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Parameter Estimation with Enhanced Performance

Stanislav Aranovskiy

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Mémoire
présenté pour obtenir le diplôme
d'Habilitation à diriger des recherches
(Génie informatique, automatique et traitement du signal)

Parameter Estimation with Enhanced Performance

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Summary

I got my Ph.D. degree in Systems Analysis and Control from the ITMO University (Russia) in 2009. For the next two years, I was an assistant researcher developing control solutions for optical telescopes. I accomplished two postdoctoral studies: at Umeå University (Sweden) and Inria Lille (France). In 2014, I joined the Adaptive and Nonlinear Control Systems Lab at ITMO University (Russia) as a researcher, where I received my Dr.Sc. degree in Automatic control in 2016. Since 2017, I am a Maître de Conférences at CentraleSupélec, campus Rennes (France).

I am a member of the IFAC Technical Committee on Adaptive and Learning Systems and the IEEE Technical Committee on System Identification and Adaptive Control; I am an IEEE Senior member since 2018. My research interests are adaptive parameter estimation, disturbance attenuation, nonlinear systems, and state estimation.

This manuscript is prepared to obtain the “Habilitation à diriger des recherches” and presents my scientific background overview and my main research direction in recent years: a method to improve transient performance in adaptive parameter estimation. The proposed procedure enhances existing parameter estimation methods and provides the following benefits: parameter estimation transients are monotonic without peaking and oscillations, the gain adjustment becomes transparent and straightforward, and asymptotic convergence can be established without persistency of excitation for some input signals.

The manuscript is organized as follows. In the first chapter, I present my CV describing my professional experience, scientific outcome and collaborations, and teaching activities. In the second chapter, I overview my research experience and background; in this chapter, I also present in brief some of my scientific activities: sinusoidal signal estimation and disturbance attenuation, state estimation for mechanical systems, human-machine interaction, as well as some industrial projects. The third chapter presents my main research activity in recent years, namely the dynamic regressor extension and mixing procedure. Finally, the fourth chapter describes my future research directions on adaptation and learning for advanced energy management. The manuscript has two appendices. Appendix A presents the used notation, and Appendix B contains the complete list of my publications.

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1 Curriculum Vitae

1.1 Education, Experience, and Mobility

1.1.1 Education, diplomas, and grades

I obtained three scientific degrees in Russia, where the last one is the full Doctor of Sciences degree, a higher scientific degree in Russia, which may be earned after the Ph.D. degree. Besides the scientific degrees, I accomplished two postdoctoral fellowships. These activities are listed below in chronological order.

Master equivalent, 2006.

- Title: Adaptive estimation for poly-harmonic signals with an irregular component.
- Degree: Engineer in control systems and informatics.
- University: ITMO University, Saint-Petersburg, Russia.

Ph.D., 2009.

- Title: Adaptive Identification of Quasi-Harmonic Disturbances.
- Degree: Ph.D. in system analysis and control.
- Supervisor: Prof. Alexey Bobtsov.
- University: ITMO University, Saint-Petersburg, Russia.

The main result of the Ph.D. thesis was a new parameter estimation method for multi-sinusoidal signals. The proposed solution had the lowest computational complexity compared to other methods available in the literature. The proposed method was successfully applied for vibration attenuation.

First postdoctoral fellowship, 2012-2013.

- Laboratory: Smart Robotics Lab, Umeå University, Sweden.
- Duration: 2 years.
- Supervisor: Prof. Leonid Freidovich.

This postdoctoral fellowship was focused on the modeling, estimation, and control of hydraulic drives in heavy robotics. The research was conducted in collaboration with heavy robotics manufacturers: Komatsu (forestry robotics) and Ålö AB (agricultural robotics). The research results were implemented in a prototype control

system delivered to stakeholders. The scientific results include 4 journal publications (2 in Q1 journals¹) and 3 conference contributions (2 in top-level conferences).

Second postdoctoral fellowship, 2015-2017.

- Laboratory: NON-A team, INRIA, Lille, France.
- Duration: 1.5 years.
- Supervisor: Dr. Denis Efimov.

The main goal of this postdoctoral fellowship was to enhance human-computer interactions with tactile devices. The main contribution is a dynamic human motion model for human-computer interaction tasks and a prediction algorithm reducing tactile devices' latency. The scientific results include 1 international patent, 2 publications (1 in a Q1 journal), and 4 conference contributions (3 in top-level conferences).

Doctor of sciences, 2016.

- Title: Adaptive Observers for Nonlinear Systems via Parameter Identification.
- Degree: Doctor of Sciences in system analysis and control.
- University: ITMO University, Saint-Petersburg, Russia.

The primary results of this work are new methods for estimation and observer design for a class of nonlinear dynamical systems based on parameter estimation. The proposed methods were successfully applied to sensorless control of electric motors, photovoltaic arrays estimation, and vibration suppression in mechanical systems.

1.1.2 Professional experience and mobility

After the Ph.D. thesis, my high mobility contributed to establishing fruitful international collaborations, developing scientific links and personal research direction.

2017 – present: Maître de Conférences (Associate Professor), CentraleSupélec, Rennes, France.

- Research activities: parameter estimation with enhanced performance and advanced control for smart energy.
- Teaching activities: teaching and supervision, new courses development.

2015 — 2017: Postdoctoral fellow, Non-A team, INRIA, Lille, France.

- Research activities: Motion prediction and estimation in human-machine interaction for touch sensors.

2014 — 2015: Associate Professor, Department of Control Systems and Informatics, ITMO University, Saint-Petersburg, Russia.

¹Here and below, Q1 journals are defined according to the [Scimago SJR](#) journal ranking.

- Research activities: nonlinear systems control and estimation.
- Industrial activities: temperature regulation in rapid thermal processes for semiconductors manufacturing.
- Teaching activities: teaching, supervision of master students and Ph.D. students.

2012 — 2013: Postdoctoral fellow, Applied Physics and Electronics Department, Umeå University, Umeå, Sweden.

- Research activities: Hydraulic systems in robotics, active vibration control.
- Industrial activities: cooperation with a manufacturer of front-end loaders, control design for hydraulic drives, and proof-of-concept prototyping.

2008–2011: Assistant Researcher, Department of Control Systems and Informatics, ITMO University, Saint-Petersburg, Russia.

- Research and industrial activities: Identification and control for high-precision electrical drives.

1.2 Research Activities

1.2.1 Main research topics

My main research topics are (primarily) focused on fundamental problems and target developing a well-grounded basis for industrial applications. For a detailed description, see Section 2.1.

- **Enhanced parameter estimation in adaptive and learning systems.** This topic consists of developing methods empowering existing learning and adaptive approaches, significantly improving transient performance, accelerating the transients, and removing peaks and oscillations.

Scientific outcome: 9 international journal publications (all 9 in Q1 journals), 8 conference contributions, and 2 publication in national journals (in Russian).

- **Velocity estimation and nonlinear observers design.** This research direction includes state estimation methods development for nonlinear systems, primarily applied to the velocity estimation problem in robotics and sensorless control in electrical drives.

Scientific outcome: 7 international journal publications (6 in Q1 journals) and 6 conference contributions.

- **Adaptive attenuation of periodic disturbances.** The results of this research activity provide adaptive solutions to attenuate unknown narrow-band disturbances, primarily applied to the vibration suppression problem in mechanical systems.

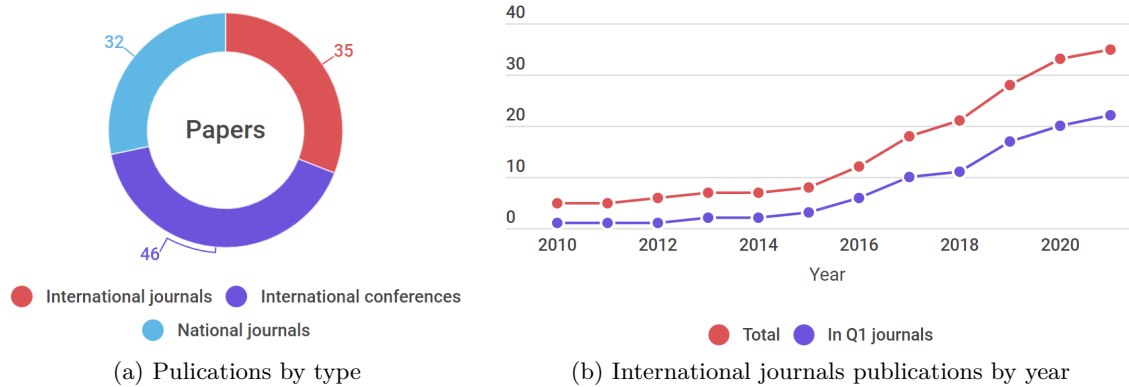


Figure 1.1: Publications by type and year.

Scientific outcome: 10 international journal publications (4 in Q1 journals), 14 conference contributions, and 13 publication in national journals (in Russian).

1.2.2 Scientific outcome

The illustrative summary of the scientific outcome is depicted in Fig. 1.1. The segmentation of the journal publications by topics is given in Section 2.2.

The total scientific outcome

- **35** peer-reviewed articles in international scientific journals including **22** publications in Q1 journals;
- **46** peer-reviewed conference contributions including **14** contributions at the top-level conferences, namely the *Conference on Decision and Control* and the *IFAC World Congress*;
- **1** international patent;
- **32** peer-reviewed articles in Russian scientific journals.

The scientific outcome for the recent years (2017–present)

- **23** peer-reviewed articles in international scientific journals including **16** publications in Q1 journals;
- **16** peer-reviewed conference contributions including **8** contributions at the top-level conferences (CDC and World Congress);
- **1** international patent;
- **3** peer-reviewed articles in Russian scientific journals.

Bibliometrics²

- h-index: **17**,
- Citations, the total number: **1312**,
- Citations in 2020: **249**.

1.2.3 Fundings and projects

Grants and fundings

Project Coordinator and Principal Investigator:

- Project funding by the Russian Ministry of Science and Education for 3 years, 2011. The project was dedicated to the design of advanced controls for high-precision drives. The team was composed of 9 members, including 4 permanent researchers and 5 non-permanents. My responsibility was to manage and direct the scientific activities, define and schedule the research tasks, coordinate the team members and monitor the overall progress.

Personal Grants and Projects:

- *L'allocation d'installation scientifique* (Scientific Facility Allocation), 2019, Rennes Metropole, France;
- *Stratégie d'attractivité durable* (Strategy of Sustainable Attractiveness), 2017, Region Bretagne, France;
- 5 personal research project grants from the Government of Saint-Petersburg, 2007-2011, Russia;
- Young Researcher Support grant by “Bortink” foundation, 2008-2010, Russia;
- 2 personal grants by the Russian Foundation of Fundamental Research, 2009 and 2011, Russia.

Participant and investigator:

- 1 ANR project ([Turbotouch](#)), 2015-2017, France;
- 11 projects funded by the Gouvernement of Russian Federaation, 2007-2015, Russia.

Industrial contracts

Scientific advisor in industrial study contracts:

- Modeling of the Rance tidal power station, EDF, 2017 and 2019;
- Control of climatic chambers, BDR Thermea France, 2018;
- Identification and modeling of synchronous motors, Renault, 2018.

²According to the [Google Scholar](#) records, May 2021.

1.2.4 Awards and distinctions

- Elected to the IEEE Senior Member grade, 2018.
- The Best Presentation winner of the “Navigation and Motion Control” conference, Russia, 2011.
- The winner of the “Young Author Support” program at the 15th IFAC Symposium on System Identification, France, 2009.

1.2.5 Scientific collaborations

National collaboration (France)

- Team Valse (NON-A), Inria, Lille. Joint research on adaptive systems, parameter estimation, and human-machine interactions.
- LORIA (*Laboratoire lorrain de recherche en informatique et ses applications*), Lorraine University, Nancy. Collaboration on control and estimation for mechanical systems.
- L2S (*Laboratoire des signaux et systèmes*), CNRS, Paris. Joint research on observer design for nonlinear and mechanical systems, adaptive control, and parameter estimation.

International collaboration

- Laboratory of Nonlinear and Adaptive Systems, ITMO University, Russia. Collaborative research on adaptive and learning systems and nonlinear control.
- School of Automation, Hangzhou Dianzi University, China. Joint research on adaptive control and parameter estimation.

1.2.6 Community activities

Technical committees

- Member of the IFAC Technical Committee on Adaptive and Learning Systems.
- Member of the IEEE Technical Committee on System Identification and Adaptive Control.

International conference program committees

- IEEE Workshop on Advanced Motion Control, 2020.
- IFAC Workshop on Adaptive and Learning Control Systems (ALCOS), 2019 and 2022.

- IEEE International Conference on Electrical Power Drive Systems, 2020.
- 34th Chinese Control Conference, 2015.

Invited lectures and seminars

- INRIA, Lille, France, 2019.
- Hangzhou Dianzi University, China, 2016-2019.

Editorial and reviewer activities, research evaluation

- Associate Editor (special issue), International Journal of Adaptive Control and Signal Processing, 2020-2021.
- Best Student Paper Award Committee member, European Control Conference, 2020.
- Reviewer for multiple international journals including IEEE Transactions on Automatic Control, Automatica, Control System Technologies, Control Engineering Practice.

1.2.7 Scientific culture dissemination

- In Russia, I presented research results on high-precision control for optical telescopes in general-audience magazines and on TV news.
- In Sweden, I participated in disseminating our results on advanced control for agricultural robotics, and a journal article was published highlighting the collaboration between the university and industry.
- In France, I disseminated the scientific challenge of advanced control for low-energy intelligent buildings; this activity was supported by Rennes Métropole. The dissemination includes **an article** in *Destination Rennes Business* media and **a video** in *Ici Rennes* media.

1.3 Teaching Activities and Supervision

1.3.1 Research supervision

- **Ph.D. theses supervision:**

- Aleksandr KAPITONOV defended his thesis “Robust control of rapid thermal processes applied to vapor deposition processes” in December 2014 at ITMO University, Saint-Petersburg, Russia. I was the only supervisor of the thesis (100% supervision rate). Aleksandr is currently an Associate Professor and research fellow at the Faculty of Control Systems and Robotics at ITMO University.

- Marina KOROTINA is funded by the CentraleSupélec scholarship, and she started her doctoral studies on “Performance Improvement in Adaptive and Learning Systems” in December 2020. I am co-supervisor (50% supervision rate).

- **Research Track supervision:**

CentraleSupélec offers a career path focused on science and research. At the end of the three years research-oriented program, the student submits a Master thesis. The Research Track enhances this expertise for those who wish to engage in a doctoral thesis or join a corporate RnD center.

- Ricardo EHLERS BINZ has successfully completed the first two years of his project “Data-driven and learning-based control” in 2020. I am the only supervisor (100% supervision rate).

1.3.2 Teaching activities

My teaching activities include lecture courses, practical and laboratory sessions, and supervision of engineer and master students. Below, I provide the list of the courses where I was involved in the course development. Then, I provide the list of the supervised master and engineer students.

Courses development

- **In CentraleSupélec**

- Learning for Modeling and Control. Responsible (100%), conception and preparation of materials on data-driven parametric and non-parametric learning. This course was first given in 2020.
- Nonlinear System Analysis. Responsible (100%), conception and preparation of materials on nonlinear systems analysis in the context of power grids and energy systems. This course was first given in 2020.
- Advanced control. Responsible (100%), conception and preparation of materials on advanced control systems (nonlinear systems, sliding-mode control, robust control). This course was first given in 2018.
- Sampled and nonlinear systems. Co-responsible (50%), conception and preparation of materials on nonlinear system analysis. This course was first given in 2018.
- Large-scale systems. Co-responsible (25%), conception and preparation of materials on linear matrix inequalities. This course was first given in 2017.

- **In ITMO University**

- Essentials of system identification. Responsible (100%), conception and preparation of materials on system identification. This course was first given in 2014.

Engineer and Master students supervision

In CentraleSupélec: co-supervision of 13 engineer students in 4 industrial studies (in collaboration with Renault, EDF, BDR Thermea France) in 2017-2020:

- Industrial project “Renovation of the Rance tidal power station - Improvement of the quality of regulation” (*Rénovation de la Conduite de La Rance – Amélioration de la qualité de la nouvelle conduite*) proposed by EDF in 2019-2020, co-supervision (50%) of 4 engineer students.
- Industrial project “Identification and modeling of synchronous motors” (*Identification d’un modèle de moteur synchrone et réduction des mesures sur banc d’essai*) proposed by Renault in 2018-2019, co-supervision (50%) of 3 engineer students.
- Industrial project “Control of climatic chambers” (*Commande de chambres climatiques*) proposed by BDR Thermea France in 2018-2019, co-supervision (50%) of 3 engineer students.
- Industrial project “Modeling of the Rance tidal power station” (*Modélisation numérique de l’usine marémotrice de la Rance*) proposed by EDF, 2017-2018, co-supervision (50%) of 3 engineer students.

In ITMO University: supervision of 2 master students and 2 engineer students in 2014-2015. The master theses are:

- Andrey LOSENKOV, master student (100% supervision), 2015: “Direct methods of multi-sinusoidal disturbances compensation”. Scientific outcome: 5 publications in national peer-reviewed journals, 1 contribution to the international peer-reviewed conference.
- Polina GRITCENKO, master student (100% supervision), 2015: “Indirect methods of multi-sinusoidal disturbances compensation”. Scientific outcome: 2 publications in international peer-reviewed journals, 2 contributions in the international peer-reviewed conference.

2 Research Experience and Background

My research field is Control Science, focusing on adaptive systems, state estimation, and prediction in the deterministic framework. I worked on several topics during my career, including periodic disturbances rejection, velocity estimation and nonlinear observer design for mechanical systems, modeling and prediction in human-computer interactions; some of these researches remain active nowadays. However, my principal current research direction is the performance enhancement in adaptive and learning systems, where I developed a novel procedure, Dynamic Regressor Extension and Mixing. My future research activities are oriented toward advanced adaptive and learning control in energy management.

In this chapter, I present the summary of my research activities, whereas the detailed description of the key ongoing research topic is given in more detail in Chapter 3. This chapter is organized as follows. In Section 2.1, I present a brief overview of my research activities, and in Section 2.2, I describe the structure of my publications by research topics. Further, in Section 2.3, Section 2.4, Section 2.5, and Section 2.6, I present short descriptions of my research activities by topics: sinusoidal signals and disturbances, state estimation and observers, human-machine interaction, and industrial applications, respectively.

In this chapter, the references starting with the letters **IJ** or **IC**, e.g., [IJ1] and [IC1], are given with respect to the list of my publications in international journals and international conferences, respectively, as given in Appendix B. And the numeric references, e.g., [1], are provided with respect to Bibliography.

2.1 Brief Overview of the Research Background

2.1.1 Research topics

My research activities can be split in the following topics.

- *Enhanced performance in parameter estimation via Dynamic Regressor Extension and Mixing* (since 2016).

Parameters estimation of a linear regression model plays a central role in many adaptive and learning control branches. This problem naturally appears in system identification, direct and indirect adaptive control, reinforcement learning systems, and other areas. Various methods solve this problem, where the most widely used are the gradient descent and the least-squares estimators. However, these methods have several drawbacks. First, even if a (weighted) estimation error norm

decays monotonically, the element-wise transients can exhibit undesirable oscillations and peaking phenomena. Second, gains tuning is somewhat complicated and typically involves numerous trial-and-error attempts. This complication arises due to two reasons: the trade-off between the transients acceleration and the peaking/oscillating behavior, and the interconnection of all transients, i.e., a gain adjustment improving the transient performance of a specific parameter inevitably affects the transients of other parameters. Finally, the convergence of the mentioned methods depends on the persistence of excitation condition that can be restrictive for some applications.

Motivated by these shortcomings, I proposed a novel *Dynamic Regressor Extension and Mixing* (DREM) procedure that introduces a nonlinear dynamic transformation decoupling the element-wise transients [IJ22]. The DREM procedure yields the following benefits:

- the transients are element-wise monotonic; thus no peaking phenomena and oscillations;
- the tuning rules are simple and transparent providing a single tuning gain for a single parameter;
- asymptotic convergence can be shown in the absence of the persistence of excitation.

The DREM procedure was successfully applied for numerous applications, including system identification [IJ11], photo-voltaic systems [IJ20], motor control [IJ23], and also robotics, power systems, and model reference adaptive control.

The development of the DREM procedure constitutes the main direction of my research activities in recent years. Its interpretation as a functional observer was shown in [IJ15], and a possible connection with the composite learning methods was discussed in [IJ12]. The analysis of excitation properties propagation is given in [UR1] and [IC5], and the generalization to the discrete-time domain can be found in [IJ3]. A fixed-time estimator under interval excitation applying the DREM procedure is proposed in [IJ8].

- *State estimation in nonlinear systems* (since 2015).

This research topic is primarily motivated by various robotics applications. The notable results include a globally exponentially stable momentum estimation method for a class of mechanical systems with a strict Lyapunov function [IJ14] and a switched observer for a class of parameter-varying systems that are unobservable for specific values of the varying parameters [IJ1]. Working on this topic, I also participated in developing a parameter-estimation-based observer design method that translates the state estimation problem to parameter estimation [IJ28].

Besides the results mentioned above, this research topic includes several case-study outcomes, such as velocity estimation for a scissored pair control moment gyroscopes [IJ5], a bias propagation analysis for a model-based homogeneous differ-

entiator in a robotics application [IJ6], and a sliding-mode observer for hydraulic drives in heavy-duty forestry robotics [IJ25].

- *Sinusoidal signal parameters estimation and adaptive compensation of sinusoidal disturbances* (since 2006).

The problem of parameter estimation for sinusoidal signals was a central topic of my Ph.D. thesis. The main contribution was the approach rewriting the frequency estimation problem as parameter estimation for a linear regression model. This approach yielded the simplified global frequency identification algorithm [IJ31]. The results on frequency identification attracted interest and were used for marine applications. The proposed frequency estimator allowed for several indirect adaptive disturbance rejection approaches [IJ19, IJ35]. I also successfully participated in the International Benchmark on Adaptive Vibrations Regulation [IJ29] organized by Professor I. D. Landau, GIPSA-Lab, France, in 2013. My recent activities in this field are devoted to estimation with enhanced convergence [IJ26, IJ21] and estimation of time-varying sinusoidal signal parameters [IJ18].

- In addition to the three main research topics mentioned above, it is also worth noting some other research activities:
 - My collaboration with an industrial partner (see Section 2.6) motivated my research on passivity-based control for rapid thermal processes in semiconductors manufacturing [IJ27].
 - The human-machine interaction was the core of my postdoctoral studies in INRIA (Lille, France). Two main outcomes of this research are i) the hybrid wrist motion dynamic model of a user performing a pointing task with a computer mouse [IJ7] and ii) an adaptive user’s trajectory prediction method for touch input devices [IJ10]. The proposed trajectory prediction method yielded the predictive display device patent [P1].

2.1.2 Selected topics included in the HDR manuscript

I performed most of my research activities on parameter estimation for sinusoidal signals and periodic disturbance attenuation during my Ph.D. study, right after that, and partially during my first postdoctoral stay (Umeå University, Sweden). My research activities on modeling and control for hydraulic systems were central to my first postdoctoral stay (Umeå University, Sweden), whereas the research on human-machine interaction was performed during the second postdoctoral stay (Inria, France). These research activities are briefly described in the following sections of this chapter.

Two main lines of my independent research activities are the observer design and the performance improvement in parameter estimation. In order to streamline the presentation, in this manuscript, I focus on the performance improvement research and present it in detail in Chapter 3. The observer design research activities are discussed in brief in Section 2.4.

Research topic	International Journals	International Conferences	National Journals
<i>Principal topics</i>			
Enhanced parameter estimation	9 (all in Q1)	8	2
State and velocity estimation	7 (6 in Q1)	6	0
Sinusoidal signals estimation and compensation	10 (4 in Q1)	14	13
<i>Other topics</i>			
Modeling and control for nonlinear systems	6 (2 in Q1)	10	11
Human-Machine interaction	2 (1 in Q1)	3	0
Other	1	5	6

Table 2.1: Structure of publications with respect to research topics (Q1 journals are defined according to the [Scimago SJR](#) journal ranking).

2.2 Structure of Publications

The structure of my publications according to the mentioned research topics is summarized in Table 2.1.

Publications in international journals by topics

- Enhanced Parameter Estimation
 - General results on the Dynamic Regressor Extension and Mixing procedure [[IJ3](#), [IJ9](#), [IJ15](#), [IJ22](#)].
 - Finite-time estimation via DREM [[IJ8](#)].
 - Performance enhancement in composite learning [[IJ12](#)].
 - Applications of the DREM procedure [[IJ11](#), [IJ20](#), [IJ23](#)].
- Velocity Estimation and Nonlinear Observers
 - Velocity observers for mechanical systems [[IJ5](#), [IJ6](#), [IJ13](#), [IJ14](#)].
 - Switched observer for parameter-varying systems [[IJ1](#)].
 - Parameter estimation-based observer [[IJ28](#)].
 - Time-varying differentiator in hydraulic drives [[IJ25](#)].
- Sinusoidal signal estimation and periodic disturbance compensation
 - Parameter estimation for sinusoidal signals with constant parameters [[IJ31](#), [IJ33](#), [IJ34](#)].
 - Parameter estimation for sinusoidal signals with time-varying parameters [[IJ18](#)].

- Performance improvement in frequency identification [IJ21, IJ26].
- Periodic disturbances compensation [IJ19, IJ29, IJ32, IJ35].
- Modeling and control for nonlinear systems
 - Delay-based adaptive output stabilization [IJ2].
 - Induction motor application [IJ4].
 - Passivity-based control [IJ16, IJ27].
 - Chattering reduction in hydraulics drive control [IJ24].
 - Friction analysis in high-precision systems [IJ30].
- Modeling and Forecasting in Human-Machine Interaction
 - Pointing motion modeling [IJ7].
 - Latency compensation and trajectory prediction [IJ10].
- Other results
 - Empowering excitation in parameter estimation [IJ17].

2.3 Sinusoidal Signals Parameter Estimation and Attenuation of Periodic Disturbances

2.3.1 Context and challenges

The problem of disturbance compensation is one of the classical problems of control theory that is important for engineering practice. Notably, such a situation arises in various applications where it is necessary to reduce the influence of noise. The broad field where such compensation often is the sole goal of control design is Active Noise Control (see, e.g., [1]). It is usually acceptable and reasonable to model noise as a deterministic narrow-band disturbance composed as a sum of a finite number of sinusoidal signals. An overview of relevant applications where the rejection of such narrow-band disturbances is required can be found in [2]; particular examples include fed-batch reactors, distributed flexible mechanical structures, and Blu-ray disc drives servomechanisms. Since the frequencies of the sinusoidal signals constituting the narrow-band disturbances are typically not known in advance and can be time-varying depending on the environment, most active disturbance rejection methods are adaptive.

Adaptive disturbance compensation is typically based on the internal model principle [3], and the compensation methods can be divided into two groups, direct and indirect. For the direct adaptive compensation, disturbance parameters are not explicitly estimated since one performs adaptive tuning of the controller's coefficients themselves [4, 5]. In contrast, in indirect adaptive methods, one first explicitly estimates disturbance parameters (frequencies) and then constructs the control law based on the resulting estimates. Thus, indirect adaptive disturbance compensation strongly relies

on the frequency identification algorithms that constitute the core of the corresponding methods. It is worth noting that besides the indirect adaptive disturbance compensation, the frequency estimation problem also arises, e.g., in servo-loops failure detection in aircraft [1], marine applications to avoid parametric roll resonance for ships [6, 7], and wind power systems control [8].

There are **two main challenges** in frequency estimation and disturbance compensation. First, these methods should be robust to inevitable noises and measurement distortions that reduce frequency estimation accuracy and cause waterbed noise amplification, as discussed in [9]. Second, such methods are often implemented in embedded devices and thus should be sufficiently computationally easy to ensure the real-time operation; this requirement excludes some methods based on signal buffering and, e.g., Fourier analysis. Finally, these methods should track parameters and reject disturbance as the frequencies drift in time.

2.3.2 Research objectives, positioning, and achievements

Frequency estimation

The adaptive frequency estimation for multi-sinusoidal signals was the core topic of my Ph.D. thesis. The **research objective** was to develop a novel method for the frequency estimation of a multi-sinusoidal signal in the presence of a bounded irregular (non-periodic) distortion component. At that time, several frequency identification methods were available in the literature, such as methods based on adaptive notch filters [10–12], on nonlinear error signal generators, e.g., the PLL-based approach [13, 14], a technique utilizing the squared measured signal [15], and more general methods capable of estimating frequencies of a sum of sinusoidal signals [16–18].

To fulfill the objective, I proposed a new frequency identification algorithm [IJ31]. The main novelty and contribution were rewriting the frequency estimation problem as a parameter estimation problem for a linear regression model. This approach yielded a simplified global frequency identification algorithm. Compared to the methods [10–13, 15], the proposed method can estimate the frequencies of a sum of sinusoidal signals, not of a single sinusoidal signal only. Compared to the works [16–18], the proposed solution had a lower dynamic dimension of the estimation algorithm, i.e., the smaller number of states in the state-space representation of the algorithm, and, to the best of my knowledge, the algorithm proposed in [IJ31] had **the lowest dynamic order** among all existing solutions. The proposed frequency estimator allowed for several indirect adaptive disturbance rejection approaches [IJ19, IJ35], and the results of [IJ31] were later extended for sinusoidal signals with time-varying parameters [IJ18].

Estimation performance

Concerning the estimation accuracy problem in the presence of noises, there are two possible scenarios for improving the performance: to improve the signal-to-noise ratio of the signal itself (filtering) or to reduce the noise sensitivity of the estimation process (averaging). Typically, the averaging leads to identification algorithms with time-varying

gains asymptotically converging to zero, for example, a least-squares algorithm with no forgetting. This approach's drawback is that the insensitive to noise algorithms also become insensitive to possible variations of the parameters to be estimated, and it is not consistent with the challenges discussed above. This drawback is usually overcome with constant-gain and constant-trace algorithms [19] or covariance-resetting [20]; however, the trade-off between averaging properties and transient time is still relevant. The second way to reduce the noise is filtering. This approach's drawback is that *a priori* knowledge of an acceptable frequency range is needed to construct a suitable band-pass filter. If the sinusoidal signal frequency can belong to an (arbitrary) wide range, no *a priori* defined filter can be applied.

My **research objective** was to design an approach that can empower existing frequency estimation methods and increase the estimation accuracy. In [IJ26], I proposed the construction of an adaptive filter cascade. The cascade consists of adaptive band-pass filters tuned by estimates of the frequency provided by a given identification algorithm. It was shown that the proposed solution significantly improves the estimation performance for a broad class of frequency estimation methods.

Direct disturbance attenuation

Besides the internal model principle [3], a large family of periodic disturbance attenuation methods (mainly attributed to acoustics) is based on adaptive feedforward compensation; see [1, 21] and references therein. It is assumed that reference signals, i.e., sinusoidal signals of the same frequencies as the disturbance, are provided to feed into the system. Then the parameters of the feedforward compensator are tuned adaptively. One classical scheme in this approach is the Filtered-x Least Mean Square (FXLMS) one (see [22, 23]), where the plant model is involved in the adaptation loop to predict the plant's output. A gradient or pseudo-gradient algorithm is often used for tuning. It can be shown, see, e.g., [23, 24], that under some conditions, the adaptive feedforward approach can be seen as an asymptotic realization of the internal model principle.

Addressing the problem of direct adaptive compensation of periodic disturbances and motivated by the adaptive feedforward concept, in [IJ29], I proposed a direct adaptive disturbance attenuation method. This method is based on a representation of the disturbance as a linear combination of outputs of stable filters applied to the measured output error signal passed through the plant model. The resulting design is similar to the classical adaptive noise control scheme, known as adaptive FXLMS [21], but differs in the parameter adaptation idea. The proposed method deals with discrete-time systems with unstable zeros challenging for adaptive regulation methods based on passivity. This method was then implemented in the **International Benchmark on Adaptive Regulation** (see Fig. 2.1). The special issue [9] devoted to this benchmark presents state-of-the-art adaptive disturbance regulation; the publication [IJ29] is also included in this issue. It is essential to highlight that the result given in [IJ29] has **the smallest computational cost** among all the benchmark approaches. The proposed method can also deal with disturbances where the exact number of sinusoidal components is unknown, contrasting with many disturbance attenuation methods, such as [5, 23, 25].



Figure 2.1: The testbed used in the Benchmark on Adaptive Regulation; see [9] and the project's [website](#).

2.3.3 Summary and scientific outcome

My research activities on sinusoidal signal parameter estimation and periodic disturbance attenuation started with my Ph.D. thesis and remained an active research field after. Most of the results in this field were obtained in 2008–2016; these results constitute a significant part of my scientific background. This research field benefited from my collaboration with **ITMO University** (Russia), **Umeå University** (Sweden), and **Hangzhou Dianzi University** (China).

In total, the scientific outcome of this research direction constitutes of **10** papers in international peer-reviewed journals (including 4 Q1 journals), **14** submissions at international peer-reviewed conferences, and **13** publications in national peer-reviewed journals (in Russian); see Section 2.2 for details.

2.4 Velocity Estimation and Observer Design

2.4.1 Context and challenges

State estimation is a common problem in control systems that has a vast of engineering applications. In this field, my motivation was mainly due to mechanical systems and robotics, where state estimation usually becomes velocity or generalized momenta estimation given the coordinates measurements.

Besides some specific applications, e.g., fault detection, state observers in mechanical systems are used to enable full-state feedback control. Thus, the estimation error transients impact the overall performance, and the exponential (rather than only asymptotic) convergence is a desirable property. Moreover, the closed-loop system analysis is

significantly simplified when a Lyapunov function for the used state observer is available, and even stronger stability results can be obtained if the Lyapunov function is strict. Furthermore, as discussed in [26, 27], globally exponentially stable systems with strict Lyapunov functions have good robustness properties. These arguments make of interest to design an observer with the **strict Lyapunov function** ensuring the **global exponential convergence** and identify a class of mechanical systems where such an observer can be applied.

Another challenge in state estimation is the observability of parameter-varying systems. Many observer design approaches assume that a plant is uniformly completely observable. Unfortunately, this assumption is violated in some applications, where a particular combination of input signals or time-varying parameters may make the plant unobservable, at least for some instances of time or for some operation regimes. Then, a single observer with constant parameters cannot track the states in all operation modes or along all trajectories.

One particular case arising, e.g., in robotics, is when the whole set of time-varying parameters, or the whole trajectory, can be divided into a finite number of subsets, or trajectory segments, such that the system is observable for these segments and loses the observability only when travelling between them. This behavior naturally yields the switched systems formulation. To this end, my goal was to design an observer capable of dealing with this class of **locally unobservable time-varying systems**. Moreover, aiming at the embedded system implementation makes it preferable to compute the observers' gains in advance, thus reducing real-time computation.

Full-state estimators can be redundant in some engineering applications, e.g., in robotics with multiple degrees of freedom (DOF), and a widely used practical approach is to estimate velocities for each DOF separately. From the signal processing point of view, this velocity estimation approach can be seen as online numerical differentiation of the measured position signal. My research in this field was motivated by the AnyWalker walking robot design [28] (see Fig. 2.2), where rapid and accurate velocity estimation is crucial to apply the stabilizing full-state feedback.

The AnyWalker robot uses an auxiliary non-anthropomorphic dynamic stabilization system that consists of two reaction wheels inside the robot's body. There are multiple solutions for walking robot stabilization, and the related simplified problem statement formulated as an inverted pendulum is included as an example in many graduate courses on control design. However, it should be noted that stabilization algorithms typically implement a full-state control law using both position and velocity measurements. Thus, the velocity estimation becomes the critical element of the vertical stabilization control design, and the main challenge here was to understand the achievable **estimation accuracy despite model mismatches** and disturbances due to the other DOFs.



Figure 2.2: AnyWalker robot uses reaction wheels as an auxiliary stabilization system; see [28, 29].

2.4.2 Research objectives, positioning, and achievements

Switched observer for locally unobservable time-varying systems

Switched observers are typically applied to systems with commutations of dynamics, where a plant can operate in a finite number of operation modes. Whereas there exist nonlinear observers for nonlinear switched systems [30, 31], linear switched system state estimation is commonly addressed through linear switched observers. The conventional approach is to construct a common Lyapunov function that is suitable for all operation modes, such as [32], and the key tools for this approach are linear matrix inequalities. However, the existence of a common Lyapunov function is a restrictive assumption, and, particularly, it does not hold if some operation modes are not observable. This problem was addressed in [33], where authors proposed conditions under which the system is observable even if some individual modes are unobservable. The same problem was considered in [34], where the authors studied when does there exist a trajectory making the system observable. The common Lyapunov function requirement can be relaxed by imposing assumptions on the average dwell time of commutation, as in [35, 36]. This concept can also be used when the switching signal is not precisely known and has to be estimated, as in [37], or is measured with a delay, as in [38].

My objective was to design a switched observer for a **parameter-varying system** that is **unobservable for some values** of time-varying parameters. The considered systems are not switched themselves and do not have a finite number of operation modes. To reformulate it as a switched system, in [IJ1], I proposed to divide the set of parameter values into a finite number of subsets, where the system is observable for each subset, and the observability is lost only when the vector of varying parameters travels from one subset to another. For such systems, I proposed a switched observer based on switched Lyapunov function with a dwell-time condition ensuring the exponential convergence. The stability condition for the proposed observer was formulated in the form of matrix

inequalities, which can be used for gains tuning.

Momentum estimation in mechanical systems

For mechanical systems applications, I considered the class of systems that are partially linearizable via a change of coordinates. This class, formally defined in [39], consists of mechanical systems whose dynamics becomes linear in velocity after a partial coordinate transformation, e.g., a linear transformation of the velocities. These systems were studied, e.g., in [40–42], because, on the one hand, observer design and controller synthesis are simplified for them while, on the other hand, many practical examples satisfy this property. My research objective was to design an exponentially converging observer for the considered class of systems with a strict Lyapunov function.

The existing solutions for this class of systems include the linearization-based observers, e.g., Kalman filter, and nonlinear observers [43], e.g., a high-gain observer [44], a sliding-mode observer [45], or an invariance-based observer [46]. My main achievement presented in [IJ14] was designing a simple globally exponentially stable Luenberger-like nonlinear momentum observer for the considered class of systems. The main contribution of the proposed solution is that it was the first observer design supported with a **strict Lyapunov function** for the considered class of systems.

Numerical differentiators

Numerical differentiators are a standard engineering solution for velocity estimation for each degree of freedom separately, and many solutions are available. To mention a few, see the first-order difference used in [47], the sliding-mode exact differentiator [48], the algebraic and annihilators-based differentiators [49,50], high-gain differentiators [51], and homogeneous differentiators [52]. Whereas differentiator-based velocity observers can be designed model-free [47,49], better performance is obtained when the observers use (at least partially) available model knowledge as in [48,50–52].

My research on model-based differentiators was motivated by the Any-Walker walking robot design (Fig. 2.2), where rapid and accurate velocity estimation is crucial to stabilizing full-state feedback control. The high-dimensional complete mechanical model of such a system makes it questionable to design a model-based observer for the whole state vector, and local velocity observers for each degree of freedom are preferred. The research objective was to study model-based differentiators' behavior in the presence of model mismatch and measurement distortions, e.g., a measurement bias. My key achievements are the experimental studies on **achievable performance** for state-of-art velocity observers [IJ5] and the **bias propagation analysis** and compensation for a class of finite-time homogeneous differentiators [IJ6].

2.4.3 Summary and scientific outcome

My research activities on velocity estimation and observer design started in 2015 and remain active. Most of the results in this field were obtained in collaboration with the

Laboratory of Signals and Systems (CNRS, France), **LORIA** (CNRS, France), **Hangzhou Dianzi University** (China), and **ITMO University** (Russia).

In total, the scientific outcome of this research direction constitutes of **7** papers in international peer-reviewed journals (including 6 papers in Q1 journals), **10** submissions at international peer-reviewed conferences, and **1** publication in a national peer-reviewed journal (in Russian); see Section 2.2 for details.

2.5 Modeling and Forecasting in Human-Machine Interaction

2.5.1 Context and challenges

Typically, human-computer interactions can be divided into two categories: direct and indirect interactions. In indirect interactions, the input device (e.g., a mouse or a trackball) and the output device (e.g., a display) are separated. In contrast, for direct interactions, the input and the output devices are coupled together, and the input and the system output (observed by a user) share the same screen. Examples of direct interactions include smartphones, pads, i.e., the touch screens.

For direct touch interactions, a significant challenge is the **latency reduction**. The detrimental impact of latency on user performance has been known for a long time [53]. Direct interactions are more sensitive to latency, and in [54], the authors found that latency greater than 25 ms can significantly affect the user performance in touch dragging tasks. Simultaneously, in [55], the authors show that latency as small as 10 ms still can be perceived in direct interactions. However, it is reasonable for modern touchscreen devices to expect the end-to-end latency of about 60 ms or more, as measured in [55]; thus, the latency reduction methods can significantly improve the performance of human-machine interactions.

Another challenge that appears in indirect human-computer interactions is the **modeling of human movements**. In indirect interactions, the user's action is typically scaled by a pointing transfer function before being displayed at the output device. Such an adjustment provides small amplification for low input velocities to improve pointing accuracy and provide high amplification for high input velocities to reduce traveling time. Experimental studies [56] report that a choice of a scaling function affects human-computer interaction, and users get better performance using switching scaling functions. However, despite all the research on evaluation and reverse-engineering of the existing scaling functions, the problem of design and optimization of such a scaling remains open due to limitations of existing dynamical models for human pointing.

Models of pointing dynamics are also of interest for endpoint prediction techniques [57], where the system attempts to predict the target cursor position from the beginning of the pointing movement. Such a prediction is further used to modify the visual interface dynamically (e.g., reduce the distance to the target or increase its width). Some techniques and methods can be found in [58], and a toolbar with dynamically expanding icons represents an example of such an approach [59]. Some other challenging applications where dynamic models of (pointing) movements can be relevant include analyses

of mouse movements for user identification [60] and human movement estimation [61], e.g., for manipulators teleoperation [62].

2.5.2 Research objectives, positioning, and achievements

Modeling of Pointing Movements

My **research objective** was a model that can describe pointing human movements in human-computer interaction, taking into account modern graphical interfaces' scaling functions.

In human-computer interactions, the Fitts' law [63] claims the logarithmic relationship between the traveled distance and the moving time, and the hybrid Optimized Initial Impulse model developed by Meyer et al. [64] is now accepted as the most well-established explanation for Fitts' law [58]. This explanation separates the pointing motion into two different stages: a rapid and large movement (ballistic phase) and a slower corrective action under feedback control (corrective phase).

The ballistic movement is typically considered an optimal control problem. A well-known result is the minimum jerk model proposed in [65], which claims that the ballistic action is performed to minimize the total jerk cost along the trajectory. As shown in [66], the jerk cost minimization leads to more plausible results than the acceleration or snap cost functions. For the tracking stage, researchers typically use linear time-varying or time-invariant models, see the crossover model [67] and the Vector Integration To Endpoint (VITE) model introduced in [68]. The first attempt to handle both ballistic and tracking phases with a single model was the Suge model [69]. It is worth noting that since the transient time of a linear system is logarithmically related to the traveled distance, the asymptotic behavior of such models reproduces Fitts' law.

In this field, **my principal achievement** is a switched dynamic model handling cursor movements in indirect pointing tasks describing both ballistic and tracking motions and taking into account motion scaling by a graphical interface system. The model also includes the intermediate commutation phase explaining the switch between the ballistic and the tracking stages. During the first stage, there is no visual guidance to the user, i.e., only sensorimotor feedback is available, and a nonlinear Lurie form system is used to model this part. When the user perceives the final cursor position is approaching, the commutation phase triggers the switching to the tracking phase. In this last phase, feedback is given by the visual perception of the user's cursor position, and an extended VITE model [68] is used to model the tracking dynamics.

I showed that both ballistic and tracking models are globally asymptotically stable under various scaling functions used in graphical interfaces and generate bounded and realistic trajectories under some established mild conditions. A series of experiments with several users performing pointing tasks were performed to validate the model showing that it fits well different pointing movements and outperforms pure ballistic and tracking models.

Latency Reduction in Direct Interactions

In this field, my **key objective** was to reduce the perceived latency, i.e., the observed lag between the user's and the cursor's motions. There are two ways to reduce the impact of latency. The first is at the hardware level, e.g., using more reactive and advanced elements and making the signal flow as fast as possible. This approach has two drawbacks, the high cost of advanced components and the increased energy consumption, which play a significant role in portable devices.

The second way to reduce latency is at the software level by using latency reduction algorithms. From the control point of view, this problem can be formulated as a trajectory prediction or forecasting problem, and convenient prediction methods can be used. However, methods based on underlying dynamic models cannot be easily applied for latency reduction since dynamic models of user behavior are not typically available for the specific user.

The lack of models motivates the use of model-free prediction methods. For instance, a trajectory prediction using a Kalman filter for a chain of integrators was proposed in [70], and a strategy based on the first-order Taylor series was used in [71]. In [72], a forecasting algorithm based on the Taylor series expansion was proposed, where the derivatives were estimated using either algebraic or homogeneous finite-time differentiators. However, these differentiators' parameter tuning is rather complicated due to their high non-linearity. Also, some model-free approaches motivated by Kalman filter, curve fitting, and heuristic considerations can be found in patents. To summarize, latency reduction can be translated to the problem of estimating the user's trajectory or, equivalently, to the further trajectory points prediction.

My achievement in this field was a **novel model-free frequency-domain-based approach** for human movements forecasting. It was shown that the proposed solution is an approximation of an ideal (not causal) predictor. The proposed forecasting algorithm can be tuned numerically as an optimization task over the available dataset on the user's movements. Moreover, an adaptive modification of the design is proposed that adapts to the changes of users and movement types. Several experimental studies with various users were performed, illustrating the proposed forecasting algorithm's applicability and performance compared to other methods.

2.5.3 Summary and scientific outcome

My research activities in this field were a part of the ANR-funded **Turbotouch** project in 2015-2017. This research benefited from my scientific collaboration with **NON-A** team, INRIA (Lille, France).

In total, the scientific outcome of this research direction constitutes of **2** papers in international peer-reviewed journals (including 1 Q1 journal), **3** submissions at international peer-reviewed conferences, and **1** international patent on latency reduction; see Section 2.2 for details.

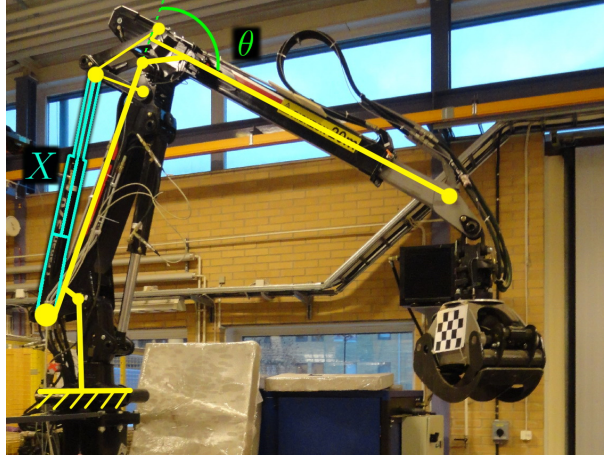


Figure 2.3: The hydraulic crane prototype at the Robotics and Control Lab, Umeå University; see [73].

2.6 Industrial Applications

Besides the scientific research activities discussed above, I also present my scientific participation and advising in some industrial projects I was involved in during my career.

2.6.1 High-precision optical telescopes for satellites tracking

In 2008–2011, I participated in a project on control system design and implementation for high-precision optical telescopes performed at ITMO University, Saint-Petersburg, Russia.

These telescopes are used in various applications, and satellite tracking is one of these tasks. Each telescope is equipped with a laser-based range sensor to measure the distance from the telescope to a satellite. The measurements from a network of telescopes are used, e.g., for GPS and GLONASS navigation. Such a telescope tracks a rotating satellite with an outstanding accuracy and precision. In this project, I was a member of the control system design team. The solutions I have designed are currently under exploitation in satellite tracking systems.

Scientific results of this project include friction modeling and analysis in high-precision electrical drives [IJ30]. The project results were also disseminated in the public sector via several general-purpose journals and TV news reports.

2.6.2 Hydraulic drives control and automation

I worked on this project in 2012–2014, being a postdoctoral member of the Robotics and Control Lab at Umeå University with professor Leonid Freidovich.

Mobile hydraulic drives are widely used in heavy-duty machines in mining, forestry, and agriculture. In contrast with stationary hydraulic systems, e.g., hydraulic presses,

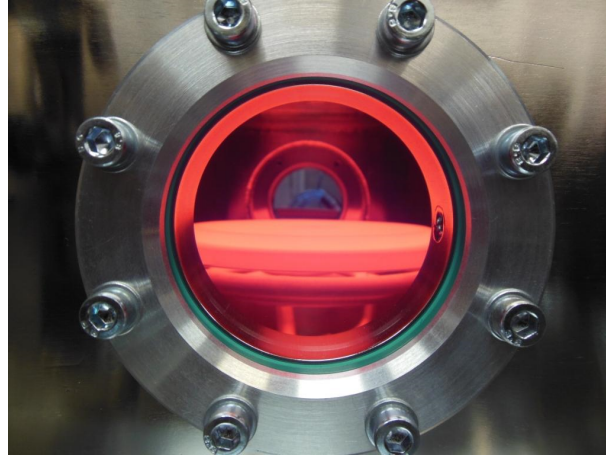


Figure 2.4: Reactor during the annealing process; see [74].

the mobile hydraulic drives are less automated. Most of such drives are nowadays controlled manually via a set of joysticks. A prototype of such a hydraulic crane is depicted in Fig. 2.3. In the Robotics and Control Lab, we have developed innovative solutions to automate mobile hydraulics: novel models of spool dynamics, velocity observers with time-varying gains, and a set of nonlinear inversion-based controllers.

Experimental studies showed that these solutions significantly improve tracking performance, and our industrial colleagues highly appreciated the practical outcome of this researches. Scientific results of this project include velocity observers with time-varying gains [IJ25] and control design with input non-linearity inversion [IJ24].

2.6.3 Temperature regulation in rapid thermal processes applied to semiconductors manufacturing

This project was performed in 2014–2015 in collaboration with a semiconductor manufacturing company in Saint-Petersburg, Russia. The goal of the project was to provide accurate temperature regulation in rapid thermal processes applied to vapor deposition processing; the experimental testbed is depicted in Fig. 2.4. We have developed a system identification procedure to solve the problem and then successfully used a robust passivity-based controller. Our industrial collaborators appreciated the provided accuracy as a significant impact on the manufacturing process.

This project’s scientific outcome includes the robust passivity-based control design for a class of nonlinear systems [IJ27, IC36]. The project results were also a part of a Ph.D. thesis prepared under my supervision by Aleksandr Kapitonov. Aleksandr successfully defended the thesis in 2015.

3 Dynamic Regressor Extension and Mixing

This chapter presents my recent research activities on parameter estimation with improved performance and the dynamic regressor extension and mixing procedure. The chapter is organized as follows. Section 3.1 presents the context of adaptive parameter estimation and discusses the related performance improvement challenges. Section 3.2 provides the positioning of my results regarding state of the art, and in Section 3.3, I present the problem statement and describe some preliminary results on parameter estimation. Section 3.4 gives the general description of the proposed procedure and discusses its use for asymptotic parameter estimation. Excitation propagation in the proposed procedure is then discussed in Section 3.5. Section 3.6 presents further advances and briefly describes various applications where the proposed procedure was used. Finally, the concluding Section 3.7 summarizes the results, provides references to related topics not discussed in the chapter, and highlights open questions. For the sake of clarity of materials presentation, proofs are collected in the appendix Section 3.A.

The content of this chapter summarizes the results previously reported in [UR1, IJ3, IJ8, IJ15, IJ22, IC5, IC25], as given in Appendix B.

3.1 Context and Challenges

To effectively face the challenges of the modern world, we should think about systems that are not rigid but flexible, about strategies that can react to changing environment, adapt and learn from experience. A proper tool to achieve these aims is adaptation and experience-based learning. Adaptive control systems represent a mature field of research, where many results on stabilization, trajectory tracking, learning from trials, and parameter estimation are available. However, adaptive control is not as widespread in practical applications as it probably deserves. The main challenges for adaptive control precluding it from being ubiquitous are questionable transient performance, complicated tuning procedures, and sufficient informational richness requirements.

Poor **transient performance** is typically associated with slow transients, peaking phenomena, and transient oscillations. The peaking phenomena mean that control signals have magnitudes significantly higher during the adaptation phase than in nominal operation when the adaptation has finished. Such control signals cannot and should not be implemented in practice since they may damage the controlled plant or drive the system away from its nominal operation, making the used mathematical model, e.g., obtained by linearization around an equilibrium, invalid. As a result, even if the closed-loop system is proven to be globally stable for a nominal system model, the modeled behavior cannot be reproduced in practice due to the peaking phenomena yielding possible instabilities during the transient phase. The peaking often arises when one wants

to accelerate the convergence rate increasing controller gains; it restricts the possibility to obtain fast adaptation and parameter learning.

In the adaptation/learning phase, control signals may oscillate. These oscillations can cause undesirable resonances in the controlled plant that are not adequately addressed by simplified mathematical models and may cause damage or instability. Transient oscillations are also associated with frequent switches in control signals that are undesirable in many practical systems.

Tuning procedures for adaptive systems generally consist of choosing a gain matrix for adaptation or parameter learning algorithms. The general idea is that high gains provide fast transients with possible peaking, whereas low gains reduce peaking and oscillations, yielding a slow convergence rate; the gain choice is also related to the tracking/filtering trade-off. However, the exact tuning rules for multiple variables are usually complicated. Since adaptation and parameter learning algorithms act on several estimated/adapted variables simultaneously, there is a strong interconnection between these variables, and it is hardly possible to accelerate/decelerate the rate of convergence for a single variable without affecting the transient performance for others. Due to these interconnections, tuning procedures are typically time-consuming and include multiple trials.

Besides transient performance, adaptive control relies on the so-called **excitation requirement** imposed for parameter estimation. The intuitive idea is that to estimate parameters, one needs sufficiently rich measurements, and this requirement is translated to the rigorous mathematical definition of Persistency of Excitation. Unfortunately, such a requirement can be hard to verify in applications. As a result, a common practice is to inject an additional instrumental signal (also known as probing noise) to get sufficient excitation. This practical solution has drawbacks: the probing noise also affects the plant's performance output, and if the probing signal is removed, then the adaptation properties are lost. Moreover, such an artificial excitement procedure can be unacceptable for users. Thus, relaxation of excitation conditions and reducing the probing signal impact are of significant interest in adaptive control systems.

To summarize, many shortcomings of adaptation and parameter learning may be associated with their transients, tuning, and excitation requirements. In this context, **my objective was to develop a solution enhancing adaptive systems' performance.**

3.2 State of the Art and Positioning

The investigation of physical phenomena by mathematical modeling frequently leads to the problem of estimating model parameters. Indeed, the differential equations appearing in the model under consideration may contain parameters that are difficult to determine in advance. Through the years, numerous research works in several disciplines have been dedicated to this fundamental problem.

The linear regression equation (LRE) plays a central role in adaptive parameter estimation and adaptive control. It can be found in system identification [75], in model-reference adaptive control [20, 76] and adaptive pole-placement [77], in filtering and pre-

diction [78], in reinforcement learning [79], and other areas. We will formally introduce the LRE in Section 3.3, but in brief, it is given by

$$y(t) = \phi^\top(t)\theta,$$

where $y(t)$ is the output signal, $\phi(t)$ is the regressor vector, and $\theta \in \mathbb{R}^n$ is the vector of unknown constant parameters. The goal is to estimate θ using the measurements of y and ϕ .

There are plenty of methods to solve this problem offline, i.e., after the complete data collection on the process is finished [80]: least squares, maximum-likelihood estimation, Bayesian linear regression, principal component regression, to mention a few. However, in adaptive and learning systems, parameters need to be estimated online, i.e., progressively and simultaneously with the collection of new data points. There exist approaches for adaptive and online estimation [19, 20, 75, 76], where two traditional strategies are the least-squares method and the gradient estimator. The applicability conditions of these methods are based on the persistence of excitation [81, 82], and thus these approaches are implicitly based on asymptotic statistics.

In standard adaptive estimation, the persistence of excitation is necessary to establish global exponential convergence. To this end, in [83, 84], authors consider if, for the gradient estimator, this condition can be relaxed in the case of asymptotic but not exponential convergence. While some analytical results are reported, the resulting requirements are somewhat technical, and it is hard to use them in applications. To the best of our knowledge, there is no known necessary and sufficient condition to conclude asymptotic stability for the standard gradient estimator without the persistency of excitation assumption.

It is a challenging theoretical problem to develop a parameter estimation method relaxing the persistency of excitation, and many research works have been devoted to it in various scenarios. One attempt to alleviate this requirement in learning and parameter estimation is based on the idea of online historical data collection, e.g., [85, 86]. This idea yielded a family of methods that are nowadays known as concurrent [87–89], or, more recently, composite learning [90–93]. Within this methodology, a dynamic data stack is built to record online historical data discretely, and the convergence of parameter estimation is shown under the interval excitation condition, a weaker requirement than the persistence of excitation. However, whereas these methods can provide learning convergence under relaxed conditions, the transient performance is typically not addressed.

Regarding the problem of transient performance, standard results [20, 76] on adaptive parameter estimators claim that (again, under the persistence of excitation condition) there exists a weighted norm of the estimation errors that can be bounded by a decaying exponential, where the rate of convergence depends on excitation characteristics [94]. However, the authors of [82] proved that the gradient estimator’s gain amplification does not always produce accelerated transients, and the worst-case bound for the rate of convergence cannot be arbitrarily accelerated; moreover, such amplification provokes pikes augmentation.

Another disadvantage of these techniques is that despite the weak monotonicity of a weighted norm of the estimation errors, the estimation transients for each component of the vector θ may be rather unpredictable, presenting notable oscillations and peaking phenomena. Furthermore, usual tuning procedures for these estimators consist of the gain matrix adjusting, which can be delicate, involving many trial-and-error attempts.

The recent results on transient performance include perturbation-based methods, where a decaying perturbation is injected during the transients to increase excitation, as in [95], and parameters-resetting techniques as in [96]. The improved transient performance was also reported in [97, 98]. However, in [97], a (robotics) application-based parameter-dependent persistence of excitation was assumed, and in [98], a finite-time identification procedure was applied; it was not studied if these results can be extended to a general linear regression model.

Another approach to addressing the performance problem is the \mathcal{L}_1 -adaptive control [99, 100], claiming that by decoupling adaptation/learning and control loops, it is possible to obtain arbitrary fast transients. However, such acceleration also increases peaking. Nevertheless, it should be highlighted that these results consider only upper bounds and the rate of convergence; the problem of transient oscillations is not addressed.

Positioning

Summarizing, when the Dynamic Regressor Extension and Mixing (DREM) procedure was proposed in [101, 102], in adaptive estimation and learning, there was no available systematic procedure able to ensure monotonic peaking-free and oscillation-free transients and simultaneously provide transparent and straightforward tuning rules for the rate of convergence. By developing the DREM procedure described in the next section, I contributed to filling this lack and provided an efficient tool addressing these challenges.

Combined with the standard gradient estimator, see Section 3.3, the DREM procedure ensures the element-wise transient monotonicity preventing oscillations and peaking, independently of the excitation conditions. For each element of the vector θ , the estimate is tuned with a separate scalar gain, which does not affect transients for other elements, making the gain tuning procedure more straightforward and transparent. Moreover, in contrast to the standard gradient methods, the DREM procedure establishes a necessary and sufficient condition for asymptotic but not exponential convergence. Finally, the DREM procedure can be efficiently combined with various estimators, e.g., with a least-squares or fixed-time estimator.

3.3 Problem Statement and Preliminaries

This section presents some preliminary results. Section 3.3.1 introduces the linear regression equation and the problem of improved parameter estimation. Section 3.3.2 discusses various types of excitation, and Section 3.3.3 presents the standard gradient estimator, its convergence conditions and transient performance.

3.3.1 Linear Regression Equation

The linear regression equation (LRE) is given by

$$y(t) = \phi^\top(t)\theta + w(t), \quad (3.1)$$

where $y(t) \in \mathbb{R}^\ell$ is the output signal, $\phi(t) \in \mathbb{R}^{n \times \ell}$ is the regressor, $w(t) \in \mathbb{R}^\ell$ is an additive distortion, e.g., a measurement noise, and $\theta \in \mathbb{R}^n$ is the vector of unknown constant parameters. The signals y and ϕ are known, e.g., they are measured, and the distortion signal w is unknown; all signals are bounded.

The problem is to estimate the vector of parameters θ using the measurements of y and ϕ . We focus on *online* parameter estimation that is the standard for adaptive systems real-time recurrent processing of the arriving measurements, in contrast with the *offline* batch processing of the previously recorded data. Whereas numerous solutions to this problem are available, **the goal** is to develop a procedure yielding improved parameter estimation performance.

Equation (3.1) considers the general case of multiple outputs, i.e., the output y is a vector and the regressor ϕ is a matrix. However, in most references, LREs are traditionally defined with a scalar output and a vector regressor. In this chapter, if the dimension ℓ is not explicitly mentioned, we assume $\ell = 1$ to stay consistent with the standard notation.

In this chapter, we focus on the continuous-time systems and methods. However, equivalent results can be easily derived for discrete-time systems; see [103].

3.3.2 Persistent, Infinite, and Interval Excitation

Definition 3.1. A bounded signal $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}^{n \times \ell}$ is *persistently excited* if there exist $T > 0$ and $\mu > 0$ such that for all $t \in \mathbb{R}_+$,

$$\int_t^{t+T} \phi(s)\phi^\top(s)ds \geq \mu I_n.$$

We further denote this property as $\phi \in \text{PE}$, or ϕ is *PE*. To mention specific values of T and μ , we write (T, μ) -*PE*.

The Persistence of Excitation property and its connection with the exponential convergence in various estimation schemes are widely known. One relaxation of this condition is *interval excitation*, also referred to as *sufficient excitation*. This relaxation is used, e.g., in concurrent and composite learning algorithms [89].

Definition 3.2. A bounded signal $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}^{n \times \ell}$ is *excited on an interval*, or *sufficiently excited*, if there exist $t_1 \geq 0$, $T > 0$, and $\mu > 0$ such that

$$\int_{t_1}^{t_1+T} \phi(s)\phi^\top(s)ds \geq \mu I_n.$$

We further denote this property as ϕ is *IE*. To mention specific values of t_1 , T and μ , we write (t_1, T, μ) -*IE*.

The fundamental difference is that the persistence of excitation is uniform in time, whereas the interval excitation holds for the particular time interval starting at t_1 . If $t_1 = 0$, then the interval excitation is also called *initial excitation* [92].

Another relaxation of the PE property is the *infinite excitation*.

Definition 3.3. A bounded signal $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}^{n \times \ell}$ is *infinitely excited* if

$$\lim_{t \rightarrow \infty} \lambda_m \left(\int_0^t \phi(s) \phi^\top(s) ds \right) = \infty.$$

The relation between different types of excitation can be summarized as follows: any persistently excited signal is infinitely excited, and any infinitely excited signal is excited on an interval.

Example 3.1. In this example, we consider the excitation definitions above for some $\mathbb{R}_+ \rightarrow \mathbb{R}$ functions.

- The signal

$$x_1(t) = \begin{cases} 1 & \text{for } t \leq 1, \\ 0 & \text{otherwise} \end{cases}$$

is excited only on the interval $[0, 1]$. It is neither infinitely excited, nor PE.

- The signal $x_2(t) = e^{-\frac{1}{2}t}$ is excited on any interval, e.g., x_2 is $(0, T, 1 - e^{-T})$ -IE for any $T > 0$. However, $\int_0^\infty x_2^2(s) ds = 1 < \infty$ and x_2 is not infinitely excited. Moreover, x_2 tends to zero, and thus it is not PE.
- The signal $x_3(t) = \frac{1}{\sqrt{t+1}}$ is excited on any interval since it remains positive for all t . Moreover, $\int_0^\infty x_3^2(s) ds = \infty$, and x_3 is infinitely excited. However, x_3 tends to zero, and thus it is not PE.
- The signal $x_4(t) = \sin(t)$ is excited on any interval and it is infinitely excited. Moreover, it is straightforward to verify that x_4 is $(\pi, \frac{\pi}{2})$ -PE. \triangle

3.3.3 Gradient Estimator

The gradient estimator for the LRE (3.1) is given by

$$\dot{\hat{\theta}}(t) = \Gamma \phi(t) \left(y(t) - \phi^\top(t) \hat{\theta}(t) \right), \quad (3.2)$$

where $\hat{\theta}$ denotes the estimate of θ and $\Gamma > 0$ is the gain matrix. Define the estimation error

$$\tilde{\theta}(t) := \hat{\theta}(t) - \theta. \quad (3.3)$$

Then the error dynamics is given by

$$\dot{\tilde{\theta}}(t) = -\Gamma \phi(t) \phi^\top(t) \tilde{\theta}(t) + \Gamma \phi(t) w(t). \quad (3.4)$$

Convergence conditions

In the noise-free scenario, i.e., assuming $w \equiv 0$, the gradient estimator ensures exponential convergence to zero of the estimation error $\tilde{\theta}$ if and only if the regressor ϕ is PE; see [20, 104, 105]. Otherwise, for $w \neq 0$, the gradient estimator is input-to-state stable with respect to w .

For the case when the regressor ϕ is not PE, no necessary and sufficient convergence condition is available. In [83, 84], authors addressed the asymptotic (but not exponential) convergence of (3.4) for $w \equiv 0$ and $\Gamma = \gamma I$ with a positive scalar γ , and without the persistence of excitation. They derived several sufficient but not necessary and necessary but not sufficient conditions. Later, the authors of [106] summarized these conditions and analyzed the integral input-to-state stability of (3.4) with respect to the noise w .

However, the conditions discussed in [83, 84, 106] are rather technical and can hardly be applied in practice. For example, the sufficient condition for global asymptotic stability (GAS) of the origin of (3.4) for $w \equiv 0$ given in [84] can be summarized as follows.

Proposition 3.1 (see [84, 106]). *Assume that, for any $t_0 \in \mathbb{R}_+$, there exist sequences of positive numbers $\{t_k\}_{k=0}^\infty$, $\{l_k\}_{k=0}^\infty$, and $\{v_k\}_{k=0}^\infty$ such that for all $k \geq 0$,*

$$\begin{aligned} t_{k+1} &\geq t_k + l_k, \\ \lambda_m \left(\int_{t_k}^{t_k+l_k} \phi(s) \phi^\top(s) ds \right) &\geq v_k, \end{aligned} \quad (3.5)$$

and

$$\sum_{k=0}^{\infty} \frac{v_k}{\gamma^{-1} + \gamma \left(\int_{t_k}^{t_k+l_k} |\phi(s)|^2 ds \right)^2} = \infty. \quad (3.6)$$

Then the origin of (3.4) with $w \equiv 0$ and $\Gamma = \gamma I$ is GAS.

Whereas the condition (3.5) has a clear interpretation, namely the regressor ϕ remains excited with a possibly decaying excitation level, the condition (3.6) is very technical.

In [84], the authors also showed that in a general case, the infinite excitation condition (see Definition 3.3) is necessary but not sufficient to ensure the GAS; the authors proved this claim by constructing a counterexample.

It is worth noting that for ϕ being (T, μ) -PE, the conditions of Proposition 3.1 are satisfied. Indeed, setting $l_k \equiv T$, $t_k = t_0 + kT$, and $v_k \equiv \mu$, for all $k \geq 0$

$$\frac{v_k}{\gamma^{-1} + \gamma \left(\int_{t_k}^{t_k+l_k} |\phi(s)|^2 ds \right)^2} \geq \frac{\mu}{\gamma^{-1} + \gamma T^2 \|\phi\|_\infty^4},$$

and thus the infinite summation in (3.6) does not converge.

Transient performance and tuning

For the noise-free case, i.e., for $w \equiv 0$, it is straightforward to show that the weighted norm $\|\tilde{\theta}\|_{\Gamma^{-1}}$ is not increasing. Indeed, for $V(\tilde{\theta}) := \tilde{\theta}^\top \Gamma^{-1} \tilde{\theta}$, we obtain¹

$$\dot{V} = -\tilde{\theta}^\top \phi \phi^\top \tilde{\theta} \leq 0.$$

However, the monotonicity of the weighted norm does not imply that the element-wise transients are monotonic. In contrast, it is well-known that the element-wise transients of the estimation error vector $\tilde{\theta}$ generally oscillate and may have significant peaks. It happens due to the interconnections induced by the off-diagonal elements of the matrix $\phi \phi^\top$.

These interconnections also complicate the gain tuning procedure. The gain matrix Γ is often chosen as a diagonal matrix, $\Gamma = \text{diag}(\gamma_i)$, where $i \in \overline{1, n}$, and $\gamma_i > 0$ are scalar parameters. Such a choice is motivated by reducing the number of tuning parameters and the intuition that i -th gain mainly affects the i -th element. However, a change in γ_i also affects the transients for $\tilde{\theta}_j$, $j \neq i$, and these interconnections yield multiple trial-and-error tuning attempts.

Regarding the speed of convergence, a standard result is that if ϕ is PE, then the weighted norm $\|\tilde{\theta}\|_{\Gamma^{-1}}$ converges to zero exponentially fast, where the rate of convergence depends on the gain Γ and the excitation characteristics of ϕ ; see, e.g., [20, 76]. However, the exponential bound cannot be arbitrary accelerated. In [82], authors show that for ϕ being (T, μ) -PE and $\Gamma = \gamma I$, solutions of (3.4) satisfy for $t \geq t_0$

$$|\tilde{\theta}(t)| \leq \sqrt{\zeta} \|\phi\|_\infty \left(e^{-\frac{1}{2}\gamma\zeta^{-1}(t-t_0)} |\tilde{\theta}(t_0)| + \gamma\zeta^{-1} \|w\|_\infty \right), \quad (3.7)$$

where

$$\zeta := \gamma\eta^{-1} e^{2\eta T}, \quad \eta := -\frac{1}{2T} \ln \left(1 - \frac{\gamma\mu}{1 + \gamma^2 T^2 \|\phi\|_\infty^4} \right).$$

The relationship between the tuning gain γ and the rate of convergence of the exponential bound function is not straightforward, and an increase of γ may decrease the rate of convergence $\frac{1}{2}\gamma\zeta^{-1}$.

3.4 DREM Procedure in Asymptotic Parameter Estimation

In this Section, we describe the DREM procedure and how this procedure yields improved transient performance in parameter estimation. The materials of this section are organized as follows. First, in Section 3.4.1 we introduce the DREM procedure and present the DREM-enhanced gradient estimator. Section 3.4.2 discusses the transient performance and the convergence conditions of the DREM-enhanced estimator, and some illustrative examples are provided in Section 3.4.3. Finally, in Section 3.4.4, we discuss the use of the DREM procedure with the least-squares estimator.

To simplify the presentation, in this section we introduce the DREM procedure for the scalar output case, $\ell = 1$. The extension to the general case $\ell \geq 1$ is straightforward, and we discuss it in Section 3.5.

¹When clear from the context, in the sequel the argument of time may be omitted.

3.4.1 DREM Procedure Description

Two steps of the DREM procedure

The first step in the DREM procedure is the *dynamic regressor extension*, where the goal is to get $n - 1$ new linear regressor equations sharing the same vector of parameters θ . To this end, we introduce $n - 1$ linear, \mathcal{L}_∞ -stable operators $H_i : \mathcal{L}_\infty \rightarrow \mathcal{L}_\infty$, $i \in \overline{1, n - 1}$. The outputs of these operators applied to a signal $x : \mathbb{R}_+ \rightarrow \mathbb{R}$ are further denoted as

$$x_{f_i}(t) := H_i[x(t)].$$

For example, the operator H_i may be an exponentially stable linear time-invariant (LTI) filter,

$$H_i = \frac{\alpha_i}{p + \beta_i}, \quad (3.8)$$

where $p := \frac{d}{dt}$ and $\alpha_i > 0$, $\beta_i > 0$. Another possible choice is the delay operator,

$$H_i[x(t)] = \begin{cases} x(t - d_i) & \text{for } t \geq d_i, \\ 0 & \text{for } t < d_i, \end{cases}$$

where $d_i \in \mathbb{R}_+$.

We apply these operators to the LRE (3.1) to get the filtered regressions

$$y_{f_i}(t) = \phi_{f_i}^\top(t)\theta + w_{f_i}(t).$$

Remark 3.1. For some choices of H_i , e.g., for an LTI filter with nonzero initial conditions, the exponentially decaying term ϵ_i may appear in the filtered regression,

$$y_{f_i}(t) = \phi_{f_i}^\top(t)\theta + w_{f_i}(t) + \epsilon_i(t).$$

To simplify the presentation, we omit these terms in the sequel, and we incorporate them in the analysis when necessary.

Combining the original LRE (3.1) with the $n - 1$ filtered regressions, we construct the **extended LRE**

$$Y(t) = \Phi(t)\theta + W(t), \quad (3.9)$$

where we define $Y : \mathbb{R}_+ \rightarrow \mathbb{R}^n$, $\Phi : \mathbb{R}_+ \rightarrow \mathbb{R}^{n \times n}$, and $W : \mathbb{R}_+ \rightarrow \mathbb{R}^n$ as

$$Y := \begin{bmatrix} y \\ y_{f_1} \\ \vdots \\ y_{f_{n-1}} \end{bmatrix}, \quad \Phi := \begin{bmatrix} \phi^\top \\ \phi_{f_1}^\top \\ \vdots \\ \phi_{f_{n-1}}^\top \end{bmatrix}, \quad W := \begin{bmatrix} w \\ w_{f_1} \\ \vdots \\ w_{f_{n-1}} \end{bmatrix}. \quad (3.10)$$

Note that Y , Φ , and W are bounded due to the \mathcal{L}_∞ -stability assumption on H_i .

The second step of the DREM procedure is the *mixing*. Recall that for any square possibly singular matrix A and its adjugate matrix $\text{adj}(A)$, it holds

$$\text{adj}(A)A = \det(A)I.$$

At this step, we multiply the extended LRE (3.9) by the adjugate matrix of $\Phi(t)$ on the left. Defining

$$\begin{aligned}\mathcal{Y}(t) &:= \text{adj}(\Phi(t))Y(t), \\ \mathcal{W}(t) &:= \text{adj}(\Phi(t))W(t),\end{aligned}\tag{3.11}$$

we get

$$\mathcal{Y}_i(t) = \Delta(t)\theta_i + \mathcal{W}_i(t),\tag{3.12}$$

where $i \in \overline{1, n}$, and we define the scalar function $\Delta : \mathbb{R}_+ \rightarrow \mathbb{R}$ as

$$\Delta(t) := \det(\Phi(t)).\tag{3.13}$$

The set of n scalar LREs (3.12) sharing the same bounded scalar regressor Δ is the outcome of the DREM procedure. Thus, the DREM procedure is a nonlinear transformation that converts the original LRE (3.1) with the vector θ containing n unknown parameters to the set of n scalar LREs (3.12) for each element θ_i separately, where the new regressor Δ is the determinant of the extended matrix Φ .

Parameter estimation and convergence properties

We can θ_i in the scalar regression (3.12) by applying the gradient estimator (3.2). It yields, for $i \in \overline{1, n}$, the DREM-enhanced gradient estimator

$$\dot{\hat{\theta}}_i(t) = \gamma_i \Delta(t) \left(\mathcal{Y}_i(t) - \Delta(t)\hat{\theta}_i(t) \right),\tag{3.14}$$

where $\gamma_i > 0$ is a scalar tuning parameter. The estimation error dynamics is then

$$\dot{\tilde{\theta}}_i(t) = -\gamma_i \Delta^2(t)\tilde{\theta}_i(t) + \gamma_i \Delta(t)\mathcal{W}_i(t),\tag{3.15}$$

where $\tilde{\theta}$ was defined in (3.3).

Consider first the noise-free case, i.e., $w \equiv 0$ implying that W and \mathcal{W} are identically zeros as well. Then the LRE (3.1) takes the form

$$y(t) = \phi^\top(t)\theta,\tag{3.16}$$

and for $i \in \overline{1, n}$, the scalar LREs (3.12) are

$$\mathcal{Y}_i(t) = \Delta(t)\theta_i.\tag{3.17}$$

The derivations above allow for establishing the following proposition (see [102]).

Proposition 3.2. *Consider the n -dimensional linear regression equation (3.1) with $\ell = 1$, $w \equiv 0$, where $y : \mathbb{R}_+ \rightarrow \mathbb{R}$ and $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}^n$ are known, bounded functions of time and $\theta \in \mathbb{R}^n$ is the vector of unknown parameters. Introduce $n-1$ linear, \mathcal{L}_∞ -stable operators $H_i : \mathcal{L}_\infty \rightarrow \mathcal{L}_\infty$, $i \in \overline{1, n-1}$. Define the vector Y and the matrix Φ as given in (3.10). Consider the estimator (3.14) with \mathcal{Y}_i and Δ defined in (3.11) and (3.13), respectively. Then the following properties hold for all $i \in \overline{1, n}$:*

P1: the estimation error $\tilde{\theta}_i$ converges to zero asymptotically if and only if Δ is a non-square-integrable function,

$$\Delta \notin \mathcal{L}_2 \Leftrightarrow \lim_{t \rightarrow \infty} \tilde{\theta}_i(t) = 0; \quad (3.18)$$

P2: the estimation error $\tilde{\theta}_i$ converges to zero exponentially fast if and only if Δ is PE;

P3: (the element-wise monotonicity) the transients $\tilde{\theta}_i$ are monotonic, i.e., for all $t_a \leq t_b$ it holds

$$|\tilde{\theta}_i(t_a)| \leq |\tilde{\theta}_i(t_b)|;$$

P4: (the element-wise tuning) variations in the gain γ_i affect the transients for $\tilde{\theta}_i$ only.

The proof of Proposition 3.2 is straightforward. For $w \equiv 0$, (3.15) becomes

$$\dot{\tilde{\theta}}_i(t) = -\gamma_i \Delta^2(t) \tilde{\theta}_i(t). \quad (3.19)$$

The exponential convergence of the linear time-varying (LTV) system (3.19) for a persistently excited Δ , i.e., property P2, is well-known; see Section 3.3. Properties P1, P3, and P4 follow directly from the solution of (3.19) given by

$$\tilde{\theta}_i(t) = e^{-\gamma_i \int_0^t \Delta^2(\tau) d\tau} \tilde{\theta}_i(0).$$

Regarding Remark 3.1, if we consider the possible presence of the exponentially decaying terms ϵ_i due to some choices of H_i , the error equation (3.19) becomes

$$\dot{\tilde{\theta}}_i(t) = -\gamma_i \Delta^2(t) \tilde{\theta}_i(t) + \epsilon_t,$$

where ϵ_t is a generic exponentially decaying term. When Δ is PE, $\tilde{\theta}_i$ converges exponentially despite ϵ_t . To prove (3.18) and establish the asymptotic (but not exponential) convergence for a non-square-integrable Δ , we note that a bounded exponentially decaying term ϵ_t is absolutely integrable, $\epsilon_t \in \mathcal{L}_1$, and apply the following Lemma (see [107]).

Lemma 3.1. *Consider the scalar system defined by*

$$\dot{x}(t) = -a^2(t)x(t) + b(t), \quad (3.20)$$

where $x(t) \in \mathbb{R}$, $a, b : \mathbb{R}_+ \rightarrow \mathbb{R}$ are piecewise continuous bounded functions, $x(t_0) = x_0$. If $a \notin \mathcal{L}_2$ and $b \in \mathcal{L}_1$ then

$$\lim_{t \rightarrow \infty} x(t) = 0. \quad (3.21)$$

The proof of Lemma 3.1 is given in Section 3.A.1.

Concerning the case $w \neq 0$, the estimator (3.14) is input-to-state stable with respect to \mathcal{W}_i if $\Delta \in \text{PE}$, which is a similar result as for the standard gradient estimator; see Section 3.3. Moreover, as it is shown in [108], if $\mathcal{W}_i \in \mathcal{L}_2$ and $\Delta \notin \mathcal{L}_2$, then $\tilde{\theta}_i$ is bounded.

3.4.2 Transients and Convergence

The improved tuning and transient performance

Two key features of the DREM procedure for the transient performance are properties P3 and P4 of Proposition 3.2. The element-wise monotonicity provides performance guarantees on the parameter estimation transients, excluding the peaking and oscillations. Even if the noise-induced components \mathcal{W}_i can deteriorate the monotonicity, these distortions are typically smaller than those induced by interconnections between elements of $\tilde{\theta}$. Another corollary is that the parameter estimate $\hat{\theta}_i$ crosses zero not more than once, up to the noise-induced component \mathcal{W}_i ; in [109], this property was used to relax the high-frequency gain sign assumption in model reference adaptive control.

The element-wise tuning property is also of great importance as it significantly simplifies the tuning procedure and allows adjusting the transient rate for a specific estimated parameter without affecting others. Such a property is valuable, e.g., for time-scale separation or filtering/tracking trade-off tuning for an individual parameter. Note that the DREM procedure allows estimating the separate component θ_i only instead of the whole parameter vector θ . In such a case, the computation of the adjoint matrix $\text{adj}(\Phi)$ can be avoided, and the elements \mathcal{Y}_i in (3.12) can be computed using Cramer's rule as

$$\mathcal{Y}_i(t) = \det(\Phi_{Y,i}(t)),$$

where $\Phi_{Y,i}$ is the matrix Φ of the extended LRE (3.9), where the i -th column is replaced by the vector Y , and $i \in \overline{1, n}$.

The novel convergence condition

The DREM procedure introduces the new convergence condition in (3.18), namely $\Delta \notin \mathcal{L}_2$. Two natural questions arise at this point.

- Q1. Can the condition $\Delta \notin \mathcal{L}_2$ hold when ϕ is not PE?
- Q2. Can a poor choice of operators H_i compromise the convergence, i.e., to yield $\Delta \in \mathcal{L}_2$ for $\phi \in \text{PE}$?

First, recall that $\Delta \notin \mathcal{L}_2$ is a weaker condition than $\Delta \in \text{PE}$. Following the definitions introduced in Section 3.3, the condition $\Delta \notin \mathcal{L}_2$ corresponds to the infinite excitation in the sense of Definition 3.3, which is weaker than the persistency of excitation. As shown in Example 3.1, $\Delta(t) = \frac{1}{\sqrt{t+1}}$ is infinitely excited, $\Delta \notin \mathcal{L}_2$, but such a signal is not persistently excited, $\Delta \notin \text{PE}$.

Answer to Q1. The answer to question Q1 is positive, as we will show with the following example (see [102]).

Example 3.2. Define $g : \mathbb{R}_+ \rightarrow \mathbb{R}$ as

$$g(t) := \frac{\sin(t)}{\sqrt{t+1}},$$

and choose the regressor $\phi = [1 \quad g + \dot{g}]^\top$. Since both g and \dot{g} tend to zero as t tends to infinity, the second element of the regressor ϕ decays, and ϕ is not persistently excited. Choose now

$$H_1 = \frac{1}{p+1}.$$

Following the DREM procedure, define $\phi_{f_1} = H_1[\phi]$. Then for the second element of ϕ_{f_1} , which is given by $\phi_{f_1,2} = H[g + \dot{g}]$, it holds

$$\dot{\phi}_{f_1,2} = -\phi_{f_1,2} + g + \dot{g}.$$

It follows then that

$$\frac{d}{dt}(\phi_{f_1,2} - g) = -(\phi_{f_1,2} - g),$$

and $|\phi_{f_1,2} - g| \rightarrow 0$ exponentially as t tends to infinity. Neglecting the exponentially decaying terms and considering the steady-state behavior, the extended regressor matrix Φ defined in (3.10) is given by

$$\Phi = \begin{bmatrix} 1 & g + \dot{g} \\ \phi_{f_1,1} & \phi_{f_1,2} \end{bmatrix} = \begin{bmatrix} 1 & g + \dot{g} \\ 1 & g \end{bmatrix},$$

and $\Delta = \det(\Phi) = -\dot{g}$. The time derivative of g is

$$\dot{g} = \frac{\cos(t)}{\sqrt{t+1}} - \frac{\sin(t)}{2\sqrt{(t+1)^3}}.$$

Since \dot{g} goes to zero as t goes to infinity, the novel regressor Δ is not persistently excited. However, it is straightforward to check that

$$\lim_{s \rightarrow \infty} \int_0^s \left(\frac{\cos(t)}{\sqrt{t+1}} - \frac{\sin(t)}{2\sqrt{(t+1)^3}} \right)^2 dt = \infty.$$

Thus, the novel regressor $\Delta \notin \mathcal{L}_2$ and the DREM-enhanced gradient estimator (3.14) converges asymptotically. \triangle

Answer to Q2. The answer to the question Q2 is also yes, and a poor choice of the operators H can compromise the convergence. We illustrate it with the following example.

Example 3.3. Consider the regressor $\phi(t) = [\sin(t) \quad \cos(t)]^\top$ and choose

$$H_1 = \frac{c(p+1)}{p^2 + p + 2},$$

where $c > 0$. Note that for the unit frequency, the operator H_1 provides the zero phase shift and the magnitude gain c . Thus, in the steady-state, $\phi_{f_1}(t) = c \sin(t)$, $\phi_{f_2}(t) = c \cos(t)$, and

$$\Phi(t) = \begin{bmatrix} \sin(t) & \cos(t) \\ c \sin(t) & c \cos(t) \end{bmatrix}.$$

Obviously, $\phi \in \text{PE}$, but $\det(\Phi) = 0$ and $\Delta \in \mathcal{L}_2$.

On the other hand, for $\phi(t) = [\sin(\sqrt{3}t) \ \cos(\sqrt{3}t)]^\top$, in the steady-state we have

$$\Phi(t) = \begin{bmatrix} \sin(\sqrt{3}t) & \cos(\sqrt{3}t) \\ c \sin(\sqrt{3}t - \frac{\pi}{3}) & c \cos(\sqrt{3}t - \frac{\pi}{3}) \end{bmatrix},$$

and $\det(\Phi) = \frac{c\sqrt{3}}{2}$, thus $\Delta \in \text{PE}$. \triangle

To summarize, the examples above illustrate that the following scenarios are possible:

- PE yields PE, i.e., the original regressor ϕ is PE, and the novel regressor Δ is PE;
- PE yields poor excitation, i.e., the original regressor ϕ is PE, but due to the poor choice of H_i , the novel regressor is not infinitely excited, $\Delta \in \mathcal{L}_2$;
- no PE yields GAS, i.e., the original regressor ϕ is not PE, but the novel regressor is infinitely excited, $\Delta \notin \mathcal{L}_2$;
- no PE yields no excitation, i.e., the original regressor ϕ is not PE, and the novel regressor is not infinitely excited, $\Delta \in \mathcal{L}_2$.

It is not yet clear if the DREM procedure requires *less excitation* than the standard (vector) gradient estimator, i.e., if the DREM-enhanced estimator (3.14) converges when the gradient estimator (3.2) does not; this question remains open due to the complicated non-exponential convergence conditions of (3.2) discussed in Section 3.3. However, the new *necessary and sufficient* asymptotic convergence condition $\Delta \notin \mathcal{L}_2$ is more transparent, and illustrative simulations in Section 3.4.3 demonstrate the better behavior of the DREM-enhanced estimator (3.14) in the $\phi \notin \text{PE}$ and $\Delta \notin \mathcal{L}_2$ scenario.

The operators H_i are crucial components of the DREM procedure. Whereas this degree of freedom can be used, e.g., to attenuate the measurement noise w , a poor choice can compromise the convergence generating a poor novel regressor Δ from a persistently excited original regressor ϕ . Thus, the resulting excitation should be studied when the DREM procedure is applied. We will alleviate this shortcoming in Section 3.5 by proposing a dynamic extension method that guarantees preservation of the original excitation.

3.4.3 Illustrative Examples

Consider the noise-free linear regression (3.16), namely

$$y(t) = \phi^\top(t)\theta,$$

where we recall that y and ϕ are known and θ is the vector of unknown parameters. Below, we use the gradient estimator (3.2) to estimate θ in (3.16), and apply the DREM procedure and the DREM-enhanced estimator (3.14) to improve the performance.

Persistently excited regressor

Consider first the case when both the original regressor ϕ and the new regressor Δ are persistently excited. Choose

$$\phi(t) = \begin{bmatrix} 1 & \sin(t) & \cos(t) \end{bmatrix}^\top. \quad (3.22)$$

Such a regressor often appears in adaptive disturbance estimation and attenuation applications. It is straightforward to show that ϕ is $(2\pi, \pi)$ -PE, i.e., for all $t \geq 0$,

$$\int_t^{t+2\pi} \phi(s)\phi^\top(s)ds \geq \pi I.$$

To apply the DREM procedure, we perform the following two steps.

Step 1. At this step, we perform the dynamic regressor extension. For $\phi(t) \in \mathbb{R}^3$, we have to introduce two operators, H_1 and H_2 . Choose H_1 as the delay,

$$H_1[x(t)] = \begin{cases} x(t-1) & \text{for } t \geq 1, \\ 0 & \text{for } t < 1, \end{cases}$$

and H_2 as the LTI filter,

$$H_2 = \frac{2}{p+1}.$$

For $t \geq 1$ and neglecting exponentially decaying terms, the extended regressor matrix Φ defined in (3.10) is given by

$$\Phi(t) = \begin{bmatrix} \phi^\top(t) \\ H_1[\phi^\top(t)] \\ H_2[\phi^\top(t)] \end{bmatrix} = \begin{bmatrix} 1 & \sin(t) & \cos(t) \\ 1 & \sin(t-1) & \cos(t-1) \\ 2 & \sqrt{2}\sin(t-\frac{\pi}{4}) & \sqrt{2}\cos(t-\frac{\pi}{4}) \end{bmatrix}.$$

Then $\det(\Phi(t)) = \cos(1) + \sin(1) - 1$, and the new regressor $\Delta = \det(\Phi)$ is also PE.

Step 2. At this step, we perform the mixing and construct new scalar LREs. Following the DREM procedure, we define the extended output vector

$$Y(t) = \begin{bmatrix} y(t) & H_1[y(t)] & H_2[y(t)] \end{bmatrix}^\top$$

and compute $\mathcal{Y} = \text{adj}(\Phi(t))Y(t)$. It yields the scalar equations (3.17) for $i = 1, 2, 3$, namely

$$\mathcal{Y}_i(t) = \Delta(t)\theta_i.$$

We are now in the position to present numerical simulations of the standard gradient estimator (3.2) applied to (3.16), and the DREM-enhanced estimator (3.14) applied to (3.17).

Choose the unknown parameter vector as $\theta = \begin{bmatrix} 1 & -1 & 2 \end{bmatrix}^\top$ and set the initial estimates $\hat{\theta}(0) = 0$ for all estimators. Consider the estimator (3.2) with $\Gamma = I$. The transients of the estimation error $\tilde{\theta}$ are given in Fig. 3.1a, where we can approximate

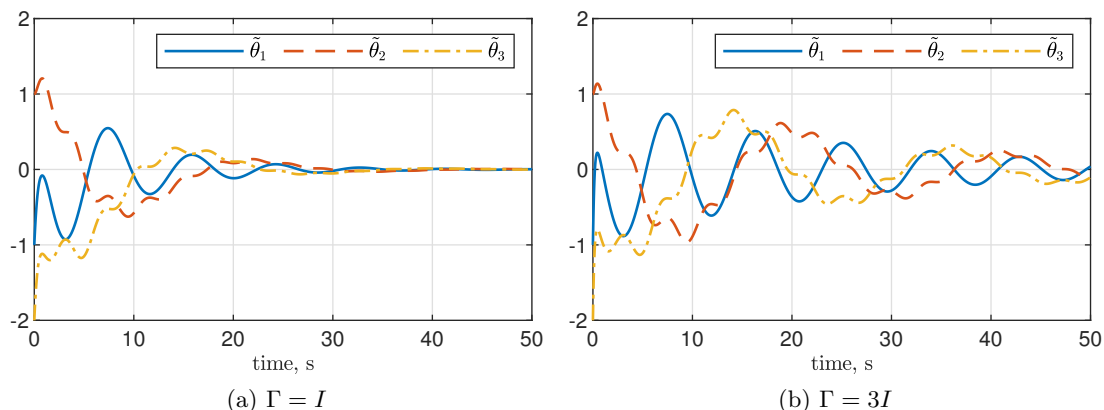


Figure 3.1: The estimation error $\tilde{\theta}$ of the gradient estimator (3.2); ϕ is given by (3.22) and persistently excited.

the transient time as 30 seconds. Suppose our goal is to have the transient time approximately equal to 10 seconds, i.e., we want to accelerate the transients. The direct gain adjustment as $\Gamma = 3I$ does not yield the desired result (see Fig. 3.1b). This adjustment makes the transients worse, increasing both the transient time and oscillations. Probably, for a more general structure of the gain matrix Γ , we can get better performance via the time-consuming trial-and-error tuning.

Let us now consider the DREM-enhanced estimator (3.14), where $\gamma_i = 1$, $i = 1, 2, 3$. The transients are given in Fig. 3.2a, where the initial absence of the response is due to the delay operator H_1 and the transient time of the LTI operator H_2 . The length of this interval depends on the choice of operators. In this example, we have chosen the operators H_1, H_2 to get approximately the same time of response for unit gains, i.e., 30 seconds, as for the gradient estimator (3.2) in Fig. 3.1a.

However, having a comparable speed of response, the DREM procedure provides monotonic estimates with no oscillations. Consider now the straightforward gains adjustment $\gamma_i = 3$ for $i = 1, 2, 3$. The resulting estimates are given in Fig. 3.2b, where the transient time is approximately 10 seconds. This example illustrates the transparent and simple tuning of the DREM-enhanced estimator; moreover, the accelerated transients remain monotonic.

Next, we consider the noised scenario, i.e., we consider the LRE (3.1) instead of (3.16) and choose w as a bounded white noise with the zero mean and the standard deviation equal to 1; note that in the noise-free case, $y(t)$ varies between -2 and 4 , and the considered additive noise is not negligible. Being applied to (3.1), the DREM procedure yields now (3.12) instead of (3.17). The plots of the estimation error $\tilde{\theta}$ for the gradient estimator (3.2) and the DREM-enhanced estimator (3.14) are depicted in Fig. 3.3a and Fig. 3.3b, respectively. The simulation results illustrate the comparable noise sensitivity of the estimators for the unit gains.

For a persistently excited regressor, both the standard gradient and the DREM-

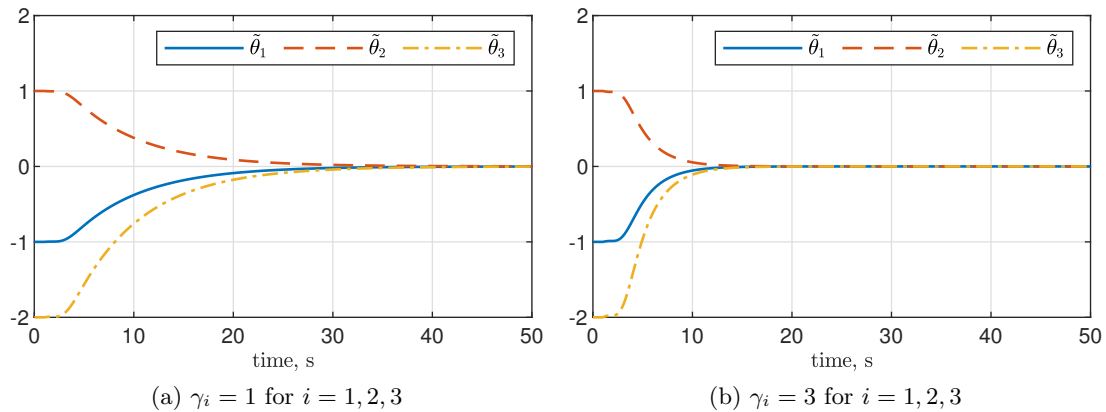


Figure 3.2: The estimation error $\tilde{\theta}$ of the DREM-enhanced estimator (3.14); Δ is persistently excited.

Tuning parameters, Γ and γ_i , $i = 1, 2, 3$	The gradient estimator (3.2)	The DREM-enhanced estimator (3.14)
$\Gamma = \frac{1}{2}I$ and $\gamma_i = \frac{1}{2}$	0.82	0.19
$\Gamma = I$ and $\gamma_i = 1$	1.68	0.40
$\Gamma = 3I$ and $\gamma_i = 3$	5.06	1.60
$\Gamma = 10I$ and $\gamma_i = 10$	16.26	8.81

Table 3.1: Mean squared norm of the estimation error $\text{MSE}(\tilde{\theta})$, $t_f = 1000$ and $L = 100$, for the noised LRE (3.1).

enhanced estimators are input-to-state stable with respect to the additive noise. Whereas the ISS gain can be estimated via the Lyapunov analysis, see, e.g., [82] and the inequality (3.7), this gain can be conservative. Below we illustrate the sensitivity of the DREM-enhanced estimator to the additive noise for various tuning parameters and compare it with the gradient estimator. To separate the transients from the noise, we perform numerical simulations for a sufficiently large time t_f seconds and then compute the mean squared norm of the estimation error $\tilde{\theta}$ over the last L seconds,

$$\text{MSE}(\tilde{\theta}) := \frac{1}{L} \int_{t_f-L}^{t_f} |\tilde{\theta}(s)|^2 ds.$$

For the considered experiment, we choose $t_f = 1000$ and $L = 100$. The numerical simulation results are given in Table 3.1 and illustrate the filtering behavior of the DREM procedure.

Finally, it is worth noting that the DREM procedure allows for the tracking/filtering trade-off tuning for the specific parameter estimate $\hat{\theta}_i$. It can be achieved by adjusting the specific gain γ_i , without affecting the other estimates' transients. To illustrate this

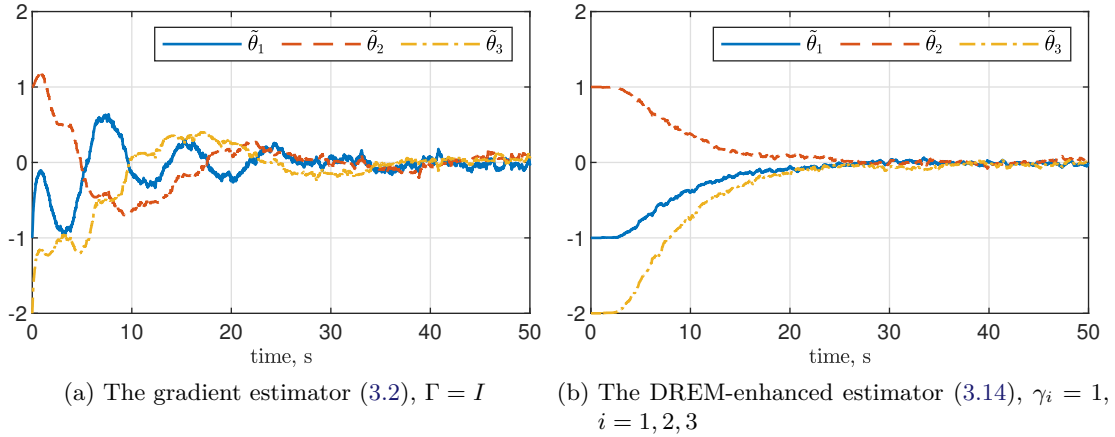


Figure 3.3: The estimation error $\tilde{\theta}$ in the presence of noise.

property, consider the DREM-enhanced estimator (3.14) applied to the noised LRE (3.12) with the same noise as in the example above, and set $\gamma_1 = \gamma_3 = 1$ and $\gamma_2 = 5$. Simulation results are given in Fig. 3.4. Whereas $\tilde{\theta}_1$ and $\tilde{\theta}_3$ behave as in the previous example, compare with Fig. 3.3, the estimate of θ_2 has a faster response (tracking) but is more sensitive to noise (filtering).

Decaying regressor

Consider the noise-free LRE (3.16), where we choose such a regressor ϕ that it is not PE but can generate a new infinitely excited regressor $\Delta \notin \mathcal{L}_2$. To this end, we recall Example 3.2 and choose $\phi = [1 \quad g + \dot{g}]^\top$, where

$$g(t) = \frac{\sin(t)}{\sqrt{t+1}}.$$

The gradient estimator (3.2) is stable, and the estimation error $\tilde{\theta}$ remains bounded. However, it is not exponentially converging since $\phi(t) \notin \text{PE}$, and PE is a necessary condition for exponential stability; moreover, we cannot conclude if the gradient estimator is asymptotically converging. The considered regressor ϕ is infinitely excited in the sense of Definition 3.3, i.e.,

$$\lim_{t \rightarrow \infty} \lambda_m \left(\int_0^t \begin{bmatrix} 1 & g(s) + \dot{g}(s) \\ g(s) + \dot{g}(s) & (g(s) + \dot{g}(s))^2 \end{bmatrix} ds \right) = \infty,$$

and thus it satisfies the necessary condition for GAS. However, it is unclear if the regressor ϕ satisfies the sufficient condition given in Proposition 3.1.

Choose $\theta = [1 \quad -1]^\top$ and $\hat{\theta}(0) = 0$. The transient behavior of the estimation error $\tilde{\theta}(t)$ for the gradient estimator (3.2) is shown in Fig. 3.5; the plots show that convergence

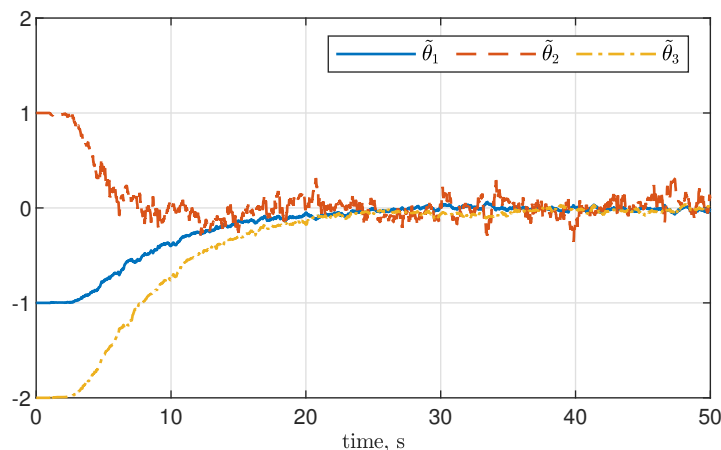


Figure 3.4: The tracking/filtering trade-off tuning; the tuning coefficients are $\gamma_1 = \gamma_3 = 1$, $\gamma_2 = 5$.

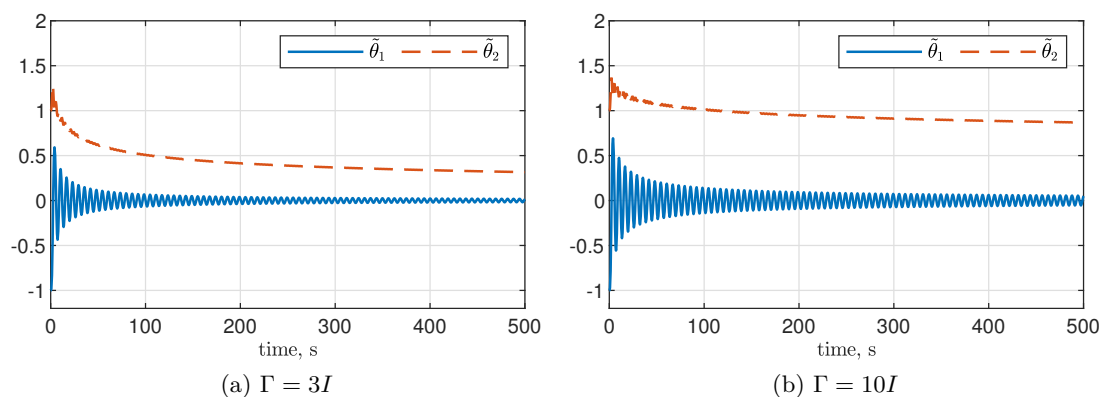


Figure 3.5: The estimation error $\tilde{\theta}$ of the gradient estimator (3.2) for $\phi \notin \text{PE}$.

has not been achieved even after a reasonably long period of 500 seconds, and the gain increase does not accelerate the convergence.

Following Example 3.2, we apply the DREM procedure with $H_1 = \frac{1}{p+1}$; then, the new regressor Δ satisfies $\Delta \notin \mathcal{L}_2$ ensuring the asymptotic convergence. The transient behavior of the estimation error $\tilde{\theta}$ for the DREM-enhanced estimator (3.14) is shown in Fig. 3.6. It illustrates the asymptotic (but not exponential) convergence and the impact of the tuning gains.

3.4.4 DREM with the Least-Squares Estimator

The least-squares estimator

In this chapter, we primarily focus on the gradient-descent type of estimators, namely the estimator (3.2). However, besides the gradient estimators, another widely used method

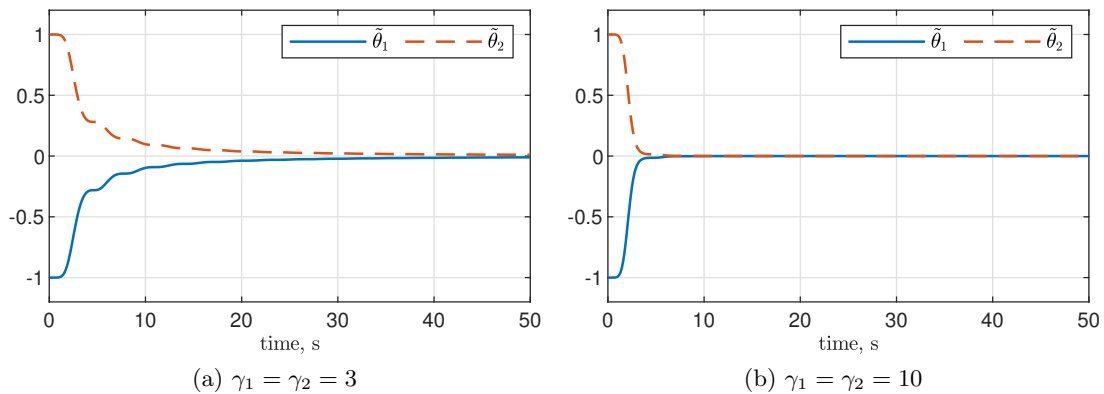


Figure 3.6: The estimation error $\tilde{\theta}$ of the DREM-enhanced estimator (3.14) for $\phi \notin \text{PE}$ and $\Delta \notin \mathcal{L}_2$.

for adaptive parameter estimation is the least-squares (LS) approach; see [20, 76, 110]. For the continuous-time linear regression model (3.1), the LS estimator is given by

$$\begin{aligned} \frac{d}{dt}\hat{\theta}(t) &= \Gamma P(t)\phi(t) \left(y(t) - \phi^\top(t)\hat{\theta}(t) \right), \\ \frac{d}{dt}P(t) &= \Gamma \left(\lambda P - P(t)\phi(t)\phi^\top(t)P(t) \right), \end{aligned} \quad (3.23)$$

where $P(0) = P_0 > 0$, $\Gamma > 0$, and $\lambda \geq 0$ are the design parameters. Here, λ is the exponential forgetting factor, where $\lambda = 0$ implies no forgetting, and $\lambda > 0$ yields the exponential forgetting of the past measurements. In contrast with the gradient estimator minimizing the instantaneous estimation error, the LS estimator minimizes the integral cost. As a result, if the LS estimator does not perform any forgetting, $\lambda = 0$, it loses the alertness, i.e., it becomes incapable of tracking variations of the parameters θ . In other words, for $\lambda = 0$, the absolute priority in the tracking/filtering trade-off is given to the filtering. Due to the loss of alertness, the no-forgetting LS estimators are not typically used in adaptive online parameters estimation, and the exponentially forgetting LS estimators are preferred. It is also worth noting that besides the exponential forgetting, there exist other methods to deal with the loss of alertness: covariance resetting, constant-trace, and other algorithms [19, 20]. However, in this section, we consider only the LS estimator with the constant forgetting factor.

The LS estimator typically has better transient performance than the gradient estimator: the transients are faster and less oscillating, and the noise attenuation is better [19]. However, the LS estimator provides neither element-wise monotonicity nor the transparent tuning recommendations for the gain Γ . The LS estimator ensures the exponential convergence if the regressor ϕ is persistently excited [20]. The drawback of the LS estimator is that for ϕ not being PE, the nonlinear dynamics of P in (3.23) can be unstable, yielding the unboundedness of P and excessive estimator's sensitivity. This effect is known as the covariance "wind-up." To deal with the wind-up, many solutions propose

to saturate, in a certain way, or reset the matrix P . Another interesting approach is the “directional forgetting” discussed in [111, 112]. The key idea is that the regressor ϕ may contain information in specific directions, i.e., for some elements of the vector θ , and this information can be incorporated in the matrix P update law. However, this approach is mainly developed for the discrete-time recursive LS estimators, and we do not address it here; this question is also discussed in Section 3.7.3.

The LS estimator in the DREM procedure

The DREM procedure renders the LRE (3.1) to the set of scalar LREs (3.12). Within this context, the LS estimator (3.23) takes the element-wise form, for $i \in \overline{1, n}$,

$$\dot{\hat{\theta}}_i(t) = \gamma_i \Delta(t) p_i(t) \left(y_i(t) - \Delta(t) \hat{\theta}_i(t) \right), \quad (3.24)$$

$$\dot{p}_i(t) = \gamma_i \left(\lambda_i p_i(t) - p_i^2(t) \Delta^2(t) \right), \quad (3.25)$$

where $p_i(0) > 0$ and $\gamma_i > 0$ are the design parameters, and $\lambda_i \geq 0$ is the element-wise forgetting factor. In the noise-free case, the error dynamics under the LS estimator is given by

$$\dot{\tilde{\theta}}_i(t) = -\gamma_i p_i(t) \Delta^2(t) \tilde{\theta}_i(t). \quad (3.26)$$

From (3.25), it follows that $p(t)$ remains nonnegative for $t \geq 0$. Together with the estimation error dynamics (3.26), it implies that the DREM-enhanced element-wise LS estimator has the same transient improvements:

- the transients of $\tilde{\theta}$ are element-wise monotonic;
- the coefficients γ_i and λ_i have the element-wise effect making the overall tuning simpler and clearer.

Let us now discuss the convergence conditions of the the element-wise LS estimator (3.24), (3.25). As well as (3.23), it converges exponentially when Δ is PE. However, it is of interest to study the convergence conditions for the DREM-specific case when Δ is not PE but belongs (or not) to \mathcal{L}_2 . Recall that, in the noise-free scenario, the gradient estimator (3.14) converges exponentially when $\Delta \in \text{PE}$, asymptotically when $\Delta \notin \mathcal{L}_2$, and ensures the boundedness of $\hat{\theta}$ when $\Delta \in \mathcal{L}_2$.

For the scalar equation (3.25), we can derive and analyze the exact solution $p_i(t)$. This analysis allows us establishing convergence properties of the element-wise LS estimator in the context of the DREM procedure, which are summarized in the following proposition (see [113]).

Proposition 3.3. *Let $i \in \overline{1, n}$. Consider the estimation algorithm (3.24), (3.25) with $p_i(0) > 0$ and $\gamma_i > 0$. The claims are:*

- (i) *If $\lambda_i = 0$ (the LS estimator without forgetting) then*
 - (C1) *if $\Delta \in \mathcal{L}_2$ then for all nonzero $\tilde{\theta}_i(0)$ the signal $\tilde{\theta}_i$ does not converge to zero;*

(C2) if $\Delta \notin \mathcal{L}_2$ then $\tilde{\theta}_i$ is monotonic and converges to zero asymptotically;

(C3) if Δ is PE, it does not imply the exponential convergence.

(ii) If $\lambda_i > 0$ (the LS estimator with forgetting) then

(C4) p_i is bound from below;

(C5) if $\Delta \in \mathcal{L}_2$ or

$$\Delta \notin \mathcal{L}_2 \text{ and } \Delta \rightarrow 0,$$

then the estimator is unstable and p_i tends to infinity;

(C6) if Δ is PE then p_i is bounded, $\tilde{\theta}_i$ is monotonic and converges to zero exponentially fast.

Whereas the LS estimator's behavior for $\Delta \in \text{PE}$ directly follows from the known properties of LS estimators, Proposition 3.3 provides also the analysis for the $\Delta \notin \mathcal{L}_2$ case and the explicit solution for p_i . The proof of Proposition 3.3 is given in Section 3.A.2.

Proposition 3.3 shows that, within the context of the DREM procedure, the use of the nonlinear (in the dynamics of p) LS estimator (3.24), (3.25) does not yield weaker convergence properties than the gradient estimator (3.14), even if the possible unboundedness of p_i can be alleviated, e.g., via projection. For the noise-free scalar LREs (3.17), both the gradient estimator and the element-wise LS ensure the transient monotonicity and the element-wise tuning. However, the benefit of the LS estimator is the better noise attenuation, which can be crucial in some applications.

3.5 Excitation Propagation in the DREM Procedure

As discussed in Section 3.4.2, dynamic extension in the first step of the DREM procedure is critical for performance. The question is how to perform such an extension that the excitation of the original regressor ϕ is preserved, either persistent or interval. As shown in Example 3.3, a poor choice can compromise the convergence even if ϕ is PE.

For a particular class of LRE, where the regressor consists of a finite sum of sinusoidal signals, one suitable choice is a series connection of delay operators. It can be proven that such a choice preserves the excitation under conditions on the delay value, but the upper frequency bound must be known in advance (see [114]). In this section, we address this problem for the more general case of multi-output linear regression with an arbitrary regressor matrix ϕ , where $\phi(t) \in \mathbb{R}^{n \times \ell}$ and $\ell \geq 1$.

This section is organized as follows. First, in Section 3.5.1, we discuss an interpretation of the DREM procedure as a linear time-varying functional observer; this interpretation generalizes finite-dimensional linear dynamic regressor extensions. Then, in Section 3.5.2, we consider one specific choice of dynamic extension, namely Kreiselmeyer's scheme, that preserves excitation of the original regressor ϕ . We provide the quantitative analysis of this excitation preservation property in Section 3.5.3, where we also analyze the dynamics of Δ , and we present an illustrative example in Section 3.5.4.

3.5.1 Functional Observer Interpretation

An interesting interpretation of the DREM procedure is to consider it as a Functional Observer for an LTV system, as discussed in [115].

Functional Observers for LTV systems

Following [116], consider the LTV system

$$\begin{aligned}\dot{x} &= A(t)x + B(t)u, \\ y &= C(t)x,\end{aligned}\tag{3.27}$$

where $x \in \mathbb{R}^{n_x}$, $u \in \mathbb{R}^{n_u}$, $y \in \mathbb{R}^{n_y}$, and the linear functional

$$v = M(t)x,\tag{3.28}$$

with $v \in \mathbb{R}^{n_v}$. The goal is to design an observer for the signal v given the measurements of y and u . In [116, Theorem 3.6], the following result is established.

Proposition 3.4. *Define a completely observable n_v -dimensional system*

$$\begin{aligned}\dot{z} &= F(t)z + G(t)u + K(t)y, \\ \vartheta &= P(t)z,\end{aligned}\tag{3.29}$$

with all the solutions of $\dot{x} = F(t)x$ converging to zero. The system (3.29) is a global asymptotic observer of the linear functional (3.28) for the system (3.27), that is, for all $x(0) \in \mathbb{R}^{n_x}$, $z(0) \in \mathbb{R}^{n_v}$ and all continuous, bounded inputs u we have

$$\lim_{t \rightarrow \infty} (v(t) - \vartheta(t)) = 0,$$

if there exists a continuously differentiable $n_v \times n_x$ matrix $\Phi(t)$ solution of the equations

$$\begin{aligned}G(t) &= \Phi(t)B(t), \\ \dot{\Phi}(t) &= F(t)\Phi(t) - \Phi(t)A(t) + K(t)C(t), \\ M(t) &= P(t)\Phi(t).\end{aligned}\tag{3.30}$$

The DREM procedure as a functional observer

The linear regression (3.16) can be seen as the following LTV system with the n -dimensional state θ and the ℓ -dimensional output y ,

$$\begin{aligned}\dot{\theta}(t) &= 0, \\ y(t) &= \phi^\top(t)\theta(t).\end{aligned}$$

This system can be written in the form (3.27) choosing $n_x = n$, $n_y = \ell$, $A = 0$, $B = 0$, and $C = \phi^\top$. The observer (3.29) and the conditions (3.30) yield

$$\begin{aligned}\dot{z}(t) &= F(t)z + K(t)y(t), \\ \dot{\Phi}(t) &= F(t)\Phi(t) + K(t)\phi^\top(t), \\ \vartheta(t) &= P(t)z(t),\end{aligned}\tag{3.31}$$

and ϑ is an asymptotic estimate of $v(t) = M(t)\theta$ if holds

$$M(t) = P(t)\Phi(t). \quad (3.32)$$

To derive the DREM procedure, choose $n_v = n$ making the matrices P and Φ square, and choose

$$M(t) = \det(\Phi(t)) I_n.$$

Then the linear functional (3.28) takes the decoupled form

$$v_i(t) = \det(\Phi(t)) \theta_i,$$

and the condition (3.32) is satisfied choosing

$$P(t) = \text{adj}(\Phi(t)). \quad (3.33)$$

Denoting $\mathcal{Y} = \vartheta$ and $\Delta = \det(\Phi)$, the LTV observer (3.31), (3.33) asymptotically generates the element-wise scalar LREs

$$\mathcal{Y}_i(t) = \Delta(t)\theta_i,$$

which are the outcome of the DREM procedure. Here, the state vector z plays the same role as the extended output matrix Y in (3.9).

Considering the LTV observer (3.31) as an implementation of the DREM procedure, the first two lines of (3.31) define the dynamics extension, and the last line performs the mixing step. Particularly, for $\ell = 1$, choosing $F(t)$ as a constant diagonal matrix with the entries α_i and $K(t)$ as a constant vector with the entries β_i , $i \in \overline{1, n}$, the LTV observer (3.31) yields the set of first-order LTI filters H_i chosen as (3.8). However, from the analysis above, it follows that the dynamic extension can also be performed with a more general choice of the time-varying matrices $F(t)$, $K(t)$ and for $\ell > 1$. Particularly, one interesting choice is $K(t) = \phi(t)$ yielding the semi-positive definite input term $\phi(t)\phi^\top(t)$ in (3.31). Together with $F(t) = -aI_n$, such a choice yields Kreisselmeier's regressor extension [105]. This scheme has the excitation preservation property that we discuss in the next section.

3.5.2 Kreisselmeier's Regressor Extension

One possible dynamics extension preserving the excitation, widely used in adaptive control, is Kreisselmeier's regressor extension introduced in [105]. For the LRE (3.1), Kreisselmeier's scheme generates the extended LRE (3.9), where the extended matrices Φ and Y are solutions of

$$\dot{\Phi}(t) = -a\Phi(t) + \phi(t)\phi^\top(t), \quad (3.34)$$

$$\dot{Y}(t) = -aY(t) + \phi(t)y(t), \quad (3.35)$$

where $\Phi(0) = \Phi_0 \geq 0$, $Y(0) = Y_0$, and $a > 0$ is a scalar tuning parameter.

Remark 3.2. *As discussed in Section 3.5.1, we can derive Kreisselmeier’s regressor extension scheme (3.34), (3.35) from the general LTV observer representation of the dynamic extension (3.31) choosing $F = -aI$ and $K = \phi$.*

The PE preservation property of (3.34) is well-known and summarized in the following implication:

$$\phi \text{ is PE} \Rightarrow \Phi(t) > 0, \forall t \geq T, \quad (3.36)$$

where T is the excitation interval of ϕ ; see Definition 3.1 in Section 3.3. This PE preservation property of (3.34) motivates its use in the DREM scheme; the authors of [117] proposed such a choice referring to it as Memory Regressor Extension.

Within the DREM context, Kreisselmeier’s scheme (3.34), (3.35) is augmented with the mixing step given by (3.11) and (3.13), namely

$$\begin{aligned} \mathcal{Y}(t) &= \text{adj}(\Phi(t)) Y(t), \\ \Delta(t) &= \det(\Phi(t)) \end{aligned}$$

yielding the scalar LREs (3.17) for $i \in \overline{1, n}$, namely

$$\mathcal{Y}_i(t) = \Delta(t)\theta_i.$$

We are interested in the new regressor Δ , for which (3.36) implies the positiveness and the PE property,

$$\phi \text{ is PE} \Rightarrow \Delta(t) > 0, \forall t > T \Rightarrow \Delta \text{ is PE.} \quad (3.37)$$

The proof of the implication (3.36) and thus (3.37) is well-known and can be found, e.g., in [76, Theorem 4.3.3], where the integral cost gradient adaptation algorithm is considered. However, these proofs provide only the *qualitative* results (3.36) and (3.37). What is more interesting for the DREM procedure is to derive *quantitative* results on the excitation preservation, i.e., given that ϕ is (T, μ) -PE, or (t_1, T, μ) -IE, what are the PE, or IE, characteristics of the new regressor $\Delta(t)$? We address this question in the next section by providing the quantitative analysis of excitation preservation via Kreisselmeier’s scheme.

More precisely, besides the discussed implication ϕ is PE $\Rightarrow \Delta$ is PE, we estimate the lower bound for Δ as a function of the excitation characteristics of ϕ and the gain a in (3.34). Then we study the Interval Excitation property of ϕ and show that it is preserved as well; we also provide the quantitative analysis of the resulting interval excitation for Δ . Moreover, we show that the inverse implication holds and Δ is PE only if ϕ is PE; this observation is somewhat intuitive and implies that the extension scheme does not bring *new* excitation.

It is worth noting that the dynamic extension scheme (3.34), (3.35) also allows establishing bounds on the dynamics of the new regressor Δ ; we present this analysis in the next section after studying the excitation preservation properties.

3.5.3 Excitation Propagation Analysis

As discussed above, the applicability of (3.34), (3.35) for the DREM procedure and the implication (3.37) can be derived from the proof of Theorem 4.3.3 in [76]. However, in that theorem, only the positiveness on the smallest eigenvalue of the matrix Φ is established. Extending that result, we present the following proposition providing precise lower bounds for the determinant of the matrix Φ . We also show that the inverse implication in (3.37) holds supporting a somewhat intuitive observation that the dynamic extension (3.34) does not create new excitation.

Proposition 3.5. *Consider the bounded signal $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}^{n \times \ell}$ and let $\Phi : \mathbb{R}_+ \rightarrow \mathbb{R}^{n \times n}$ be a solution of (3.34) for some initial value $\Phi(0) = \Phi_0 \geq 0$. Let $\Delta : \mathbb{R}_+ \rightarrow \mathbb{R}$ be the determinant of Φ . Then if ϕ is (T, μ) -PE, then for any positive integer $q \geq 1$ and for all $t \geq qT$, it holds*

$$\Delta(t) \geq \mu^n \left(\sum_{k=1}^q e^{-akT} \right)^n \quad (3.38)$$

and

$$\liminf_{t \rightarrow \infty} \Delta(t) \geq \left(\frac{\mu}{e^{aT} - 1} \right)^n. \quad (3.39)$$

Moreover, the following implication holds

$$\phi \in \text{PE} \Leftrightarrow \Delta \in \text{PE}. \quad (3.40)$$

The proof of Proposition 3.5 is given in Section 3.A.3.

In the vein of Proposition 3.5, it can be also shown that the dynamic extension (3.34) preserves the interval excitation in the sense of Definition 3.2 as well. To this end, let the signal ϕ be (t_1, T, μ) -IE for some $t_1 \geq 0$, $T > 0$, and $\mu > 0$. Define $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ as

$$f(t) := \lambda_m \left(\int_{t_1}^t \phi(s) \phi^\top(s) ds \right). \quad (3.41)$$

The function f is continuous and nondecreasing. It is not necessarily differentiable; however, due to the boundedness of ϕ , it admits a Lipschitz constant, i.e., there exists a positive constant $\rho > 0$ such that for any $s \geq t_1$ and $h \geq 0$ it holds

$$0 \leq f(s+h) - f(s) \leq \rho h.$$

With this definition, we can formulate the following proposition establishing the interval excitation preservation.

Proposition 3.6. *Consider the bounded signal $\phi : \mathbb{R}_+ \rightarrow \mathbb{R}^{n \times \ell}$ and let $\Phi : \mathbb{R}_+ \rightarrow \mathbb{R}^{n \times n}$ be a solution of (3.34) for some initial value $\Phi(0) = \Phi_0 \geq 0$. Let the signal $\Delta : \mathbb{R}_+ \rightarrow \mathbb{R}$ be the determinant of Φ . If the signal ϕ is (t_1, T, μ) -interval excited for some $t_1 \geq 0$, $T > 0$, and $\mu > 0$, then the signal Δ is (t_1, T, α) -interval excited for*

$$\alpha := \left(\frac{\rho}{a^2} e^{-aT} \left(e^{\frac{a\mu}{\rho}} - 1 - \frac{a\mu}{\rho} \right) \right)^{2n} > 0, \quad (3.42)$$

where ρ is the Lipschitz constant of the function f defined in (3.41).

The proof of Proposition 3.6 is given in Section 3.A.4.

Proposition 3.5 and Proposition 3.6 justify using (3.34), (3.35) for the dynamic regressor extension step of the DREM procedure. Under this choice, the persistence of excitation and the interval excitation properties of the original regressor are always preserved. The obtained bounds (3.38), (3.39), and (3.42) allow for performance evaluation of the DREM-enhanced estimation algorithms, e.g., the convergence rate and noise sensitivity gains estimation.

It is also of interest to study the dynamics of the DREM-generated new regressor Δ for Kreisselmeier's regressor extension scheme. Such an analysis can be established using Jacobi's formula [118, Theorem 8.1]

$$\dot{\Delta}(t) = \text{tr}\left(\text{adj}(\Phi(t)) \dot{\Phi}(t)\right),$$

and the relation between eigenvalues of Φ and $\text{adj}(\Phi)$. Recalling that $\lambda_M(\Phi)$ is the maximum eigenvalue of Φ , we establish the following proposition, whose proof is given in Section 3.A.5.

Proposition 3.7. *Let Φ be a solution of (3.34) and let $\Delta = \det(\Phi)$. Then $\forall t \geq 0$:*

- if $\lambda_M(\Phi(t)) = 0$, then $\dot{\Delta} = 0$;
- if $\lambda_M(\Phi(t)) > 0$, then

$$\dot{\Delta}(t) \geq \left(-an + \frac{\|\phi(t)\|^2}{\lambda_M(\Phi(t))}\right) \Delta(t).$$

It is also worth noting that the upper bound of the maximum eigenvalue of Φ can be estimated given the upper bound of ϕ .

3.5.4 Illustrative Example

To illustrate the excitation preservation and the lower bounds (3.38) and (3.39), we consider the problem of magnitude and phase estimation for sinusoidal signals with known frequencies. In this example, we also illustrate that the DREM procedure is also applicable for multiple-output linear regression models. To this end, chose $\ell = 2$, $n = 3$, and consider

$$\begin{aligned} y_1(t) &= A \sin(\pi t + \psi), \\ y_2(t) &= B + A \cos(2t + \psi) \end{aligned}$$

where B , $A > 0$, and $\psi \in [-\pi, \pi)$ are the unknown parameters to be estimated. These signals can be rewritten as the LRE (3.16) with

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}, \quad \phi(t) = \begin{bmatrix} 0 & \sin(\pi t) & \cos(\pi t) \\ 1 & \cos(2t) & -\sin(2t) \end{bmatrix}^\top, \quad \theta = \begin{bmatrix} B \\ A \cos(\psi) \\ A \sin(\psi) \end{bmatrix}, \quad (3.43)$$

and $w \equiv 0$. Obviously, the values A , B , and ψ can be reconstructed given θ .

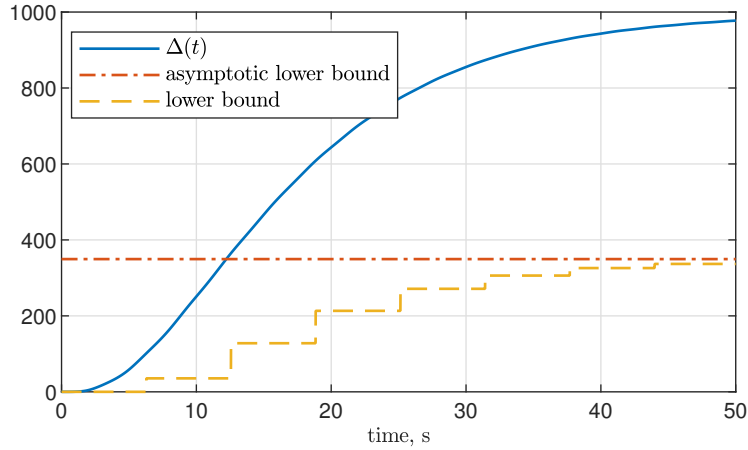


Figure 3.7: The new regressor $\Delta(t)$, the lower bound (3.38), and the asymptotic lower bound (3.39).

It is straightforward to verify that the regressor ϕ is $(2\pi, \mu)$ -PE, where

$$\mu = 2\pi - \frac{\sin(2\pi^2)}{2\pi},$$

i.e., for all $t \geq 0$

$$\int_t^{t+2\pi} \phi(s)\phi^\top(s)ds \geq \mu I.$$

We apply the DREM procedure with the dynamic regressor extension (3.34), (3.35), where the only tuning parameter is chosen as $a = 0.1$. The new regressor Δ is depicted in Fig. 3.7 with the lower bound (3.38) and the asymptotic lower bound (3.39).

Whereas the goal of this example is to illustrate the lower bounds established in Proposition 3.5 and the positiveness of the regressor $\Delta(t)$, for the completeness we present also the estimation error $\tilde{\theta}$ for the gradient estimator (3.2) and the DREM-enhanced estimator (3.14), where we obtain Δ and \mathcal{Y} with Kreisselmeier's scheme, i.e., we compute Δ and \mathcal{Y} as (3.11) and (3.13) with Φ and Y generated by (3.34), (3.35), respectively.

Choosing $B = 1$, $A = 2\sqrt{2}$, and $\psi = -\frac{\pi}{2}$ yields

$$\theta = \begin{bmatrix} 1 & 2 & -2 \end{bmatrix}^\top.$$

For the estimation algorithms, we choose $\Gamma = I$ for (3.2), $\gamma_i = 1$, $i = 1, 2, 3$, for (3.14), and $\hat{\theta}(0) = 0$. The plots of the estimation error $\tilde{\theta}$ are depicted in Fig. 3.8 and illustrate the improved transient performance due to the use of the DREM procedure. The initial transient phase of the DREM-enhanced estimator is due to the initial transients of the regressor Δ , see Fig. 3.7, where the positiveness of Δ is guaranteed only after T seconds for a (T, μ) -PE signal ϕ .

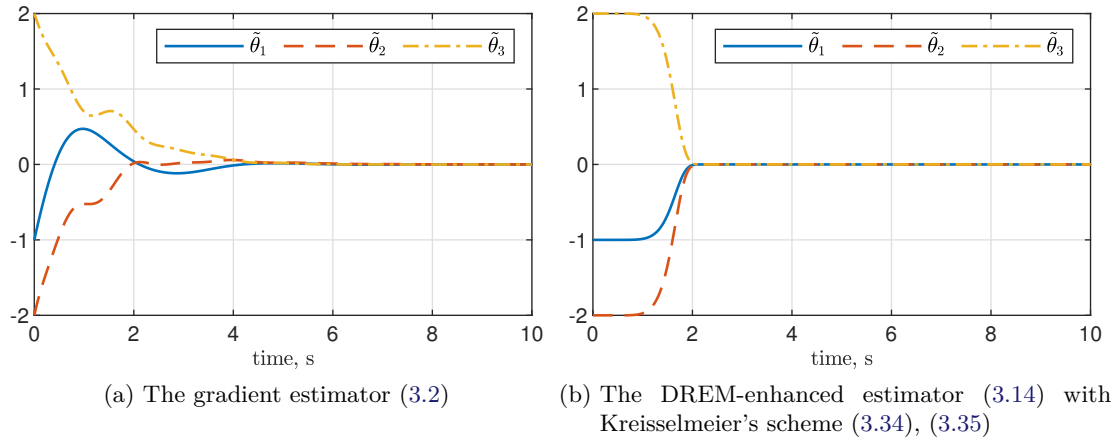


Figure 3.8: The estimation error $\tilde{\theta}$ for (3.1) with $w \equiv 0$ and y, ϕ, θ defined in (3.43).

3.6 Going Further

This section presents some further developments of the DREM procedure and briefly discusses various applications where the DREM procedure was used. Section 3.6.1 discusses how the DREM procedure can be used for parameter estimation of a nonlinear in parameters regression model where some of the nonlinearities constitute a monotone operator. Section 3.6.2 introduces the fixed-time parameter estimation under interval excitation via the DEM procedure. Finally, in Section 3.6.3, we briefly describe some applications where the DREM procedure was successfully applied.

3.6.1 Partially Monotonic Nonlinearly Parametrized Regressions

In previous sections, we considered linear in parameters regression models (3.1) and (3.16). However, in many practical problems, parameters enter nonlinearly in the regression form yielding the nonlinear regression equation

$$y(t) = \phi^\top(t)\psi(\theta), \quad (3.44)$$

where we now assume that $\theta \in \mathbb{R}^p$ and $\psi : \mathbb{R}^p \rightarrow \mathbb{R}^n$ is a nonlinear function; we also recall that $y(t) \in \mathbb{R}^\ell$ and $\psi(t) \in \mathbb{R}^{n \times \ell}$.

Designing parameter identification algorithms for nonlinearly parameterized regressions is difficult, and one common solution is to overparametrize it. To this end, we define $\eta := \psi(\theta)$ and transform the nonlinear (in θ) equation (3.44) into the linear one, namely

$$y(t) = \phi^\top(t)\eta.$$

Then, the tools and methods for linear regression parameter estimation can be applied. However, overparametrization is not always applicable when the true parameter θ is required since the mapping ψ must be invertible, at least locally. Moreover, typically

$n > p$, i.e., the parameter space of the linear equation is bigger, which complicates prior knowledge incorporation concerning the domain of validity of parameters θ .

An interesting case is when the function ψ exhibits some monotonicity properties, and estimation in such a case is discussed, e.g., in [119]. Unfortunately, it is often the case that this property holds only for some of the functions entering in the regression. In this section, we consider using the DREM technique to separate and isolate the monotonic nonlinearities and exploit the monotonicity to achieve consistent parameter estimation for nonlinearly parameterized regressions.

Before presenting the general result, we first consider the following example.

Example 3.4. Consider the nonlinearly parametrized regression for the scalar parameter $\theta \in \mathbb{R}$,

$$y(t) = \phi_1(t) (\theta - e^{-\theta}) + \phi_2 \cos(\theta) = \begin{bmatrix} \phi_1(t) & \phi_2(t) \end{bmatrix} \begin{bmatrix} \psi_1(\theta) \\ \psi_2(\theta) \end{bmatrix} = \phi^\top(t) \eta,$$

where $\psi_1(\theta) := \theta - e^{-\theta}$, $\psi_2(\theta) := \cos(\theta)$, and $\eta := \begin{bmatrix} \psi_1(\theta) & \psi_2(\theta) \end{bmatrix}^\top$ is the over-parametrized parameter vector.

Using standard parameter estimation methods, we can estimate η and then reconstruct θ . Note, however, that ψ_2 is not bijective, and thus we use only ψ_1 to estimate θ or apply a more advanced estimation approach, e.g., the nonlinear least squares. On the other hand, using the DREM procedure, we get the scalar linear equation

$$\mathcal{Y}_1(t) = \Delta(t) \eta_1 = \Delta(t) \psi_1(\theta), \quad (3.45)$$

where \mathcal{Y}_1 and Δ are generated using, e.g., Kreisselmeier's scheme as discussed in Section 3.5.2. Then we can estimate only η_1 and reconstruct θ inverting the ψ_1 function,

$$\hat{\theta}_1 = \hat{\eta}_1 + \mathcal{W}_0(e^{-\hat{\eta}_1}),$$

where \mathcal{W}_0 is the principal branch of the Lambert (product logarithm) function.

However, we can also exploit the monotonicity property of ψ_1 combining it with the DREM procedure and estimate θ directly from (3.45).

To this end, we note that ψ_1 is strictly monotonically increasing,

$$\psi_1'(\theta) \geq \rho > 0,$$

where $\rho = 1$ in the example. It implies that for all $a, b \in \mathbb{R}$, it holds

$$(a - b) (\psi_1(a) - \psi_1(b)) \geq \rho(a - b)^2. \quad (3.46)$$

Choose the estimation law

$$\dot{\hat{\theta}} = \gamma \Delta \left(\mathcal{Y}_1 - \Delta \psi_1(\hat{\theta}) \right)$$

for some initial condition $\hat{\theta}(0)$. Then for the estimation error $\tilde{\theta} = \hat{\theta} - \theta$ it holds

$$\dot{\tilde{\theta}} = -\gamma \Delta^2 \left(\psi_1(\hat{\theta}) - \psi_1(\theta) \right),$$

and for the Lyapunov function

$$V = \frac{1}{2}\tilde{\theta}^2$$

using (3.46), we obtain

$$\begin{aligned}\dot{V} &= -\gamma\Delta^2(\hat{\theta} - \theta)(\psi_1(\hat{\theta}) - \psi_1(\theta)) \\ &\leq -\gamma\rho\Delta^2\tilde{\theta}^2 = -2\gamma\rho\Delta^2V.\end{aligned}$$

Then $\Delta \notin \mathcal{L}_2$ implies that $\tilde{\theta}$ converges to zero asymptotically, and moreover, if Δ is PE, then $\tilde{\theta}$ converges to zero exponentially fast. \triangle

Let us now generalize the result of Example 3.4 for the nonlinear regression (3.44). Assuming $n \geq p$, suppose that among n functions ψ_i , $i \in \overline{1, n}$, there exist p functions satisfying the monotonicity condition. Without loss of generality, assume that these functions are ψ_1, \dots, ψ_p , i.e.,

$$\psi = \begin{bmatrix} \psi_g \\ \psi_b \end{bmatrix}, \quad \psi_g = \begin{bmatrix} \psi_1 \\ \vdots \\ \psi_p \end{bmatrix}, \quad \psi_b = \begin{bmatrix} \psi_{p+1} \\ \vdots \\ \psi_n \end{bmatrix},$$

where the “good” functions ψ_g constitute a P -monotone operator $\mathbb{R}^p \rightarrow \mathbb{R}^p$, i.e., there exists a positive definite matrix $P \in \mathbb{R}^{p \times p}$ such that²

$$P \frac{\partial \psi_g(\theta)}{\partial \theta} + \left(\frac{\partial \psi_g(\theta)}{\partial \theta} \right)^\top P > 0 \quad (3.47)$$

for all θ uniformly. This property implies, see [120], that there exists $\rho > 0$ such that for all $a, b \in \mathbb{R}^p$

$$(a - b)^\top P (\psi_g(a) - \psi_g(b)) \geq \rho(a - b)^\top P (a - b).$$

Apply the DREM procedure, e.g., Kreisselmeier’s scheme (3.34), (3.35) with the mixing (3.11), (3.13), and consider only the first p elements. Then we get

$$\bar{y}(t) = \Delta(t)\psi_g(\theta), \quad (3.48)$$

where \bar{y} consists of the first p elements of y .

Proposition 3.8. *Consider (3.48), where the function $\psi_g : \mathbb{R}^p \rightarrow \mathbb{R}^p$ satisfies (3.47) for some positive-definite matrix P . Apply the estimator*

$$\dot{\hat{\theta}} = \gamma\Delta(\bar{y} - \Delta(t)\psi_g(\hat{\theta})), \quad (3.49)$$

where $\gamma > 0$ is a scalar. Then $\Delta \notin \mathcal{L}_2$ implies that $\hat{\theta}$ converges to θ asymptotically, and if Δ is PE, then the convergence is exponential.

²This section considers monotonically increasing functions, and extension to the monotonically decreasing functions is straightforward.

The proof is trivial noting that (3.49) yields

$$\dot{\hat{\theta}} = -\gamma\Delta^2 \left(\psi_g(\hat{\theta}) - \psi_g(\theta) \right).$$

Consider the Lyapunov function

$$V = \frac{1}{2} \tilde{\theta}^\top P \tilde{\theta}.$$

Then

$$\dot{V} = -\gamma\Delta^2 \tilde{\theta}^\top P \left(\psi_g(\hat{\theta}) - \psi_g(\theta) \right) \leq -2\gamma\rho\Delta^2 V$$

and

$$V(t) \leq e^{-2\gamma\rho \int_0^t \Delta^2(s) ds} V(0)$$

completing the proof.

Remark 3.3. *It is worth noting that in the considered nonlinear parametrization, the element-wise monotonicity of the transients is not guaranteed, in contrast to the linear (in parameters) equations.*

3.6.2 Fixed-time Convergence Under Interval Excitation

Estimation algorithms discussed in Section 3.4 require either persistent or infinite excitation³ and provide asymptotic convergence. In this section, we consider a method allowing fixed-time parameter estimation under the interval excitation.

As discussed in Section 3.2, some parameter estimation methods relax the PE condition, e.g., the concurrent and composite learning. In a certain sense, these methods “prolongate” the (finite in time) interval excitation over the infinite time horizon, i.e., the data collected during the initial interval is then used in the absence of excitation. In contrast to that idea, in [108], authors proposed to estimate parameters during the initial interval, i.e., before the excitation decays. To this end, they proposed several finite-time algorithms providing parameter estimation under the interval excitation. However, the algorithms proposed in [108] require *a priori* knowledge of the admissible parameter values interval, i.e., the authors assume that $|\theta_i| \leq \bar{\theta}$ for $i \in \overline{1, n}$, where $\bar{\theta}$ is a known constant. Extending that approach, in [121], we proposed two fixed-time estimation algorithms that provide estimation convergence under interval excitation and do not require the knowledge of $\bar{\theta}$. To streamline the presentation, in this section, we only briefly discuss one of that results; see [121] for more details and illustrative examples.

As in [108], the discussed method involves the DREM procedure. First, a fixed-time estimation method is constructed for a scalar linear regression, i.e., when a single scalar parameter is estimated. Then, the DREM procedure is applied allowing to estimate the vector of parameters θ . I.e., the linear regression for the vector θ is transformed to the set of scalar equations, and the fixed-time estimation algorithm is then applied element-wise. The excitation preservation properties of the DREM procedure discussed

³Recall that the infinite excitation for a scalar signal is equivalent to the non-square-integrability.

in Section 3.5, see Proposition 3.6, justify the applicability of the proposed solution under the interval excitation of the original regressor ϕ .

To present the result, we first introduce the following definition.

Definition 3.4. *A continuous function $\kappa : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ belongs to the class \mathcal{K} if $\kappa(0) = 0$ and it is strictly increasing. A function $\varpi : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ belongs to the class \mathcal{GKL} if $\varpi(s, 0)$ belongs to the class \mathcal{K} , $\varpi(s, \cdot)$ is decreasing, and for each $s \in \mathbb{R}_+$ there exists $T_s \in \mathbb{R}_+$ such that $\varpi(s, t) = 0$ for all $t \geq T_s$.*

Now we can present how the DREM procedure is used to ensure the fixed-time estimation. Consider the LRE (3.1), namely

$$y(t) = \phi^\top(t)\theta + w(t),$$

where ϕ is $(0, T, \mu)$ -IE, see Definition 3.2, and w is bounded. Apply the DREM procedure with Kreisselmeier's scheme (3.34), (3.35) and the mixing (3.11), (3.13) transforming the vector LRE to the set of n scalar LREs (3.12), namely

$$\mathcal{Y}_i(t) = \Delta(t)\theta_i + \mathcal{W}_i(t),$$

for $i \in \overline{1, n}$. Due to the properties of the DREM procedure and recalling Proposition 3.6, we get that \mathcal{W}_i are bounded for all $i \in \overline{1, n}$, Δ and the time derivative of Δ are bounded, and Δ is $(0, T, \alpha)$ -IE for α given by (3.42).

Consider the estimation algorithm

$$\dot{\hat{\theta}}_i(t) = \Delta(t) \left(\gamma_{1,i} \left[\mathcal{Y}_i(t) - \Delta(t)\hat{\theta}_i(t) \right]^{1-\eta_i} + \gamma_{2,i} \left[\mathcal{Y}_i(t) - \Delta(t)\hat{\theta}_i(t) \right]^{1+\eta_i} \right), \quad (3.50)$$

where $\eta_i \in [0, 1)$ and $\gamma_{1,i} > 0$, $\gamma_{2,i} > 0$ are the tuning parameters, $i \in \overline{1, n}$, and

$$[x]^\eta := \text{sign}(x)|x|^\eta.$$

Then there exists $T_{f,i} \in (0, T]$ and $\gamma_{0,i} > 0$ such that if $\min(\gamma_{1,i}, \gamma_{2,i}) \geq \gamma_{0,i}$ then the estimation error dynamics $\tilde{\theta}_i$ is fixed-time input-to-state stable for $T_{f,i}$, i.e., the exist functions $\varpi : \mathbb{R}_+ \times \mathbb{R}_+ \rightarrow \mathbb{R}_+$ belonging to the class \mathcal{GKL} and $\kappa : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ belonging to the class \mathcal{K} , such that

$$|\tilde{\theta}_i(t)| \leq \varpi \left(|\tilde{\theta}_i(0)|, t \right) + \kappa(\|\mathcal{W}_i\|_\infty), \quad \forall t \in [0, T_{f,i}], \quad (3.51)$$

and $\varpi \left(|\tilde{\theta}_i(0)|, T_{f,i} \right) = 0$.

The values of $T_{f,i}$ and $\gamma_{0,i}$ depend on the tuning coefficient η_i and the excitation characteristics T and α . The proof of (3.51) and expressions for the functions ϖ and κ can be found in [121]. Moreover, it can be shown, that in the noise-free case, the transients $\tilde{\theta}$ are element-wise monotone.

The inequality (3.51) implies that in the noise-free case, the estimate $\hat{\theta}$ converges to the true value θ in the fixed time T_f , and for the noised case, the estimation error remains bounded, where the upper bound depends on the noise magnitude $\|\mathcal{W}_i\|_\infty$.

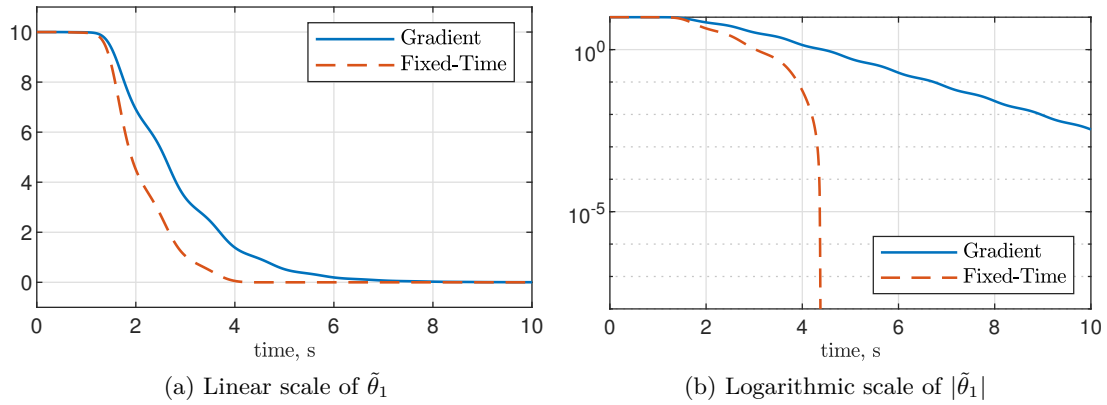


Figure 3.9: Estimation of θ_1 in (3.52) via the DREM-enhanced gradient estimator (3.14) and the fixed-time estimator (3.50) for ϕ being PE.

This result illustrates that the DREM procedure can be fruitfully combined not only with the asymptotic gradient and least-square estimators, as discussed in Section 3.4, but also with more advanced approaches, such as finite/fixed-time estimators discussed in [108, 121].

Example 3.5. Consider the linear regression

$$y(t) = \begin{bmatrix} \phi_1(t) & \sin(3t) \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \phi^\top(t)\theta, \quad (3.52)$$

where the signal ϕ_1 will be defined later. The goal is to estimate the first element of the vector θ , namely θ_1 , and we compare the DREM-enhanced gradient estimator (3.14) and the fixed-time estimator (3.50).

To this end, we first apply Kreisselmeier's scheme (3.34), (3.35) with $a = 1$ and the mixing (3.11), (3.13) transforming (3.52) into

$$\mathcal{Y}_1(t) = \Delta(t)\theta_1.$$

The true value of the vector θ is $\theta = [-10 \ 5]^\top$, and for all estimators we set zero initial conditions, $\hat{\theta}_1(0) = 0$.

Persistent excitation. Choose $\phi_1(t) = 1$ for all t , then the regressor ϕ in (3.52) is PE. For the DREM-enhanced gradient estimator (3.14), we choose $\gamma_1 = 5$, and for the fixed-time estimator (3.50), we choose $\eta_1 = \frac{1}{2}$ and $\gamma_{1,1} = \gamma_{2,1} = 5$.

The transients of $\hat{\theta}_1$ are depicted in Fig. 3.9, both in linear and logarithmic scale. Whereas both estimators are capable of estimating the true value, the estimator (3.50) converges in fixed time.

Interval excitation. Choose now $\phi_1(t) = 1$ for $t \leq 5$ and zero otherwise, then the regressor ϕ in (3.52) is not PE, but it is excited on the interval from zero to five seconds. We use the same tuning coefficients as above. The transients of $\hat{\theta}_1$ are depicted

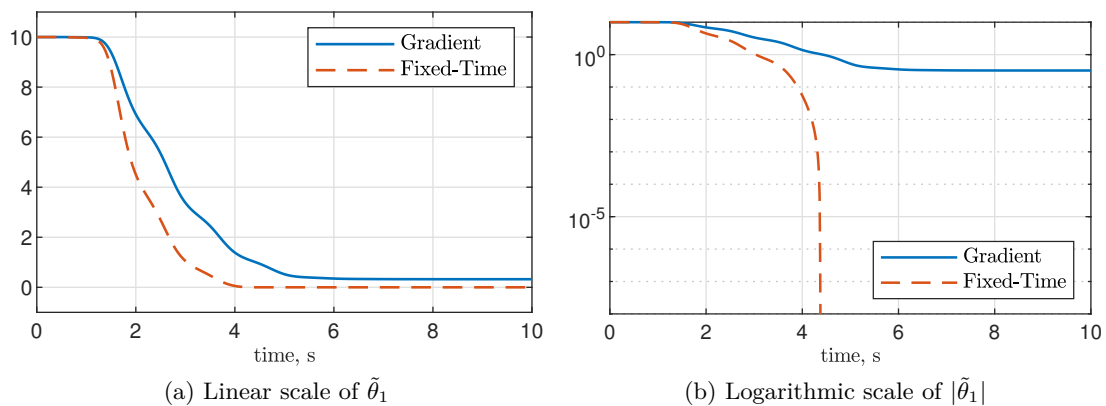


Figure 3.10: Estimation of θ_1 in (3.52) via the DREM-enhanced gradient estimator (3.14) and the fixed-time estimator (3.50) for ϕ not PE but excited on the interval $[0, 5]$.

in Fig. 3.10, both in linear and logarithmic scale. Whereas the gradient estimator fails to converge due to the absence of the persistent excitation, the fixed-time estimator converges under the interval excitation. \triangle

3.6.3 Applications

This chapter mainly focuses on the theoretical aspects of the DREM procedure and its properties. At the same time, the DREM procedure was used in various applications, and this section provides a brief overview of some of them.

Adaptive systems

Regarding indirect adaptive control, an improved online parameter estimation method for linear time-invariant systems identification is presented in [114]. In that work, it is also proven that for an input signal consisting of a finite number of sinusoidal components, delay operators used for the dynamic regressor extension step preserves the persistence of excitation; this property is valuable for embedded applications where the delay operators are implemented as a memory buffer without additional computational costs.

In the recent work [122], authors apply the DREM procedure to a class of nonlinearly parametrized regressions in Euler–Lagrange models and discrete-time indirect adaptive pole-placement.

Considering direct adaptive control, in [109], authors use the DREM procedure to relax the high-frequency gain sign assumption in model reference control; the key idea is that due to the monotonicity properties of the DREM procedure, the estimate of the high-frequency gain crosses zero at most once enabling singularity avoidance.

State estimation

In [123], authors use the DREM procedure to enhance parameter estimation-based observers applying it to power systems and chemical–biological reactors. The DREM procedure is also used to develop an adaptive state observer [124], where the linear regression separation allows to isolate unknown parameters; this approach is extended to time-varying parameters in [125]. Finally, the authors of [126] use the DREM procedure to get the fixed-time estimation for delayed linear time-varying systems.

Sinusoidal signals estimation and disturbance rejection

The DREM procedure was successfully applied for parameter estimation of multi-sinusoidal signals. In this class of problems, regressors are typically persistently excited due to the nature of studied signals, and the benefit of the DREM procedure was the improved transient performance and transient time acceleration. Starting from the straightforward application of the DREM procedure in [127], more advanced results were further derived, including signals with time-varying magnitude [128] and finite-time frequency estimation [129]. Regarding the direct adaptive disturbance attenuation, the work [130] reports how the DREM procedure can be used to empower adaptive regulation.

Robotics and Sensorless Control

In robotics, the DREM procedure is applied for parameters estimation with accelerated convergence rate; see [131] for a quadrotor example and [132] for the attitude control problem of a rigid body. The DREM is also used in the simultaneous localization and mapping problem reformulated as parameter estimation [133] and vision-based position control, where the camera’s orientation was treated as an unknown parameter [134].

The use of the DREM procedure in sensorless control of permanent magnet synchronous motors is attributed to the position and flux estimation via the DREM-enhanced state observers discussed above [135, 136].

Power systems

In [137], authors use the DREM procedure to monitor the power system inertia, and the paper describes several test cases using the 1013-machine European Network of Transmission System Operators for Electricity dynamic model. In [138], authors integrate the DREM procedure into a photovoltaic arrays’ maximum power extraction algorithm for current-voltage characteristic estimation and update. In [139], authors apply the DREM procedure to improve transient response and get finite-time convergence for a DC-DC buck converter regulation.

Other applications

Two applications not included in the groups above are the works [140, 141]. In [140], authors consider a high-frequency noninvasive valvometry device in an autonomous biosensor system using bivalve mollusks valve-activity measurements for ecological monitoring

purposes, and the DREM procedure is used to allow the decoupled fixed-time estimation. In [141], authors address the state estimation problem for a bioreactor containing a single substrate and several competing species, where the total biomass is the only available measurement, and the challenge is to estimate the concentration of the competing species. As in the previous example, the DREM procedure is used to get the fixed-time estimation of decoupled system parameters.

3.7 Conclusion

3.7.1 Summary

In this chapter, we have discussed the DREM procedure, namely the Dynamic Regressor Extension and Mixing. The DREM procedure consists of two steps, where the first one is a linear dynamic extension of the original LRE, and the second is the nonlinear signal transformation given by the adjugate matrix multiplication. Thus, in a nutshell, the DREM procedure is a transformation that renders a linear regression equation for a vector of unknown parameters into a set of scalar linear regressions for each unknown parameter separately.

The straightforward application of the DREM procedure is to combine it with a gradient estimator. The resulting DREM-enhanced estimator has the following nice properties:

- the transients are element-wise monotonic;
- the gain adjustment becomes simple and direct;
- the asymptotic convergence can be established without the persistency of excitation.

The DREM-enhanced estimator outperforms the standard gradient one, which is illustrated in examples. Furthermore, the DREM procedure can also be fruitfully combined with different types of estimators, e.g., the least-squares estimator or finite/fixed-time estimators, providing element-wise monotonicity and tuning. It also can be used for nonlinearly parameterized regression models, where some nonlinearities satisfy the monotonicity condition.

The DREM procedure gives rise to the new regressor, namely the extended matrix's determinant, and the excitation of this new regressor is a crucial question. Interpretation of the DREM procedure as a linear time-varying observer motivates using Kreisselmeier's scheme for the dynamic extension step. This specific choice has good excitation preservation properties and alleviates the risk of excitation loss; it also allows to evaluate the new excitation as a function of the original regressor's characteristics, both for the persistent and interval excitation.

The DREM procedure was applied in various applications, including robotics, sensorless control, power systems, and periodic signals estimation, and is proven to be instrumental in providing improved parameter estimation and relaxing excitation conditions.

3.7.2 See Also

This chapter focused on the principal aspects of the DREM procedure and did not include several other DREM-related results to streamline the presentation. Below, we mention some results not included in this chapter.

- In this chapter, we discuss only the continuous-time formulation of the DREM procedure. Similar results can be derived in discrete-time as well. For the discrete-time formulation of the DREM procedure and discussion on the convergence conditions, see [103]. The excitation-preservation properties of the discrete-time equivalence of Kreisselmeier's scheme are shown in [130].
- Kreisselmeier's scheme guarantees excitation preservation; however, the excitation properties can be further improved. One such extension of Kreisselmeier's scheme with improved convergence is discussed in [103].
- In Section 3.6.2, we discussed the fixed-time parameter estimation with the DREM procedure. It is worth noting, that despite the results discussed in Section 3.6.2, the finite-time convergence can be also achieved combining the DREM procedure with algebraic estimators. The authors of [142] present examples of such estimators.

3.7.3 Open Questions

Several questions remain open for the DREM procedure, and in this section, we discuss three of them.

As discussed in Section 3.4.2, the DREM procedure yields a necessary and sufficient condition for asymptotic convergences of the DREM-enhanced gradient estimator. This condition is weaker than the persistence of excitation of the original regressor and differs from necessary but not sufficient and sufficient but not necessary conditions for the standard gradient estimator. The open question remains if the DREM-enhanced estimator converges under the conditions weaker than the standard gradient estimator. In other words, does there exist a regressor ϕ such that the standard gradient estimator does not converge, and the DREM-enhanced does? This question is complicated by rather technical sufficient convergence conditions for the gradient estimator.

Another open question is related to the parameter estimation problem when some elements of the regressor vector ϕ in (3.1) are linearly dependent, i.e., there exists a constant vector $c \in \mathbb{R}^n$, such that $\phi^\top(t)c = 0$ for all t . Obviously, such a regressor is not excited on any interval, and the extended regressor matrix Φ in the DREM procedure is always singular; the vector of unknown parameters θ cannot be estimated. However, from the practical point of view, it is of interest to estimate a part of the vector θ , i.e., those elements that have enough excitation. This research direction has similarities to the directional forgetting methods in the least-squares framework, which is performed via the singular value decomposition of the covariance matrix. It is an envisioned research direction to see if this approach can be applied to the DREM procedure while keeping the element-wise monotonicity of the transients.

Finally, the practical implementation of the DREM procedure involves the determinant computation, which is computationally demanding. Thus, one interesting research direction is the recurrent reformulation of the discrete-time DREM procedure, both for the new regressor Δ and the new output \mathcal{Y} providing computationally efficient implementation of the DREM procedure.

3.A Proofs

3.A.1 Proof of Lemma 3.1

The proof was first presented in [107].

Proof. Solution $x(t)$ of the scalar linear time-varying system (3.20) is given by

$$x(t) = \phi(t, t_0)x(t_0) + \int_{t_0}^t \phi(t, s)b(s)ds, \quad (3.53)$$

where

$$\phi(t, \tau) = \exp\left(-\int_{\tau}^t a^2(s)ds\right).$$

For $a \notin \mathcal{L}_2$, i.e.,

$$\int_{t_0}^t a^2(s)ds \rightarrow \infty \text{ as } t \rightarrow \infty,$$

the function $\phi(t, \tau)$ has the following properties:

$$0 < \phi(t, \tau) \leq 1, \quad \forall t \geq \tau, \quad (3.54)$$

$$\phi(t, \tau) \rightarrow 0 \text{ as } t \rightarrow \infty, \quad (3.55)$$

and for any $\delta \geq 0$, for all $t \geq \tau + \delta$,

$$\phi(t, \tau) \leq \phi(t, \tau + \delta). \quad (3.56)$$

Recalling that $b \in \mathcal{L}_1$, i.e.,

$$\int_{t_0}^{\infty} |b(s)|ds =: C < \infty, \quad (3.57)$$

we also state that for any arbitrary small $\varepsilon_b > 0$ there exists $T_b > t_0$ such that

$$\int_{T_b}^{\infty} |b(s)|ds < \varepsilon_b. \quad (3.58)$$

Due to (3.55), we conclude that $\phi(t, t_0)x(t_0) \rightarrow 0$ as $t \rightarrow \infty$, and to prove (3.21) it is sufficient to prove that the integral term in (3.53) tends to zero as well.

Define

$$I(t) := \int_{t_0}^t \phi(t, s)b(s)ds.$$

For an arbitrary $T \in [t_0; t]$, the function $I(t)$ can be divided into two parts:

$$I(t) = \int_{t_0}^T \phi(t, s)b(s)ds + \int_T^t \phi(t, s)b(s)ds.$$

Due to (3.56), for $t \geq T$ and any $s \leq T$ it holds $\phi(t, s) \leq \phi(t, T)$. Recalling (3.57), the following holds:

$$\int_{t_0}^T \phi(t, s)b(s)ds \leq \int_{t_0}^T \phi(t, T)|b(s)|ds \leq C\phi(t, T),$$

where C was defined in (3.57). Recalling (3.54), we obtain

$$\int_T^t \phi(t, s)b(s)ds \leq \int_T^t |b(s)|ds,$$

which results in

$$|I(t)| \leq C\phi(t, T) + \int_T^t |b(s)|ds$$

for any $T \geq t_0$.

Now we can show that $I(t) \rightarrow 0$. Choose arbitrary small $\varepsilon > 0$. According to (3.58), there exists $T_b = T_b(\varepsilon) \geq t_0$ such that $\forall t \geq T_b$

$$\int_{T_b}^t |b(s)|ds < \frac{1}{2}\varepsilon.$$

Recalling (3.55), there exists $t_\phi = t_\phi(\varepsilon, T_b) \geq T_b$ such that $\phi(t, T_b) < \frac{1}{2C}\varepsilon$ for all $t \geq t_\phi$. Then $\forall t \geq t_\phi$

$$|I(t)| \leq C\phi(t, T_b) + \int_{T_b}^t |b(s)|ds < \varepsilon,$$

which implies $I(t) \rightarrow 0$ as $t \rightarrow \infty$ and completes the proof. \square

3.A.2 Proof of Proposition 3.3

The proof was first presented in [113].

Proof. Part 1. Consider first the case $\lambda_i = 0$. We obtain

$$\dot{p}_i(t) = -\gamma_i p_i^2(t) \Delta^2(t)$$

yielding

$$p_i(t) = \frac{1}{c_1 + \gamma_i \int_0^t \Delta^2(s)ds},$$

where $c_1 := \frac{1}{p_i(0)}$. Obviously, for $\Delta \notin \mathcal{L}_2$ we have $p_i \rightarrow 0$, as it is expected for an LS estimator without forgetting. The error dynamics (3.26) can be now written as

$$\dot{\tilde{\theta}}(t) = -\beta(t)\tilde{\theta}, \tag{3.59}$$

where

$$\beta(t) = \frac{\gamma_i \Delta^2(t)}{c_1 + \gamma_i \int_0^t \Delta^2(s)ds} = \frac{\Delta^2(t)}{c_2 + \int_0^t \Delta^2(s)ds}, \tag{3.60}$$

and $c_2 := \frac{c_1}{\gamma_i}$. Since Δ is bounded, the LTV system (3.59) has the unique solution

$$\tilde{\theta}_i(t) = \frac{c_3}{c_2 + \int_0^t \Delta^2(s)ds}, \tag{3.61}$$

where $c_3 := \tilde{\theta}(0)c_2$. Indeed, taking the time derivative of (3.61) we obtain

$$\begin{aligned}\dot{\tilde{\theta}}(t) &= -\frac{c_3\Delta^2(t)}{(c_2 + \int_0^t \Delta^2(s)ds)^2} \\ &= -\frac{\Delta^2(t)}{(c_2 + \int_0^t \Delta^2(s)ds)} \frac{c_3}{(c_2 + \int_0^t \Delta^2(s)ds)} = -\beta(t)\tilde{\theta}.\end{aligned}$$

From (3.61), we observe that $\tilde{\theta}_i$ does not converge to zero if $\Delta \in \mathcal{L}_2$ and $c_3 \neq 0$, which proves the claim (C1) of Proposition 3.3.

On the other hand, for $\Delta \notin \mathcal{L}_2$ it follows from (3.61) that $\tilde{\theta}$ converges to zero asymptotically. Moreover, since the function β defined in (3.60) is nonnegative, the convergence is monotonic, which proves the claim (C2).

Finally, to prove the claim (C3) of Proposition 3.3, notice that $\Delta \in \text{PE}$ implies that β defined in (3.60) converges to zero. Thus, β is not PE, and $\tilde{\theta}$ does not converge exponentially since the PE of β is a necessary condition of the exponential convergence of (3.59).

Part 2. Consider now the case $\lambda_i > 0$. The nonlinear ODE (3.25) has the solution

$$p_i(t) = \frac{e^{\lambda_i\gamma_i t} p_i(0)}{1 + p_i(0)\gamma_i \int_0^t e^{\lambda_i\gamma_i s} \Delta^2(s) ds}.$$

The rest of proof is performed in three steps. First, we show that for all bounded Δ , p_i is bounded from below by a positive constant proving the claim (C4) of Proposition 3.3. Second, we show that Δ being PE implies that p_i is bounded from above $\tilde{\theta}$ converges exponentially proving the claim (C6). Finally, we show that p_i tends to infinity if $\Delta \in \mathcal{L}_2$ or if Δ tends to zero, which proves the claim (C5).

Step 1. Consider the inverse function

$$\frac{1}{p_i(t)} = \frac{1}{p_i(0)} e^{-\lambda_i\gamma_i t} + \gamma_i z(t), \quad (3.62)$$

where

$$z(t) = e^{-\lambda_i\gamma_i t} \int_0^t e^{\lambda_i\gamma_i s} \Delta^2(s) ds.$$

Recalling that Δ is bounded, say $\Delta^2(t) \leq \bar{\Delta}$, the function z is bounded as

$$z(t) \leq \bar{\Delta} e^{-\lambda_i\gamma_i t} \int_0^t e^{\lambda_i\gamma_i s} ds = \frac{\bar{\Delta}}{\lambda_i\gamma_i} (1 - e^{-\lambda_i\gamma_i t}) \leq \frac{\bar{\Delta}}{\lambda_i\gamma_i}.$$

It follows that

$$\frac{1}{p_i(t)} \leq \frac{1}{p_i(0)} + \frac{\bar{\Delta}}{\lambda_i},$$

and p_i is bounded from below as

$$p_i(t) \geq \frac{p_i(0)\lambda_i}{\lambda_i + \bar{\Delta}p_i(0)} = p_m. \quad (3.63)$$

Step 2. Assume that Δ is (T, μ) -PE. Then for $t \geq T$ the function z is bounded from below as

$$\begin{aligned} z(t) &\geq e^{-\lambda_i \gamma_i t} \int_{t-T}^t e^{\lambda_i \gamma_i s} \Delta^2(s) ds \\ &\geq e^{-\lambda_i \gamma_i t} e^{\lambda_i \gamma_i (t-T)} \int_{t-T}^t \Delta^2(s) ds \geq \mu e^{-\lambda_i \gamma_i T}. \end{aligned}$$

Then we have two bounds,

$$\begin{aligned} \frac{1}{p_i(t)} &\geq \frac{1}{p_i(0)} e^{-\lambda_i \gamma_i T} && \text{for } 0 \leq t \leq T, \\ \frac{1}{p_i(t)} &\geq \gamma_i z(t) \geq \gamma_i \mu e^{-\lambda_i \gamma_i T} && \text{for } t > T, \end{aligned}$$

and p_i is bounded by

$$p_i(t) \leq e^{\lambda_i \gamma_i T} \max\left(p_i(0), \frac{1}{\mu \gamma_i}\right) = p_M. \quad (3.64)$$

From (3.63) and (3.64) it follows that if Δ is PE, then the signal $t \mapsto \sqrt{p_i(t)}\Delta(t)$ is bounded and PE as well. Therefore the exponential convergence of $\hat{\theta}$ follows from (3.26). Moreover, the convergence is monotonic since $p_i(t)\Delta^2(t) \geq 0, \forall t \geq 0$.

Step 3. Assume now that $\Delta \in \mathcal{L}_2$. Rewriting z as

$$z(t) = \int_0^t e^{-\lambda_i \gamma_i (t-s)} \Delta^2(s) ds,$$

it can be noted that z is the solution of the differential equation

$$\dot{z}(t) + \lambda_i \gamma_i z(t) = \Delta^2(t), \quad z(0) = 0. \quad (3.65)$$

Note that $\Delta \in \mathcal{L}_2$ implies $\Delta^2 \in \mathcal{L}_1$. It is known that the considered stable first order LTI system (3.65) has a finite \mathcal{L}_1 gain, thus $z \in \mathcal{L}_1$. Noting also that \dot{z} is bounded and applying Barbalat's lemma, we conclude $z \rightarrow 0$. Then from (3.62) it follows that p_i tends to infinity and the estimator (3.24), (3.25) is unstable.

For the case when $\Delta \notin \mathcal{L}_2$ but $\Delta \rightarrow 0$, we note that the LTI system (3.65) is exponentially stable, therefore for $\Delta^2 \rightarrow 0$ we have $z \rightarrow 0$ and p_i tends to infinity. \square

3.A.3 Proof of Proposition 3.5

Proof. The proof consists of two parts. First, we show that if ϕ is (T, μ) -PE, then the inequalities (3.38) and (3.39) hold proving the direct implication in (3.40). Next, we show that the inverse implication in (3.40) also holds.

Part 1: $\phi \in \text{PE}$ implies the inequalities (3.38), (3.39) and $\Delta \in \text{PE}$.

The solution of (3.34) is given by

$$\Phi(t) = e^{-at}\Phi(0) + \int_0^t \psi(t, s) ds, \quad \forall t \in \mathbb{R}_+, \quad (3.66)$$

where

$$\psi(t, s) := e^{-a(t-s)}\phi(s)\phi^\top(s).$$

Consider $t \geq T$ and let $q \geq 1$ be a positive integer number such that $t \geq qT$. The integral term in (3.66) can be rewritten as

$$\int_0^t \psi(t, s)ds = \int_0^{t-qT} \psi(t, s)ds + \sum_{k=1}^q \int_{t-kT}^{t-kT+T} \psi(t, s)ds.$$

For any positive integer $k \leq q$ it holds

$$\begin{aligned} \int_{t-kT}^{t-kT+T} \psi(t, s)ds &= e^{-at} \int_{t-kT}^{t-kT+T} e^{as}\phi(s)\phi^\top(s)ds \\ &\geq e^{-at}e^{a(t-kT)}\mu I = \mu e^{-akT}I. \end{aligned}$$

Then

$$\Phi(t) \geq \mu \sum_{k=1}^q e^{-akT}I + \int_0^{t-qT} \psi(t, s)ds + e^{-at}\Phi(0). \quad (3.67)$$

For $\Phi(0) \geq 0$, the sum of the last two terms in the right-hand side of this inequality is a semi positive-definite matrix,

$$\int_0^{t-qT} \psi(t, s)ds + e^{-at}\Phi(0) \geq 0.$$

Then from (3.67) it follows that for all $t \geq qT$ the smallest eigenvalue of $\Phi(t)$ is not less than $\mu \sum_{k=1}^q e^{-akT}$. Thus

$$\det(\Phi(t)) \geq \mu^n \left(\sum_{k=1}^q e^{-akT} \right)^n,$$

and (3.38) follows.

The PE property follows by noting that for $t \geq T$ it holds

$$\Delta(t) \geq \mu^n e^{-anT} > 0,$$

and Δ is strictly separated from zero for all $t \geq T$.

To get the inequality (3.39), we choose q as the largest integer such that $t \geq qT$. Then $q \rightarrow \infty$ as $t \rightarrow \infty$. Since

$$\lim_{q \rightarrow \infty} \sum_{k=1}^q e^{-akT} = \frac{1}{e^{aT} - 1},$$

the asymptotic lower bound (3.39) for $\Delta(t)$ follows.

Part 2: $\Delta \in \text{PE} \Rightarrow \phi \in \text{PE}$. Now we will show that if Δ is PE, then ϕ is also PE. More precisely, we will show that if ϕ is bounded and there exist $T > 0$ and $\mu > 0$ such that for all $t \in \mathbb{R}_+$

$$\int_t^{t+T} \Delta(s)^2 ds \geq \mu,$$

then there exist $L > 0$ and $\alpha > 0$ such that for all $t \in \mathbb{R}_+$

$$\int_t^{t+L} \phi(s)\phi^\top(s)ds \geq \alpha I.$$

Since the matrix Φ given by (3.66) is bounded for bounded ϕ , it follows that all its eigenvalues are non-negative and also bounded, and there exists a constant $c > 0$ such that for all $t \in \mathbb{R}_+$

$$c\lambda_m(\Phi(t)) \geq \Delta^2(t),$$

where a conservative estimate of c is

$$c = \left(\sup_t \lambda_M(\Phi(t)) \right)^{2n-1}.$$

Then

$$\int_t^{t+T} \lambda_m(\Phi(s)) ds \geq \frac{\mu}{c}. \quad (3.68)$$

From (3.66), it follows that for $s \geq t$

$$\Phi(s) = e^{-a(s-t)}\Phi(t) + \int_t^s e^{-a(s-\tau)}\phi(\tau)\phi^\top(\tau)d\tau.$$

Then recalling (3.68), for any positive integer k it holds

$$\begin{aligned} & \int_t^{t+kT} \lambda_m(\Phi(s)) ds \\ &= \int_t^{t+T} \lambda_m \left(e^{-a(s-t)}\Phi(t) + \int_t^s e^{-a(s-\tau)}\phi(\tau)\phi^\top(\tau)d\tau \right) ds \\ &\geq \int_t^{t+kT} \lambda_m \left(e^{-a(s-t)}\Phi(t) \right) ds \\ &+ \int_t^{t+kT} \lambda_m \left(\int_t^s e^{-a(s-\tau)}\phi(\tau)\phi^\top(\tau)d\tau \right) ds \geq k\frac{\mu}{c}. \end{aligned}$$

Note that

$$\begin{aligned} & \int_t^{t+kT} \lambda_m \left(e^{-a(s-t)}\Phi(t) \right) ds \\ &= \lambda_m(\Phi(t)) \int_t^{t+kT} e^{-a(s-t)} ds \leq \frac{1}{a} \lambda_m(\Phi(t)) \end{aligned}$$

for $a > 0$. Choose c_0 as

$$c_0 := \frac{1}{a} \sup_t \lambda_m(\Phi(t)).$$

Then we have that for any k

$$\int_t^{t+kT} \lambda_m \left(\int_t^s e^{-a(s-\tau)}\phi(\tau)\phi^\top(\tau)d\tau \right) ds \geq k\frac{\mu}{c} - c_0. \quad (3.69)$$

Note that

$$\lambda_m \left(\int_t^s e^{-a(s-\tau)} \phi(\tau) \phi^\top(\tau) d\tau \right) \leq \lambda_m \left(\int_t^s \phi(\tau) \phi^\top(\tau) d\tau \right),$$

and for all s satisfying $t \leq s \leq t + kT$ it holds

$$\lambda_m \left(\int_t^s \phi(\tau) \phi^\top(\tau) d\tau \right) \leq \lambda_m \left(\int_t^{t+kT} \phi(\tau) \phi^\top(\tau) d\tau \right).$$

Thus

$$\begin{aligned} & \int_t^{t+kT} \lambda_m \left(\int_t^s e^{-a(s-\tau)} \phi(\tau) \phi^\top(\tau) d\tau \right) ds \\ & \leq \int_t^{t+kT} \lambda_m \left(\int_t^s \phi(\tau) \phi^\top(\tau) d\tau \right) ds \\ & \leq \int_t^{t+kT} \lambda_m \left(\int_t^{t+kT} \phi(\tau) \phi^\top(\tau) d\tau \right) ds \\ & = kT \lambda_m \left(\int_t^{t+kT} \phi(\tau) \phi^\top(\tau) d\tau \right). \end{aligned} \tag{3.70}$$

Finally, combining (3.69) and (3.70) yields

$$\lambda_m \left(\int_t^{t+kT} \phi(\tau) \phi^\top(\tau) d\tau \right) \geq \frac{\mu}{cT} - \frac{c_0}{kT}.$$

Since c and c_0 are constants and do not depend on k , we can choose the positive integer $k \geq 1$ such that $k > \frac{c_0 c}{\mu}$. Then $\phi(t)$ is PE with $L = kT$ and $\alpha = \frac{\mu}{cT} - \frac{c_0}{kT} > 0$. \square

3.A.4 Proof of Proposition 3.6

Proof. As it is discussed in the proof of Proposition 3.5, see Section 3.A.3, the solution of (3.34) is given by (3.66). Then for $t_1 \leq t \leq t_1 + T$

$$\Phi(t) = e^{-a(t-t_1)} \Phi(t_1) + \int_{t_1}^t e^{-a(t-s)} \phi(s) \phi^\top(s) ds,$$

and

$$\begin{aligned} \int_{t_1}^{t_1+T} \lambda_m(\Phi(t)) dt & \geq \int_{t_1}^{t_1+T} \lambda_m \left(\int_{t_1}^t e^{-a(t-s)} \phi(s) \phi^\top(s) ds \right) dt \\ & \geq \int_{t_1}^{t_1+T} e^{-a(t-t_1)} f(t) dt, \end{aligned}$$

where f is defined in (3.41),

$$f(t) = \lambda_m \left(\int_{t_1}^t \phi(s) \phi^\top(s) ds \right).$$

The function f is continuous and nondecreasing, $f(t_1) = 0$ and, due to the interval excitation, $f(t_1 + T) \geq \mu$. The function f is not necessarily differentiable; however, since ϕ is bounded it admits a Lipschitz constant, i.e., there exists a positive constant $\rho > 0$ such that for any $s \geq t_1$ and $h \geq 0$ it holds

$$0 \leq f(s + h) - f(s) \leq \rho h,$$

and $\rho T \geq \mu$. Define $\tau := \frac{\mu}{\rho} \leq T$. Since $f(t_1 + T) \geq \mu$ and $f(t)$ cannot grow faster than a linear function with the slope equal to ρ , it follows that for all $t \in [t_1 + T - \tau, t_1 + T]$ it holds

$$f(t) \geq \rho(t - (t_1 + T - \tau)).$$

Thus

$$\begin{aligned} \int_{t_1}^{t_1+T} e^{-a(t-t_1)} f(t) dt &\geq \int_{t_1+T-\tau}^{t_1+T} e^{-a(t-t_1)} \rho(t - (t_1 + T - \tau)) dt \\ &= \frac{\rho}{a^2} e^{-aT} (e^{a\tau} - 1 - a\tau). \end{aligned}$$

Finally, recalling that $\Delta(t) \geq (\lambda_m(\Phi(t)))^n$, the (t_1, T, α) -interval excitation follows:

$$\int_{t_1}^{t_1+T} \Delta^2(t) dt \geq \alpha,$$

where $\alpha > 0$ is as defined in (3.42),

$$\alpha = \left(\frac{\rho}{a^2} e^{-aT} \left(e^{\frac{a\mu}{\rho}} - 1 - \frac{a\mu}{\rho} \right) \right)^{2n}.$$

□

3.A.5 Proof of Proposition 3.7

Proof. Consider the time evaluation of $\Delta = \det(\Phi)$. The matrix Φ is a solution of (3.34), so Δ obeys Jacobi's formula [118, Theorem 8.1]:

$$\dot{\Delta}(t) = \text{tr}(\text{adj}(\Phi(t)) \dot{\Phi}(t)),$$

where $\Delta(0) = \det(\Phi(0))$.

Substituting (3.34), we obtain for all $t \in \mathbb{R}_+$:

$$\begin{aligned} \dot{\Delta}(t) &= \text{tr}(-a \text{adj}(\Phi(t)) \Phi(t) + \text{adj}(\Phi(t)) \phi(t) \phi(t)^\top) \\ &= -an\Delta(t) + \text{tr}(\text{adj}(\Phi(t)) \phi(t) \phi(t)^\top), \end{aligned}$$

where we recall that the dimension of ϕ is $n \times \ell$. Let $\phi_k \in \mathbb{R}^n$ be the k -th column of ϕ , $k \in \overline{1, \ell}$. Due to elementary properties of the matrix trace function, it follows for all $t \geq 0$,

$$\text{tr}(\text{adj}(\Phi(t)) \phi(t) \phi(t)^\top) = \sum_{k=1}^{\ell} \phi_k(t)^\top \text{adj}(\Phi(t)) \phi_k(t),$$

and we obtain

$$\dot{\Delta}(t) = -an\Delta(t) + \sum_{k=1}^{\ell} \phi_k(t)^\top \text{adj}(\Phi(t)) \phi_k(t). \quad (3.71)$$

Recall that the eigenvalues of an adjoint matrix can be estimated as follows. Let $\lambda_{1,\Phi}, \dots, \lambda_{n,\Phi}$ denote the eigenvalues of Φ . Applying Schur's Lemma, it is then straightforward to show that the eigenvalues of $\text{adj}(\Phi)$ are given by

$$\lambda_{i,\text{adj}(\Phi)} = \prod_{j \neq i} \lambda_{j,\Phi}, \quad \forall i = 1, \dots, n,$$

and for all $i \in \overline{1, n}$ it holds

$$\lambda_{i,\text{adj}(\Phi)} \lambda_{i,\Phi} = \det(\Phi).$$

In particular

$$\lambda_m(\text{adj}(\Phi)) \lambda_M(\Phi) = \det(\Phi) \quad (3.72)$$

Since $\Phi(t) \geq 0$, then $\lambda_M(\Phi(t)) = 0$ implies that all eigenvalues of $\Phi(t)$ are zeros, and so are the eigenvalues of $\text{adj}(\Phi(t))$. That implies for all $t \geq 0$ and all $k \in \overline{1, \ell}$,

$$\phi_k(t)^\top \text{adj}(\Phi(t)) \phi_k(t) = 0.$$

On the other hand, if $\lambda_M(\Phi) > 0$, then due to (3.72)

$$\phi_k^\top \text{adj}(\Phi) \phi_k \geq \lambda_m(\text{adj}(\Phi)) |\phi_k|^2 = |\phi_k|^2 \frac{\det(\Phi)}{\lambda_M(\Phi)}$$

and for all $t \geq 0$

$$\sum_{k=1}^{\ell} \phi_k^\top(t) \text{adj}(\Phi(t)) \phi_k(t) \geq \frac{\Delta(t)}{\lambda_M(\Phi(t))} \sum_{k=1}^{\ell} |\phi_k(t)|^2.$$

Recall that for the induced matrix norm $\|\phi\|$ it holds

$$\sum_{k=1}^{\ell} |\phi_k|^2 \geq \|\phi\|^2,$$

and substituting it in (3.71), we obtain for $\lambda_M(\Phi) > 0$

$$\dot{\Delta}(t) \geq \left(-an + \frac{\|\phi(t)\|^2}{\lambda_M(\Phi(t))} \right) \Delta(t),$$

which completes the proof. □

4 Future Research Direction

My future research activities can be divided into two main lines, where the short/mid-term research is a continuation of my current activities on the DREM procedure, and the mid/long-term research is more oriented towards new challenges of energy management applications.

First, in the short/mid-term, I continue working on the further development of the DREM procedure. For this topic, the envisaged research directions are discussed in the Open Questions section of Chapter 3 (see Section 3.7.3). It includes parameter estimation under insufficient excitation, computationally-efficient formulation of the DREM procedure, and other related topics. These activities benefit from my collaboration with **ITMO University** and **INRIA Lille**. Particularly, the parameter estimation under insufficient excitation is considered in the ongoing research on “Performance Improvement in Adaptive and Learning Systems” by my Ph.D. student Marina KOROTINA; Marina started her studies in December 2020. I believe that pursuing this direction will yield interesting results, further extending the scope of the DREM procedure.

The DREM development research direction naturally follows from the materials discussed in the previous chapter, and I already discussed it briefly in Section 3.7.3. In this chapter, I focus on presenting the mid/long-term research activities. As such, I intend to orient the achieved results on parameter estimation and adaptation towards advanced energy management challenges, including energy-efficient smart buildings.

This chapter is organized as follows. First, I discuss the energy transition context, the related challenges, and my research objectives in Section 4.1. Section 4.2 describes the state of the art and positioning, and I discuss the ongoing activities in Section 4.3. Finally, in Section 4.4, I discuss the expected impacts of the proposed research.

4.1 Context, Challenges, and Research Objectives

Nowadays, vast concerns of sustainability, climate change, and carbon emission motivate multiple research initiatives on the energy management front, including the Systems and Control domain [143]. Due to the increasing penetration level and production share of renewable sources, energy management becomes more sensitive to inevitable environmental variations, such as weather conditions. On the other hand, renewable power plants typically provide a faster response to control input, and they are now providing more and more ancillary services balancing demand and supply in power grids. These systems must be flexible to stay efficient; they must react and adapt to operating conditions changes, such as weather or daily energy consumption patterns.

Another concept in energy management is building energy regulation; globally, buildings produce about 30% of CO₂ emissions and consume approximately 40% of the world’s

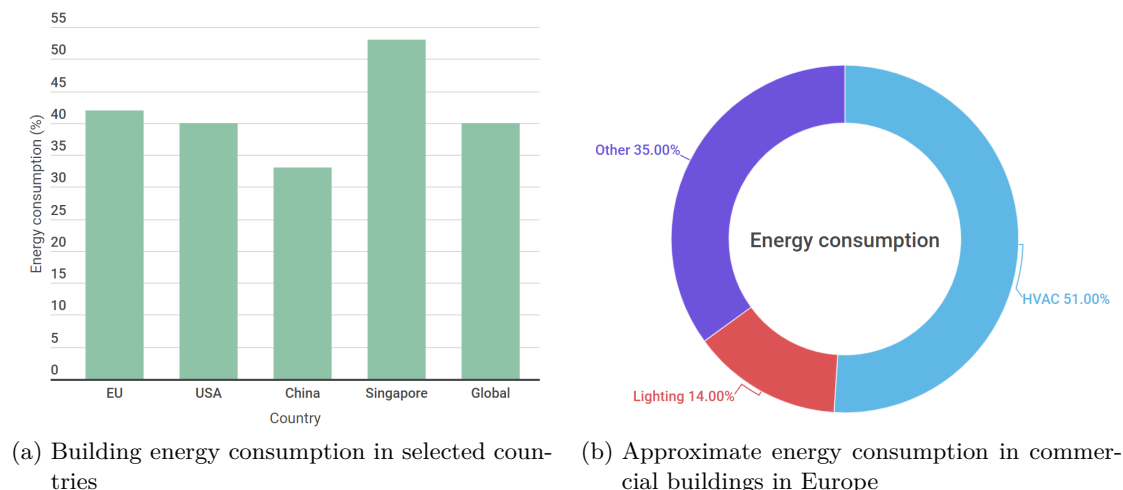


Figure 4.1: Buildings energy consumption statistics [144].

total energy [144] (see Fig. 4.1a). That makes them one of the critical priorities in sustainable energy developments. Approximately half of the buildings' energy consumption is associated with heating, ventilation, and air conditioning (HVAC) (see Fig. 4.1b). However, energy savings should not compromise indoor comfort and air quality, impacting inhabitants' health, working efficiency, and general satisfaction. In various cases, people may decline to work or live in a particular environment; moreover, poor indoor conditions may cause a mix of illnesses called the sick building syndrome.

At the same time, the study [145] shows that climate regulation systems based on prescribed temperature profiles are not as efficient as expected since users do not typically update the schedule when circumstances change; thus, either energy consumption is not optimal, or the indoor comfort level degrades.

Control Theory or, more precisely, adaptive and learning control can address these challenges. However, to obtain the real added value to existing solutions, the adaptation and learning must be fast and accurate, providing at the same time transparent and straightforward tuning rules. As discussed in Chapter 3, these requirements can be addressed by empowering existing learning and adaptive methods with the performance-improving DREM procedure. Thus, it is an exciting challenge to orient the DREM procedure towards providing an efficient solution for advanced control of modern energy management systems.

Specifically, my **research objective** is to contribute to advanced control strategies for energy management by developing appropriate adaptive and learning-based approaches enhancing the overall control performance. To achieve the objective, I aim at applying parameter learning-based solutions in the following scenarios: adaptive model-predictive climate control with the online estimation of model parameters and estimation, learning, and prediction of building occupants' behavior.

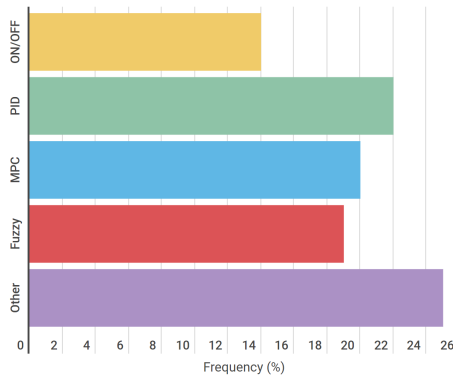


Figure 4.2: Distribution of regulation methods based on the survey [151].

4.2 State of the Art, Positioning, and Ongoing Activities

Energy management of recent buildings poses challenging control problems [146], such as the automated load shaping to reduce the peak consumption or the simultaneous control of all climate variables (indoor temperature and humidity, airflow, or CO₂ concentration). Conventional PID controllers and simple ON-OFF structures cannot cope with these challenges, and advanced control methods are applied.

Model predictive control (MPC) is probably the most popular advanced control method used in buildings energy management; see, e.g., [147–150]. According to the survey [151], the MPC is used in approximately 20% of all applications, whereas for PID this value is 22% and 14% for ON/OFF control (see Fig.4.2). Other regulation methods include heuristic designs [152], feedback linearization [153], robust and quadratic-optimal control of linearized systems, adaptive control for real-time energy balancing [154], optimal feedforward control [155], and other solutions; however, the frequency of use of each of these methods is relatively low compared with MPC.

MPC relies on a model of building dynamics. It naturally handles multiple variables and explicitly manages constraints on input and output signals. However, one of the main obstacles to the practical implementation of predictive control is the need for a sufficiently accurate building model. Model uncertainty (in a broad sense, including perturbations) is generally cited as the most critical issue for the control performance [146]. Mitigating this uncertainty requires adaptation because model parameters always drift with time, e.g., due to changes in occupants' behavior. Therefore, predictive control should be combined with adaptive online estimation of model parameters yielding the indirect adaptive model predictive control.

Several successful results are reported on adaptive MPC, e.g., in [156], an adaptive MPC solution is applied to building climate control, and in [157], another adaptive MPC is used for energy management, where load fluctuations are estimated online using least-squares estimation. In [158], the underlying model parameters of a linear discrete-time system are estimated online and used to update the MPC gains adaptively. In [159], adaptive predictive control is performed for a hybrid model that incorporates the heating

effects due to occupancy, and in [160], a model is approximated by a neural network trained by the recursive least-squares algorithm. However, the adaptive MPC designs used nowadays, like those listed above, typically use standard adaptation techniques like gradient-based or recursive least-squares update laws. Thus, these designs suffer from the drawbacks discussed in Chapter 3 and related to transient performance and excitation conditions. From this perspective, **the DREM procedure is the right tool** to address these challenges and enhance adaptive MPC solutions used in buildings' control systems.

Another family of adaptive/learning methods worth noting in energy management and smart buildings is Reinforcement Learning (RL). In [161], the problem of power allocation in hybrid energy storage was considered, and the RL-based real-time power management strategy was shown to be more efficient than a rule-based solution. An RL-based control was successfully used in building applications in [162], achieving comfort in buildings with minimal energy consumption. Also, a promising hybrid between MPC and RL named "Direct MPC" was recently proposed for energy management [163]; it uses a short-horizon MPC for its natural ability to handle constraints along with a Bellman-like penalization of the final state to bring optimality related to uncertain inputs. In my mid/long-term research, I intend to explore this direction, and specifically to pay attention to the concept of "safe RL," which is based on the Lyapunov function approach and was introduced in the machine learning community [164] to tackle the issue of the respecting constraints during the learning phase.

4.3 Ongoing Activities

Regarding the proposed research directions, I coordinate the project entitled "Adaptation and Learning for Smart Buildings." The project aims to initiate collaborative research activities and networking establishing; *Rennes Metropole* supported the project in 2019. Unfortunately, due to the pandemic, all project-related travel activities were suspended in 2020, and the project will hopefully resume in 2021/2022.

The learning-based temperature regulation in a building is also a part of the *research track*-student Ricardo EHLERS BINZ's project entitled "Data-driven and learning-based control," where I am the only supervisor.

Moreover, I believe that my current position is well-suited for the envisaged research activity. Members of the Automatic Control Team work in the control for energy management domain, including the predictive and optimal frameworks, and the energy transition and sustainable development are in the focus of Rennes Campus of CentaleSupélec. From 2019, the campus participates in the joint project of "Smart and Secure Room." In this project, a room at the campus is equipped with a renewable energy source, namely solar panels, multiple sensors, and corresponding power electronics and interfaces. After finishing the installations, this room will serve as a comprehensive testbed for experiments on advanced control for smart energy.

4.4 Impact and benefits

My future research direction benefits from combining fundamental scientific aspects and clear application domain orientation addressing social challenges. Thus, I firmly believe it will be attractive for Ph.D. students and give rise to outstanding theses. Besides this educational aspect, my research direction also has the following impacts.

Scientific impact. Despite a clear relevance to industry concerns in the sustainable energy domain, my planned research activity is essentially fundamental research considering the problems of adaptive and learning systems. The expected scientific results will contribute to the theoretical basis of the field, making adaptive control more attractive for practical applications.

Economic and social impact. The envisaged research activities will establish an interdisciplinary link between control theory and energy management applications, such as energy production/storage/distribution and energy-efficient buildings.

An excellent opportunity is the SMILE (Smart Ideas to Link Energies) initiative launched in 2016 in the Region Brittany, France. This initiative involves numerous enterprises working in the field of smart energy, and in the mid/long-term perspective, my research activities may trigger academia-industry collaboration.

Regarding the social impact, I address the social challenge of clean, safe, and efficient energy. The development of smart and energy-saving buildings is essential to reduce the households' costs, while the implementation of energy-efficient control strategies will also increase the energy autonomy of small cities and isolated areas.

General public dissemination. The topic of sustainable and efficient energy is timely and attractive, and the envisaged research is a good way to present and advertise control theory and its possible applications to a broad audience easily and convincingly.

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A Notation

- \mathbb{R} is the set of reals, and \mathbb{R}_+ is the set of nonnegative reals;
- \mathbb{N} is the set of positive integers;
- I_n is the $n \times n$ identity matrix, for all $n \in \mathbb{N}$ (when clear from the context, the subscript n can be omitted);
- for $m, n \in \mathbb{N}$, $\overline{m, n} := \{p \in \mathbb{N} \mid m \leq p \leq n\}$ if $m \leq n$ and \emptyset , otherwise;
- for a symmetric matrix P , $\lambda_m(P)$ and $\lambda_M(P)$ are the minimum and the maximum eigenvalues of P , respectively;
- for a square matrix A , we denote the adjugate matrix as $\text{adj}(A)$;
- for a square matrix A , we denote its trace as $\text{tr}(A)$;
- for a vector $x \in \mathbb{R}^n$, $|x|$ is the Euclidean vector norm, and for a positive definite matrix W , we denote the weighted norm as $\|x\|_W := \sqrt{x^\top W x}$;
- for a square matrix A , we denote the induced matrix norm as $\|A\|$, $\|A\| := \sup_{|x|=1} |Ax|$;
- for a signal $x : \mathbb{R}_+ \rightarrow \mathbb{R}^n$, we denote

$$\|x\|_1 := \int_0^\infty |x(s)| ds, \quad \|x\|_2 := \left(\int_0^\infty |x(s)|^2 ds \right)^{\frac{1}{2}}, \quad \text{and} \quad \|x\|_\infty := \text{ess sup}_{t \geq 0} |x(t)|;$$

- the set of all signals $x : \mathbb{R}_+ \rightarrow \mathbb{R}$, such that $\|x\|_p$ is finite, where $p \in \{1, 2, \infty\}$, we denote as \mathcal{L}_p ;
- an operator $H : \mathcal{L}_\infty \rightarrow \mathcal{L}_\infty$ applied to a signal $x : \mathbb{R}_+ \rightarrow \mathbb{R}$, we denote as $H[x(t)]$; if $x : \mathbb{R}_+ \rightarrow \mathbb{R}^n$, then $H[x(t)]$ denotes the element-wise application of the operator.

B List of Publications

Submitted, under review (UR)

- UR1 Aranovskiy S., Ushirobira R., Korotina M., Vedyakov. A, On preserving-excitation properties of a dynamic regressor extension scheme, submitted to **IEEE Transactions on Automatic Control**, 2021. Preprint available at <https://hal-centralesupelec.archives-ouvertes.fr/hal-03245139>.

International Peer-Reviewed Journals (IJ)

- IJ1 Aranovskiy S., Efimov D., Sokolov D., Wang J., Ryadchikov I., Bobtsov A., Switched Observer Design For a Class of Locally Unobservable Time-Varying Systems, **Automatica**, 2021. Accepted, available online.
- IJ2 Wang J., Aranovskiy S., Fridman E., Sokolov D., Efimov D., Bobtsov A., Robust adaptive stabilization by delay with parametric uncertainty in state and output equations, **IEEE Transactions on Automatic Control**, 2021. Accepted, available online.
- IJ3 Ortega R., Aranovskiy S., Pyrkin A., Astolfi A., Bobtsov A., New Results on Parameter Estimation via Dynamic Regressor Extension and Mixing: Continuous and Discrete-time Cases, **IEEE Transactions on Automatic Control**, 66(5), pp. 2265–2272, 2020.
- IJ4 Ortega R., Pyrkin A., Bobtsov A., Efimov D., Aranovskiy S., A Globally Convergent Adaptive Indirect Field-Oriented Torque Controller For Induction Motors, **Asian Journal of Control**, 22(1), pp. 11–24, 2020.
- IJ5 Aranovskiy S., Ryadchikov I., Nikulchev E., Wnag J., Sokolov D., Experimental comparison of velocity observers: a scissored pair control moment gyroscope case study, **IEEE Access**, 8, pp. 21694-21702, 2020.
- IJ6 Aranovskiy, S., Ryadchikov, I., Nikulchev, E., Wnag, J., Sokolov, D., Bias Propagation and Estimation in Homogeneous Differentiators for a Class of Mechanical Systems, **IEEE Access**, 8, pp. 19450-19459, 2020.
- IJ7 Aranovskiy S., Ushirobira R., Efimov D., Casiez G., A switched dynamic model for pointing tasks with a computer mouse, **Asian Journal of Control**, 22, pp. 137-1400, 2020.

- IJ8 Wang J., Efimov D., Aranovskiy S., Bobtsov A., Fixed-time estimation of parameters for non-persistent excitation, **European Journal of Control**, 55, pp. 24-32, 2020.
- IJ9 Pyrkin A., Bobtsov A., Ortega R., Vedyakov A., Aranovskiy S., Adaptive State Observer Design Using Dynamic Regressor Extension and Mixing, **Systems and Control Letters**, 133, pp. 104519, 2019.
- IJ10 Aranovskiy S., Ushirobira R., Efimov D., Casiez G., An adaptive FIR filter for trajectory prediction and latency reduction in direct Human-Computer interactions, **Control Engineering Practice**, 91, pp. 104093, 2019.
- IJ11 Aranovskiy S., Belov A., Ortega R., Barabanov N., Bobtsov A., Parameter Identification of Linear Time-Invariant Systems Using Dynamic Regressor Extension and Mixing, **International Journal of Adaptive Control and Signal Processing**, 33(6), pp. 1016-1030, 2019.
- IJ12 Pan Y., Aranovskiy S., Bobtsov A., Yu H., Efficient learning from adaptive control under sufficient excitation, **International Journal of Robust and Nonlinear Control**, 29(10), pp. 3111-3124, 2019.
- IJ13 Aranovskiy S., A. Biryuk, E. Nikulchev, Ryadchikov I., Sokolov D., Observer Design for an Inverted Pendulum with Biased Position Sensors, **Journal of Computer and Systems Sciences International**, 58(2), pp. 297–304, 2019.
- IJ14 Aranovskiy S., Ortega R., J. G. Romero, Sokolov D., A Globally Exponentially Stable Speed Observer for a Class of Mechanical Systems: Experimental and Simulation Comparison with High-Gain and Sliding Mode Designs, **International Journal of Control**, 92(7), pp. 1620-1633, 2019.
- IJ15 Ortega R., Praly L., Aranovskiy S., Yi B., Zhang W., On Dynamic Regressor Extension and Mixing Parameter Estimators: Two Luenberger Observers Interpretations, **Automatica**, 95(9), pp. 548-551, 2018.
- IJ16 Pyrkin A., Aranovskiy S., Bobtsov A., Kolyubin S., Nikolaev N., Fradkov Theorem-Based Control of MIMO Nonlinear Lurie Systems, **Automation and Remote Control**, 79(6), pp. 1074–1085, 2018.
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- IJ26 Aranovskiy S., Bobtsov A., Pyrkin A., Gritcenko P., Adaptive filters cascade applied to a frequency identification improvement problem, **International Journal of Adaptive Control and Signal Processing**, 30(5), pp. 677-689, 2016.
- IJ27 Aranovskiy S., Ortega R., Cisneros R., A robust PI passivity-based control of nonlinear systems and its application to temperature regulation, **International Journal of Robust and Nonlinear Control**, 26(10), pp. 2216-2231, 2016.
- IJ28 Ortega R., Bobtsov A., Pyrkin A., Aranovskiy S., A parameter estimation approach to state observation of nonlinear systems, **Systems and Control Letters**, 85, pp. 84-94, 2015.
- IJ29 Aranovskiy S., Freidovich L., Adaptive Compensation of Disturbances Formed as Sums of Sinusoidal Signals with Application to an Active Vibration Control Benchmark, **European Journal of Control**, 19(4), pp. 253-265, 2013.
- IJ30 Ruderman M., Aranovskiy S., Bobtsov A., Bertram T., Nonlinear dynamics of drives with elasticities and friction, **Automation and Remote Control**, 73(10), pp. 1604-1615, 2012.
- IJ31 Aranovskiy S., Bobtsov A., Kremlev A., Nikolaev N., Slita O., Identification of frequency of biased harmonic signal, **European Journal of Control**, 16(2), pp. 129-139, 2010.

- IJ32 Aranovskiy S., Bobtsov A., Pyrkin A., Adaptive observer of an unknown sinusoidal output disturbance for linear plants, **Automation and Remote Control**, 70(11), pp. 1862-1870, 2009.
- IJ33 Aranovskiy S., Bobtsov A., Kremlev A., Luk'anova G., Nikolaev N., Identification of frequency of a shifted sinusoidal signal, **Automation and Remote Control**, 69(9), pp. 1447-1453, 2008.
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- IJ35 Aranovskiy S., Bobtsov A., Kremlev A., Compensation of a finite-dimensional quasi-harmonic disturbance for a nonlinear object, **Journal of Computer and Systems Sciences International**, 45(4), pp. 518-525, 2006.

Patent (P)

- P1 Casiez, G., Efimov, D., Ushirobira, R., Aranovskiy, S., and Roussel, N. (2016). Predictive display device. Patent No. WO2018055280A1, filing date (FR3056317B1): 09/20/2016.

International Peer-Reviewed Conferences (IC)

- IC1 Aranovskiy S., Efimov D., Sokolov D., Wang J., Ryadchikov I., Bobtsov A., State estimation for a locally unobservable parameter-varying system: one gradient-based and one switched solutions, **IFAC World Congress**, 2020.
- IC2 Efimov D., Aranovskiy S., Fridman E., Sokolov D., Wang J., Bobtsov A., Adaptive stabilization by delay with biased measurements, **IFAC World Congress**, 2020.
- IC3 Bobtsov A., Pyrkin A., Aranovskiy S., Nikolaev N., Slita O., Kozachek O., Stator flux and load torque observers for PMSM, **IFAC World Congress**, 2020.
- IC4 Efimov D., Aranovskiy S., Bobtsov A., Raissi T., On fixed-time parameter estimation under interval excitation, **European Control Conference**, 2020.
- IC5 Korotina M., Aranovskiy S., Ushirobira R., Vedyakov A., On parameter tuning and convergence properties of the DREM procedure, **European Control Conference**, 2020.
- IC6 Sokolov D., Aranovskiy S., Gusev A., Ryadchikov I., Experimental comparison of velocity estimators for a control moment gyroscope inverted pendulum, **IEEE Workshop on Advanced Motion Control**, 2020.

- IC7 Bobtsov A., Pyrkin A., Aranovskiy S., Nikolaev N., Slita O., Kozachek O., Stator Flux Finite-time Observer for Non-Salient Permanent Magnet Synchronous Motors, **Mediterranean Conference on Control and Automation**, 2020.
- IC8 Le Van Tuan, Korotina M., Bobtsov A., Aranovskiy S., Pyrkin A., Online Estimation of time-varying frequency of a sinusoidal signal, **IFAC Workshop on Adaptation and Learning in Control and Signal Processing**, 2019.
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