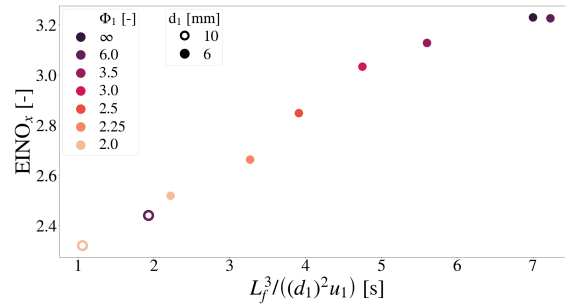


Fig. 11. EINO<sub>x</sub> as a function of residence time  $L_f^3 / (d_1^2 u_1)$ .  $\Phi_{tot}=0.4$ ,  $\mathcal{P}_{th} = 10$  kW.

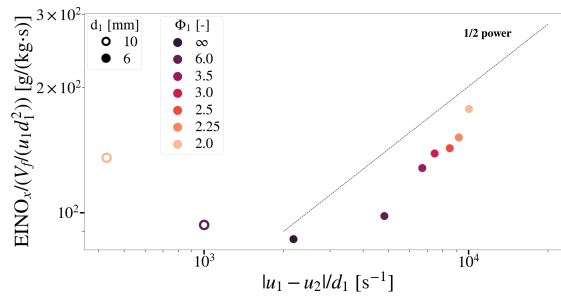


Fig. 12. Scaled EINO<sub>x</sub> following [20] as a function of the coaxial global strain rate in a logarithmic plot.  $\Phi_{tot}=0.4$  and  $\mathcal{P}_{th} = 10$  kW.

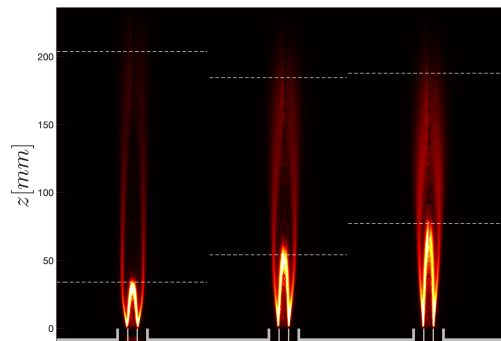


Fig. 13. Abel-inverted OH\* chemiluminescence images at various thermal powers for  $d_1 = 6$  mm,  $\Phi_1=2$ ,  $\Phi_{tot}=0.4$ . From left to right:  $\mathcal{P}_{th}=5, 10, 15$  kW.



REFERENCES

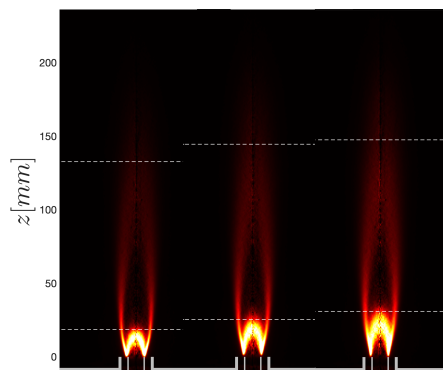


Fig. 14. Abel-inverted OH\* chemiluminescence images at various thermal powers for  $d_1 = 10$  mm.  $\Phi_1=2$ ,  $\Phi_{tot}=0.4$ ,  $\mathcal{P}_{th}=6.5, 10, 15$  kW.

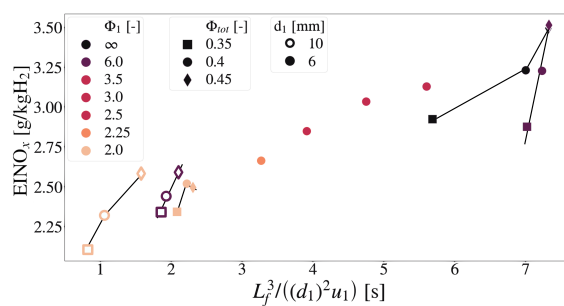


Fig. 15.  $EINO_x$  as a function of flame residence time  $L_f^3 / (d_1^2 u_1)$  for different values of  $\Phi_{tot}$ .

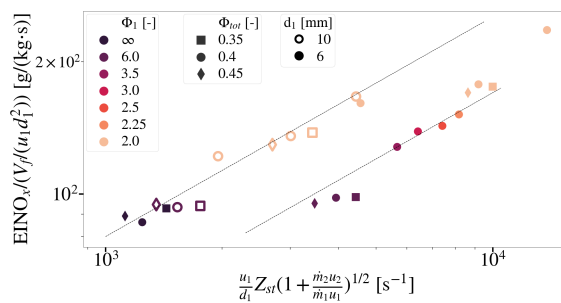


Fig. 16. Scaled  $EINO_x$  as a function of a modified strain rate  $u_1 / d_1$  (see corrective factor in axis title) plotted in a logarithmic plot.

REFERENCES

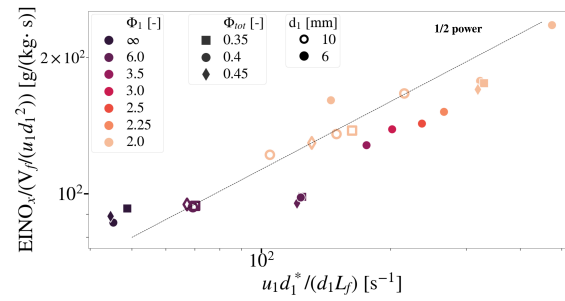


Fig. 17. Scaled  $EINO_x$  as a function of modified strain rate  $\frac{u_1}{d_1} \frac{d_1^*}{L_f}$  plotted in a logarithmic plot.